REINFORCED CONCRETE DESIGN HANDBOOK

BY
PROF.DR.SHAKER EL BEHAIRY
FACULTY OF ENGINEERING
AIN SHAMS UNIVERSITY
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CHAPTER VIII YIELD LINE ANALYSIS

PART (II)

Applications to The New Egyptian Code 2001 for Design and Construction of R.C.structures

PART (III)

DESIGN AIDS

TABLES AND CHARTS FOR DESIGN OF SECTIONS (U.S.D)

CHAPTER I

GEOMETRY

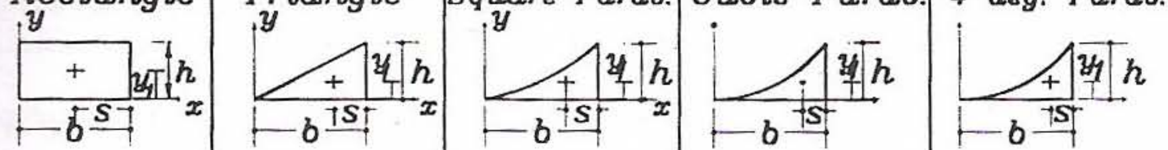
I.1. AREAS AND CIRCUMFERENCE & WEIGHTS OF REINFORCING STEEL BARS

a. Steel Bars Used in U.A.R.

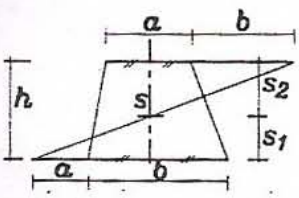
ϕ mm	Weight Kg/m	CIRCUM (cm)	Area of Cross-Section in cm^2									
			1	2	3	4	5	6	7	8	9	10
6	.222	1.88	.283	.566	.848	1.13	1.41	1.70	1.98	2.26	2.54	2.83
8	.395	2.51	.503	1.01	1.51	2.01	2.51	3.02	3.52	4.02	4.52	5.03
10	.617	3.14	.785	1.57	2.36	3.14	3.93	4.71	5.50	6.28	7.07	7.85
13	1.04	4.08	1.33	2.66	3.98	5.31	6.64	7.96	9.29	10.6	11.9	13.3
16	1.58	5.03	2.01	4.02	6.03	8.04	10.1	12.1	14.1	16.1	18.1	20.1
19	2.23	5.97	2.83	5.67	8.50	11.3	14.2	17.0	19.9	22.7	25.5	28.4
22	2.98	6.91	3.80	7.60	11.4	15.2	19.0	22.8	26.6	30.4	34.2	38.0
25	3.85	7.85	4.91	9.82	14.7	19.6	24.5	29.5	34.4	39.3	44.2	49.1
28	4.83	8.80	6.16	12.3	18.5	24.6	30.8	37.0	43.1	49.3	55.4	61.6
32	6.31	10.05	8.04	16.1	24.1	32.2	40.2	48.3	56.3	64.3	72.4	80.4
38	8.90	11.94	11.3	22.6	33.9	45.2	56.5	67.8	79.1	90.4	102	113

b. Steel Bars Used in Other Countries.

ϕ mm	Weight Kg/m	CIRCUM (cm)	Area of Cross-Section in cm^2									
			1	2	3	4	5	6	7	8	9	10
5	.154	1.57	.196	.393	.589	.785	.982	1.18	1.37	1.57	1.77	1.96
7	.302	2.20	.385	.770	1.15	1.54	1.93	2.31	2.69	3.08	3.46	3.85
12	.888	3.77	1.13	2.26	3.39	4.52	5.65	6.79	7.92	9.05	10.2	11.3
14	1.21	4.40	1.54	3.08	4.62	6.16	7.70	9.24	10.8	12.3	13.9	15.4
18	2.00	5.65	2.54	5.09	7.63	10.2	12.7	15.3	17.8	20.4	22.9	25.4
20	2.47	6.28	3.14	6.28	9.42	12.6	15.7	18.8	22.0	25.1	28.3	31.4
24	3.55	7.54	4.52	9.04	13.6	18.1	22.6	27.1	31.7	36.2	40.7	45.2
26	4.17	8.17	5.31	10.6	15.9	21.2	26.5	31.9	37.2	42.5	47.0	53.1
30	5.55	9.42	7.07	14.1	21.2	28.3	35.3	42.4	49.5	56.6	63.6	70.7
34	7.13	10.68	9.08	18.2	27.2	36.3	45.4	54.5	53.6	72.6	81.7	90.8
36	7.99	11.31	10.2	20.4	30.6	40.8	50.7	61.2	71.4	81.6	91.8	102

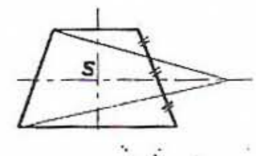


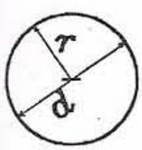
	Rectangle	Triangle	Square Parab.	Cubic Parab.	4th deg. Parab.
A	1.b.h	1/2.b.h	1/3.b.h	1/4.b.h	1/5.b.h
s	1/2 b	1/3 b	1/4 b	1/5 b	1/6 b
y_1	1/2 h	2/6 h	3/10 h	4/14 h	5/18 h
Eqn.	$y = h$	$y = \frac{h}{b} x$	$y = \frac{h}{b^2} x^2$	$y = \frac{h}{b^3} x^3$	$y = \frac{h}{b^4} x^4$



Trapezium $A = \frac{a + b}{2} . h$

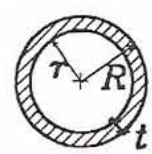
$S_1 = \frac{h}{3} \frac{(2a+b)}{(a+b)}$, $S_2 = \frac{h}{3} \frac{(2b+a)}{(a+b)}$



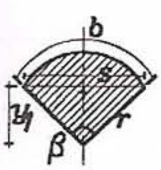


$A = \pi \ r^2 = \pi \ \frac{d^2}{4}$

Circum. $= 2 \pi r$
$= \pi \ d$



$A = \pi \ (R^2 - r^2)$
$= \pi \ (D^2 - d^2)/4$
$= \pi \ (R + r) \ t$



$A = br/2 = \pi r^2 \beta/360$

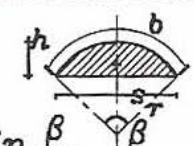
$b = \pi r \beta \ /180$

$y_1 = \frac{2}{3} \frac{r \ s}{b}$

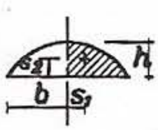
$A = \frac{br}{2} - \frac{s.(r-h)}{2}$

$s = 2\sqrt{h(2r-h)} = 2r.\sin\frac{\beta}{2}$

$A \simeq \frac{2}{3} s.h$ (for small β)



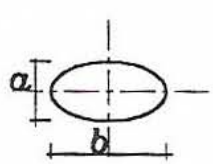
Square parab.



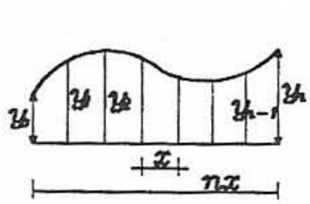
$A = 2/3 \ b.h$

$S_1 = 3/8 \ b$, $S_2 = \frac{2}{5} h$

Ellipse



$A = \pi \ a \ b \ /4$



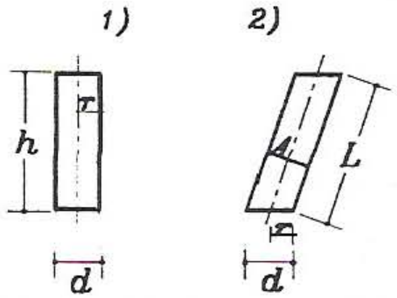
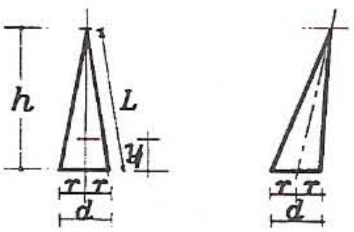
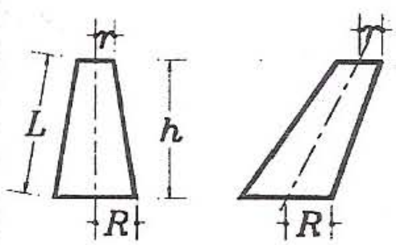
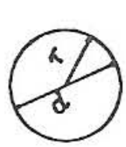
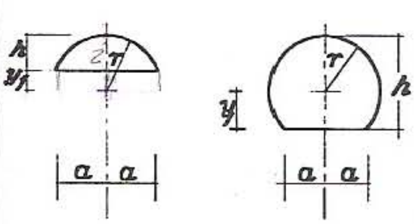
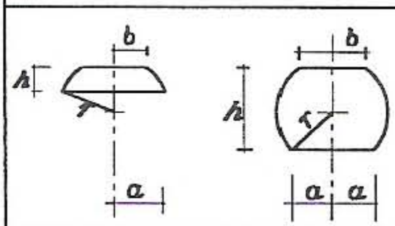
1. **Simpson's Rule** : (n-even)

$A = \frac{x}{3} [y_o + 2(y_2 + y_4 + \ldots) + 4(y_1 + y_3 + \ldots) + y_n]$

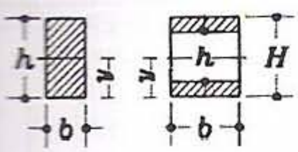
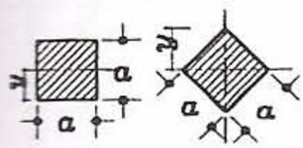
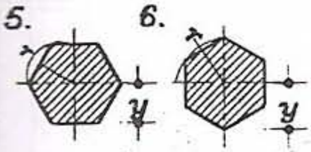
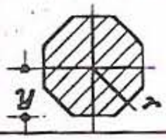
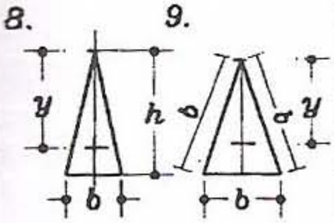
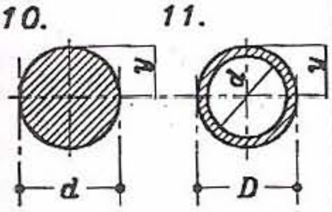
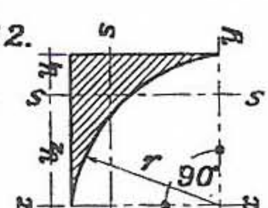
2. **Approximate Rule** : (Trapezium)

$A = \frac{x}{2} [y_o + 2(y_1 + y_2 + y_3 + \ldots) + y_n]$

I.4. VOLUME AND SURFACE AREA OF DIFFERENT BODIES

Body	V = Volume , S = Surface area of body.
	Cylinder :1) $V = \pi r^2 h$, $S = 2\pi r h$ 2) $V = A.l = \pi r^2 . \frac{h}{l} . l = \pi r^2 h$ S = Length of circumference at A * Length l
	Cone : $V = \frac{1}{3} A.h = \pi r^2 (h/3)$ $S = \pi r.l = \pi r \sqrt{r^2 + h^2}$ $Y_1 = h/4$
	Part of Cone : $V = \frac{\pi h}{3} (R^2 + r^2 + R.r)$ $S = \pi .L. (R+r)$ $= \pi (R+r) \sqrt{(R-r)^2 + h^2}$
	Sphere : $V = (\frac{4}{3}) \pi r^3 = \pi^3 d /6$ $S = \pi d^2 = 4\pi .r^2$
	Part of Sphere: $V = \pi h^2 (r - \frac{h}{3}) = \frac{\pi h}{6} (3a^2 + h^2)$ $S = 2\pi r h = \pi (a^2 + h^2)$ $Y_1 = (3/4) \frac{(2r-h)^2}{3r-h}$, $a = \sqrt{h(2r-h)}$
	Part of Sphere : $V = \frac{\pi h}{6} (3a^2 + 3b^2 + h^2)$ $S = 2\pi r.h$

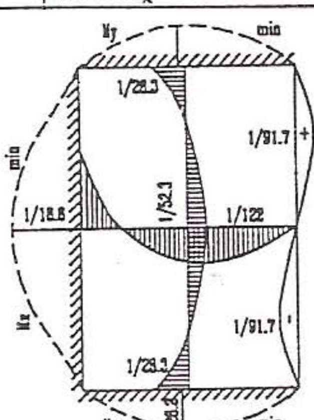
I.7. CENTER OF GRAVITY, SECTION MODULUS & MOMENT OF INERTIA FOR DIFFERENT SECTIONS.

Section	C.G. (Y)	I	Z = I/Y
1. 2. 	1) $\frac{h}{2}$ 2) $\frac{H}{2}$	1) $\frac{bh^3}{12}$ 2) $\frac{b}{12}(H^3-h^3)$	1) $\frac{bh^2}{6}$ 2) $\frac{b}{6H}(H^3-h^3)$
3. 4. 	3) $\frac{a}{2}$ 4) $\frac{a}{2}\sqrt{2}$	3) $\frac{a^4}{12}$ 4) $\frac{a^4}{12}$	3) $\frac{a^3}{6}$ 4) $0.1179\ a^3$
5. 6. 	5) $0.866\ r$ 6) r	5) $0.5413\ r^4$ 6) $0.5413\ r^4$	5) $\frac{5}{8}r^3 = 0.625\ r^3$ 6) $= 0.5413\ r^3$
7. 	7) $0.924\ r$	7) $0.6381\ r^4$	7) $0.6906\ r^3$
8. 9. 	8) $\frac{2}{3}h$ 9) $0.5778\ b$	8) $I_x = \frac{bh^3}{36}$ $I_y = \frac{hb^3}{48}$ 9) $\approx 0.018\ b^4$	8) $Z_x = \frac{bh^2}{24}$ $Z_y = \frac{hb^2}{24}$ 9) $\approx 0.0313^3\, b$
10. 11. 	10) $\frac{d}{2}$ 11) $\frac{D}{2}$	10) $\frac{\pi d^4}{64} \approx 0.05 d^4$ 11) $\frac{\pi}{64}.(D^4-d^4)$	10) $\frac{\pi d^3}{32} \approx 0.1 d^3$ 11) $\frac{\pi}{32} \cdot \frac{D^4-d^4}{D}$
12. 	$Y_1 = 0.2234r$ $Y_2 = 0.7766r$	$I_s = 0.0075r^4$ $I_x = I_y \approx 0.137r^4$ $I_{xy} = \frac{r^4}{8}$ $I_{ss} = 0.0044r^4$	$Z_{s2} = \frac{0.0075r^4}{y_2}$ $= 0.00966r^3$ $\approx 0.01\ r^3$

B) SLABS SUPPORTED ON FOUR SIDES AND SUBJECTED TO TRIANGULAR LOADS

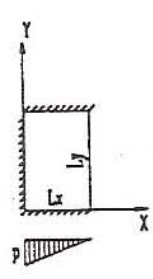
B-5) SLAB SIMPLY SUPPORTED ON A LONGER SIDE AND FIXED ON THE OTHER THREE SIDES

Position	L_y / L_x	1.0	1.1	1.2	1.3	1.4	1.5	2.0
$y=0.5L_y$	M_x min $= -PL^2_x$:	29.0	25.5	22.9	21.1	19.6	18.8	16.1
$y=0$ & $y=L$	M_y min $= -PL^2_x$: at x/L_x =	34.5 0.45	32.1 0.47	30.3 0.48	29.2 0.49	28.5 0.49	28.2 0.49	27.3 0.49
$y=0.5L_y$	M_x max $= PL^2_x$: at x/L_x =	104 0.38	86.9 0.42	75.2 0.45	64.9 0.47	57.8 0.50	52.3 0.50	39.4 0.52
$y=0.5L_y$	M_x m $= PL^2_x$:	118	91.7	76.3	65.3	57.8	52.3	39.7
$x=0.5L_x$	M_y max $= PL^2_x$:	95.2 0.5	95.2 0.5	98.0 0.5	103 0.5	112 0.5	122 0.5	182 0.3 & 0.7
$x=0.5L_x$	M_y m $= PL^2_x$:	95.2	95.2	98.0	103	112	1122	233
$x=L_x$	M_{xy} max $= \pm PL^2_x$: at y/L_y =	149 0.18 & 0.82	128 0.18 & 0.82	114 0.18 & 0.82	103 0.18 & 0.82	97.1 0.18 & 0.82	91.7 0.17 & 0.83	85.5 0.14 & 0.86
$x=0$	$\overline{Q}_x$ m, $\overline{R}_x$ m } $= Pl_x$:	3.1	2.9	2.8	2.7	2.7	2.6	2.5
$x=L_x$	Q_x max $= -PL_x$: R_x max $= PL_x$:	21.3 12.7	18.1 11.2	15.8 10.3	14.2 9.7	13.1 9.3	12.3 9.1	10.4 8.9
$y=0$ & $y=0.5L_y$	$\pm\overline{Q}_y$ max, $\overline{R}_y$ max } $= Pl_x$: at x/L_x =	4.1 0.3	4.1 0.4	3.9 0.5	3.8 0.5	3.8 0.5	3.8 0.5	3.7 0.5

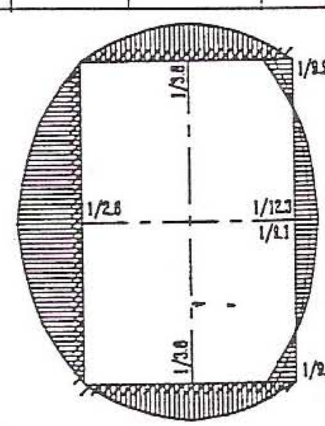


FACTOR pLx^2

MOMENTS



Ly/Lx = 1.50

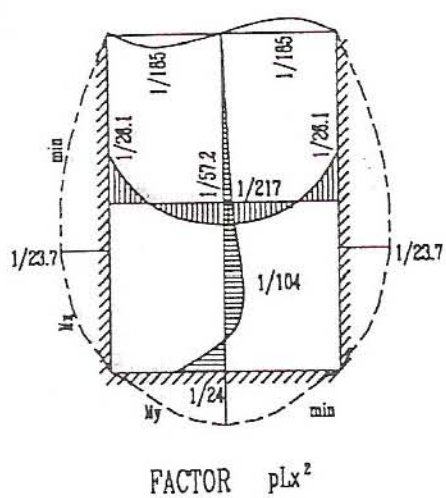


FACTOR pLx

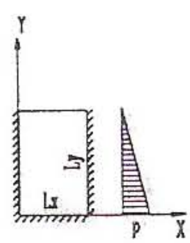
REACTIONS & S.F.

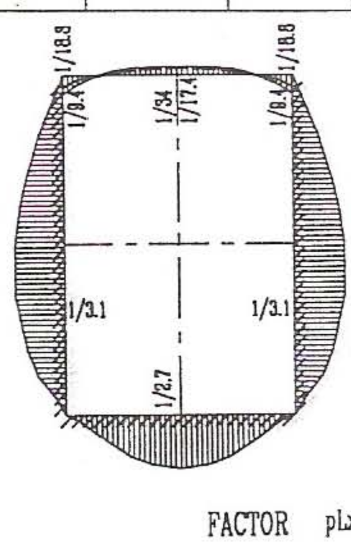
B-5`) SLAB SIMPLY SUPPORTED ON A SHORTER SIDE AND FIXED ON THE OTHER THREE SIDES

Position	L_y / L_x	1.0	1.1	1.2	1.3	1.4	1.5	2.0
x=0 & x=L_x	M_x min $= -PL_x^2$: at x/L_x =	34.5 0.45	31.1 0.44	28.7 0.43	26.7 0.42	25.3 0.40	23.7 0.38	20.2 0.31
x=0.5L_x	M_y min $= -PL_x^2$:	29.0	26.9	25.8	24.9	24.1	23.8	21.9
y=0.5L_y	M_x max $\cong M_x m = PL_x^2$:	95.2	81.3	71.4	64.9	60.6	57.2	40.5
x=0.5L_x	M_y max $= PL_x^2$: at y/L_y =	104 0.38	104 0.35	104 0.34	104 0.33	104 0.32	104 0.27	104 0.22
x=0.5L_x	M_y m $= PL_x^2$:	118	128	143	161	185	217	588
y=L_y	M_{xy} max $= \pm PL_x^2$: at x/L_x =	149 0.18 & 0.82	154 0.2 & 0.8	159 0.2 & 0.8	167 0.2 & 0.8	175 0.2 & 0.8	185 0.2 & 0.8	244 0.2 & 0.8
x=0 & x=L_x	$\overline{\pm Q_x}$ max, R_x max } $= Pl_x$: at y/L_y =0.3	4.1	3.8	3.6	3.4	3.2	3.1	2.8
y=0	$\overline{Q_y}$ m, $\overline{R_y}$ m } $= Pl_x$:	3.1	3.0	2.9	2.8	2.8	2.7	2.6
y=L_y	Q_y m $= -PL_x$: R_y m $= PL_x$:	21.3 12.7	23.4 13.3	25.8 14.1	28.5 15.1	31.3 16.3	34.0 17.4	47.4 23.8
x=0.5L_x y=0.5L_y	$\delta_m = \dfrac{PL_x^4}{Et^3}$.	.009	.010	.011	.012	.013	.014	.015



MOMENTS





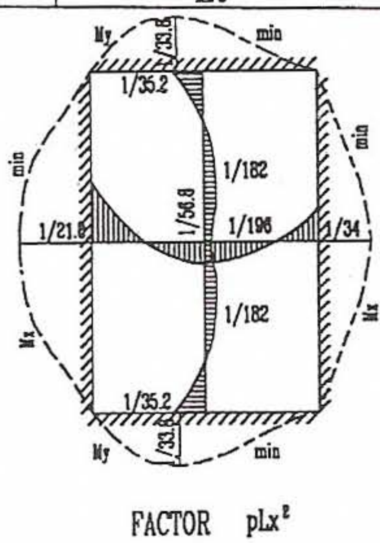
REACTIONS & S.F.

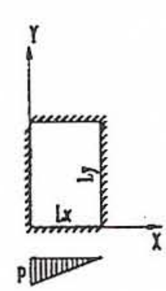
Ly/Lx = 1.50

B) SLABS SUPPORTED ON FOUR SIDES AND SUBJECTED TO TRIANGULAR LOADS

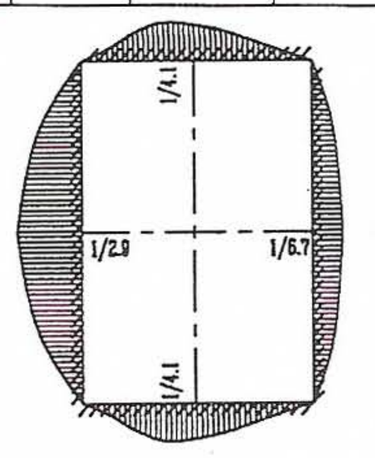
B-6) SLAB FIXED ON ALL FOUR SIDES.

Position	L_y / L_x	1.0	1.1	1.2	1.3	1.4	1.5	2.0
x=0 y=0.5L_y x=L_x	M_x min $= -PL^2_x$: M_x min $= -PL^2_x$:	30.0 56.2	26.7 47.2	24.7 41.7	23.3 38.0	22.2 35.5	21.8 34.0	20.2 30.4
y=0 y=L_y	M_y min $= -PL^2_x$: at x/L_x = 0.4	36.9	36.0	35.1	34.6	34.4	33.8	33.8
y=0.5L_y	M_x max $= PL^2_x$: at x/L_x =	98.0 0.34	82.6 0.38	73.0 0.40	65.8 0.41	60.6 0.42	56.8 0.44	48.5 0.45
x=0.5L_x	M_y max $= PL^2_x$: at y/L_y =	114 0.5	120 0.5	132 0.5	147 0.5	167 0.40 & 0.60	185 0.34 & 0.66	213 0.23 & 0.77
x=0.5L_x	M_y m $= PL^2_x$:	114	120	132	147	170	196	526
y=0.5L_y	$-\bar{Q}_x$ m, $\bar{R}_x$ m } $= Pl_x$: at x =0	3.1	2.9	2.9	2.9	2.9	2.9	2.9
y=0.5L_y	$-\bar{Q}_x$ m, $\bar{R}_x$ m } $= Pl_x$: at x =Lx	8.3	7.4	6.7	6.7	6.7	6.7	6.7
y=0 & y=L_y	$\pm\bar{Q}_y$ max, $\bar{R}_y$ max } $= Pl_x$: at x/L_x =	4.1 0.3	4.1 0.3	4.1 0.4	4.1 0.4	4.1 0.4	4.1 0.4	4.1 0.4
x=0.5L_x y=0.5L_y	$\delta_m = \frac{PL_x^4}{Et^3}$.	.009	.010	.011	.012	.013	.014	.015





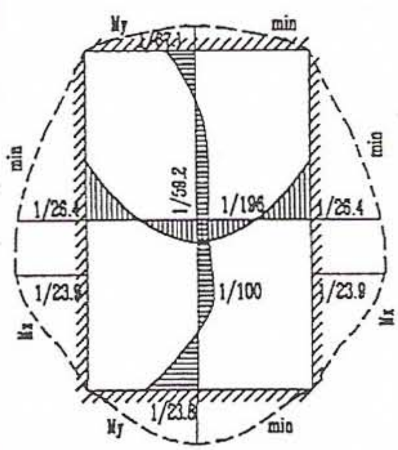
Ly/Lx = 1.50



B) SLABS SUPPORTED ON FOUR SIDES AND SUBJECTED TO TRIANGULAR LOADS

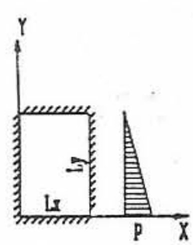
B-6) SLAB FIXED ON ALL FOUR SIDES.

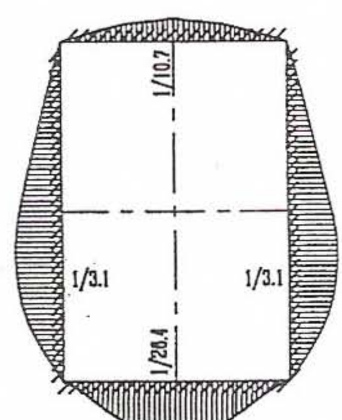
Position	L_y / L_x	1.0	1.1	1.2	1.3	1.4	1.5	2.0
x=0 & x=L_x	M_x min $= -PL^2_x$:	36.9	33.1	30.8	27.5	25.6	23.9	20.2
	at y/L_y =	0.42	0.40	0.40	0.40	0.38	0.37	0.33
y=0 x=0.5L_x	M_y min $= -PL^2_x$:	30.0	27.5	26.1	25.0	24.1	23.8	21.9
y=L_y	M_y min $= -PL^2_x$:	56.2	55.5	57.1	59.5	62.9	67.1	92.6
y=0.5L_y	M_x max $\cong M_x m = PL^2_x$:	114	91.7	78.7	69.9	63.7	59.2	50.0
x=0.5L_x	M_y max $= PL^2_x$:	98.0	98.0	98.0	99.0	99.0	100	100
	at y/L_y =	0.37	0.35	0.33	0.33	0.33	0.32	0.24
x=0.5L_x	M_y m $= PL^2_x$:	114	121	132	147	170	196	526
x=0 & x=L_x	$\pm\overline{Q}_x$ max $= \overline{R}_x$ max } $= Pl_x$: at y/L_y = 0.3	4.1	3.9	3.6	3.4	3.3	3.1	2.8
y=0 x=0.5L_x	$\overline{Q}_y = \overline{R}_y = PL_x$:	3.1	2.9	2.8	2.8	2.7	2.6	2.5
y=L_y	$\overline{Q}_y = \overline{R}_y = PL_x$:	8.3	8.4	8.8	9.4	10.0	10.7	14.3
x=0.5L_x y=0.5L_y	$\delta_m = \dfrac{PL_x^4}{Et^3}$.	.008	.009	.010	.012	.012	.013	.015



FACTOR pLx^2

MOMENTS





FACTOR pLx

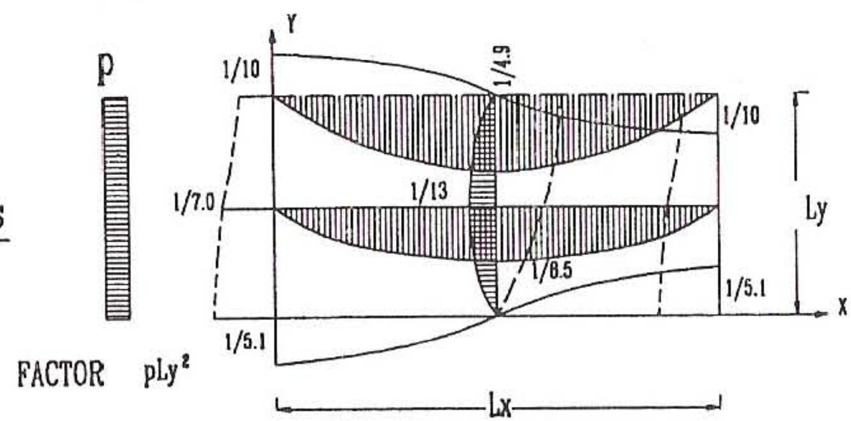
REACTIONS & S.F.

Ly/Lx = 1.50

C-1) SLAB SIMPLY SUPPORTED ON THREE SIDES AND SUBJECT TO UNIFORM LOAD

Position	$L_y / L_x =$:	0.25	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
$x = 0.5L_x$	$M_y m = PL^2_y$:	8.4	8.9	10.5	13.0	16.5	21.3	27.6	35.7	46.2	59.4	75.9	96.5	12.2	15.4
$x = 0.5L_x$	$M_y max = PL^2_y$:	8.4	8.9	10.5	13.0	16.4	20.9	26.6	33.5	41.7	50.4	60.0	70.4	81.7	93.8
	at $y/L_y =$	0.5	0.5	0.5	0.48	0.45	0.42	0.4	0.38	0.33	0.28	0.26	0.25	0.24	0.22
$y = 0.5L_y$	$M_x m = PL^2_y$:	7.9	7.9	8.0	8.5	9.1	10.0	11.0	12.3	13.7	15.2	17.0	18.8	20.9	23.0
$y = L_y$	$M_x max = M_x m = PL^2_y$:	4.0	4.1	4.4	4.9	5.5	6.3	7.3	8.5	9.8	11.3	12.9	14.7	16.7	18.9
$y = 0$	$M_x corner = \pm PL^2_y$:	2.2	2.7	3.8	5.1	6.6	8.3	10.3	12.7	15.3	18.3	21.6	25.2	29.1	33.3
$y = 0.5L_y$	$M_{xy} m = \pm PL^2_y$:	2.6	3.2	4.9	7.0	9.8	13.4	18.1	24.1	31.8	41.5	53.8	69.2	88.4	112
$y = L_y$	$M_{xy} corner = \pm PL^2_y$:	2.9	3.8	6.3	10.1	15.8	24.5	37.6	57.2	86.5	129	193	286	422	618
$x = 0.5L_x$, $y = L_y$	$\delta_m = pL^4_x / E t^3$	.022	.030	.048	.067	.085	.100	.113	.120	.126	.132	.137	.148	.153	.164

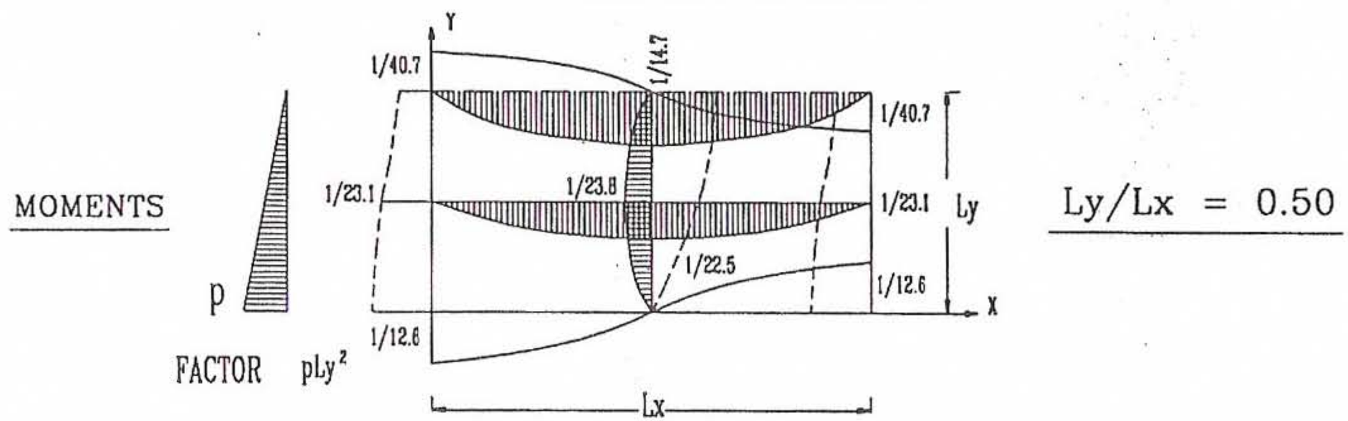
MOMENTS



Ly/Lx = 0.50

C-2) SLAB SIMPLY SUPPORTED ON THREE SIDES AND SUBJECT TO TRIANGULAR LOAD

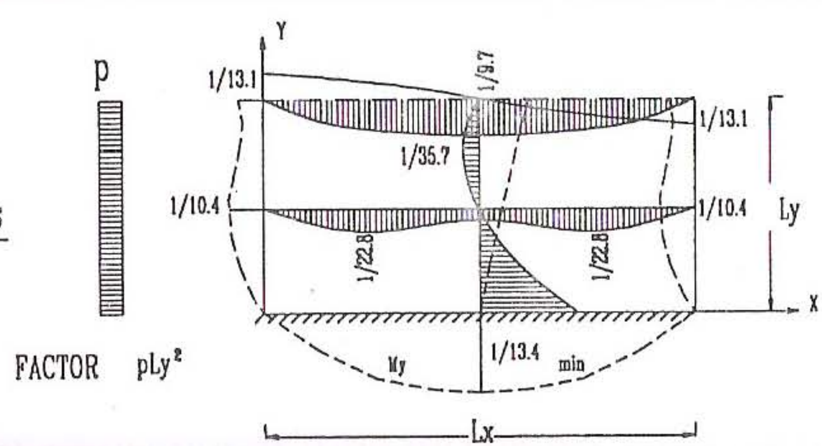
Position	$L_y / L_x =$:	0.25	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
$x = 0.5L_x$	$M_{ym} = PL^2_y$:	16.7	17.5	20.2	24.0	29.5	36.6	45.8	57.4	71.9	89.7	112	138	171	210
$x = 0.5L_x$	$M_{y}max = PL^2_y$: at y/L_y =	16.2 0.4	16.9 0.4	19.1 0.4	22.5 0.4	26.7 0.35	31.6 0.32	37.6 0.3	43.8 0.28	51.3 0.25	60.5 0.23	70.2 0.22	79.8 0.2	89.9 0.19	101 0.18
$x = 0.5L_x$	$M_{y}min = -PL^2_y$: at y/L_y =										4566 0.9	1642 0.9	1175 0.9	1016 0.9	960 0.9
$y = 0.5 L_y$	$M_{xm} = PL^2_y$:	23.5	23.2	23.2	23.8	25.0	26.8	28.9	31.4	34.3	37.4	40.9	44.7	48.6	52.9
$y = L_y$	$M_{xm} = PL^2_y$:	12.1	12.3	13.2	14.7	16.8	19.6	23.0	27.3	32.4	38.5	45.6	54.0	63.6	74.7
$x = 0.5L_x$	$M_{x}max = PL^2_y$: at y/L_y =	12.1 1.0	12.3 1.0	13.2 1.0	14.7 1.0	16.8 1.0	19.6 1.0	23.0 1.0	27.3 1.0	31.7 0.8	35.9 0.65	40.1 0.6	44.5 0.55	48.6 0.5	52.9 0.5
$y = 0$ $y = 0.5L_y$ $y = L_y$	M_{xy} corner $= \pm PL^2_y$: M_{xy} m $= PL^2_y$: M_{xy} corner $= PL^2_y$:	6.4 ±7.9 ±9.4	7.5 ±10.1 ±12.8	10.0 ±15.6 ±22.7	12.6 ±23.1 ±40.7	15.5 ±33.8 ±77.3	18.7 ±49.4 ±168	22.2 ±72.7 ±559	26.0 ±109 ±1678	30.1 ±171 ±527	34.4 ±285 ±394	39.1 ±544 ±360	44.0 ±1558 ±359	49.0 84483 ±374	54.5 ±1274 ±402
$x = 0.5L_x$ $y = L_y$	$\delta_m = pL^4_x / Et^3$:	.007	.010	.016	.021	.027	.031	.034	.037	.038	.039	.039	.039	.038	.038



C-3) SLAB SIMPLY SUPPORTED ON TWO SIDES, FIXED ON ONE SIDE AND SUBJECT TO UNIFORM LOAD

Position	$L_y / L_x =$:	0.25	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
$x = 0.5L_x$	$M_y min = -PL^2_y$:	2.3	2.4	2.9	3.4	4.1	4.9	6.0	7.1	8.5	10.0	11.8	13.7	15.8	18.1
	$M_y m = PL^2_y$:	-17.2	-17.2	-43.6	∞	82.0	57.0	52.2	54.0	59.2	67.5	78.7	92.9	110	132
	$M_y max = PL^2_y$:	170	90	46.8	35.7	33.3	35.4	39.7	45.4	54.5	66.4	79.7	93.7	110	127
	at y/L_y =	0.88	0.85	0.79	0.75	0.71	0.68	0.65	0.62	0.6	0.57	0.53	0.5	0.46	0.43
$y = L_y$	$M_x m = PL^2_y$:	26.4	18.0	11.8	9.7	9.0	9.1	9.6	10.4	11.4	12.6	14.0	15.7	17.6	19.6
	$M_x max = PL^2_y$:	14.3	13.3	11.4	9.7	9.0	9.1	9.6	10.4	11.4	12.6	14.0	15.7	17.6	19.6
	at x/L_x =	0.15	0.2	0.3	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
$y = 0.5L_y$	$M_x m = PL^2_y$:	73.2	52.3	31.7	24.0	20.6	18.9	18.6	18.9	19.7	20.7	22.0	23.5	26.2	27.2
	$M_x max = PL^2_y$:	27.5	27.3	25.6	22.8	20.6	18.9	18.6	18.9	19.7	20.7	22.0	23.5	26.2	27.2
	at x/L_x =	0.15	0.15	0.2	0.3	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
$y = L_y$	M_{xy} corner $= \pm PL^2_y$:	8.7	9.0	10.5	13.1	17.1	23.2	32.2	45.4	64.5	92.3	133	191	274	394
$x = 0.5L_x$, $y = L_y$	$\delta_m = pL^4_x / Et^3$	.005	.009	.020	.035	.051	.070	.087	.098	.109	.120	.131	.142	.153	.164

MOMENTS

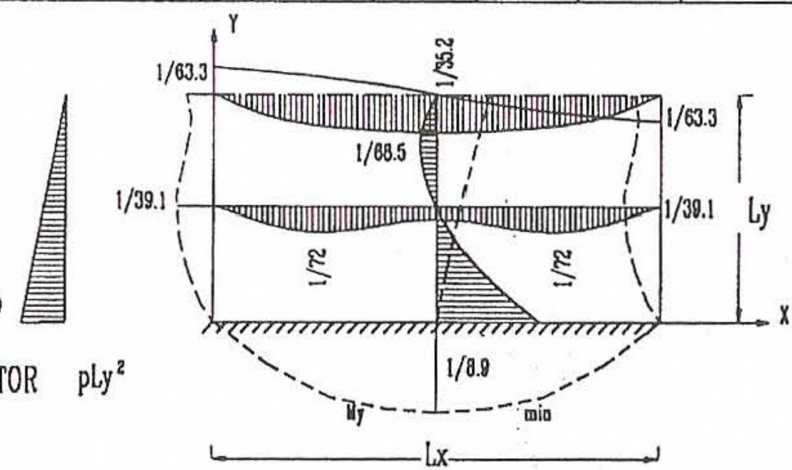


Ly/Lx = 0.50

C-4) SLAB SIMPLY SUPPORTED ON TWO SIDES, FIXED ON ONE SIDE AND SUBJECT TO TRIANGULAR LOAD

Position	L_y / L_x	0.25	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
$x = 0.5L_x$	$M_y min = -PL_y^2$:	6.6	6.9	7.8	8.9	10.3	11.7	13.5	15.4	17.5	19.8	22.2	24.8	27.5	30.5
	$M_y m = PL_y^2$:	-114	-299	175	87.8	71.4	68.3	70.9	77.3	86.7	99.2	115	134	158	186
	$M_y max = PL_y^2$:	220	135	84.0	68.5	64.2	65.1	68.6	76.5	85.5	95.7	107	119	132	145
	at y/L_y =	0.75	0.75	0.69	0.62	0.58	0.53	0.49	0.46	0.45	0.43	0.4	0.38	0.36	0.35
$y = L_y$	$M_x m = PL_y^2$:	94.7	66.0	42.8	35.2	32.7	32.9	34.7	37.9	42.6	47.6	54.2	620	71.1	81.9
	$M_x max = PL_y^2$:	49.5	48.4	41.2	35.2	32.7	32.9	34.7	37.9	42.6	47.6	54.2	620	71.1	81.9
	at x/L_x =	0.15	0.2	0.3	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
$y = 0.5L_y$	$M_x m = PL_y^2$:	252	184	110	80.5	66.0	59.2	56.0	54.9	55.0	56.3	58.2	60.7	63.6	67.0
	$M_x max = PL_y^2$:	650	75.0	76.8	72.0	64.8	59.2	56.0	54.9	55.0	56.3	58.1	60.6	63.6	66.9
	at x/L_x =	0.1	0.15	0.15	0.2	0.3	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
$y = L_y$	M_{xy} corner $= \pm PL_y^2$:	37.0	39.1	47.3	63.3	90.1	143	258	618	15152	1122	678	560	524	521
$x = 0.5L_x$, $y = L_y$	$\delta_m = pL_x^4 / E t^3$	.001	.003	.006	.010	.015	.019	.024	.026	.031	.033	.034	.035	.035	.035

MOMENTS



FACTOR pLy^2

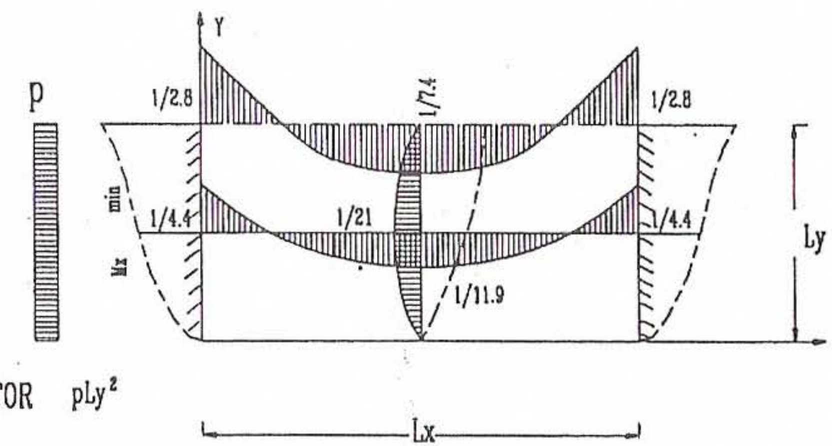
Ly/Lx = 0.50

El-Behairy R.C. Design Handbook Chapter -4- Rectangular Plates

C-5) *SLAB FIXED ON TWO SIDES, SIMPLY SUPPORTED ON ONE SIDE AND SUBJECT TO UNIFORM LOAD*

Position	L_y / L_x	0.25	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
$y = L_y$	$M_x min = -PL_y^2$:	1.0	1.3	1.9	2.8	3.8	5.2	6.8	8.7	10.8	13.2	15.7	18.6	21.6	24.9
	$M_x m = PL_y^2$:	4.4	4.6	5.8	7.4	9.6	12.2	15.4	19.1	23.4	28.2	33.5	39.4	45.8	52.6
$y = 0.5L_y$	$M_x min = -PL_y^2$:	2.0	2.5	3.4	4.4	5.6	7.0	8.6	10.3	12.2	14.4	16.8	19.4	22.3	25.4
	$M_x m = PL_y^2$:	8.1	8.4	9.9	11.9	14.4	17.3	20.5	24.0	28.2	32.8	37.5	43.0	48.8	55.4
$x = 0.5L_x$	$M_y m = PL_y^2$:	9.6	11.4	15.3	21.6	30.5	43.2	61.1	85.0	122	171	240	331	462	652
	$M_y max = PL_y^2$:	9.6	11.9	15.2	21.0	28.3	36.6	48.2	61.2	75.8	90.9	104	118	133	150
	at y/L_y =	0.5	0.48	0.47	0.43	0.38	0.33	0.28	0.25	0.22	0.19	0.16	0.13	0.1	0.1
$x = 0.5L_x$ $y = L_y$	$\delta_m = pL_x^4 / Et^3$	.015	.020	.025	.030	.033	.034	.034	.034	.034	.034	.034	.034	.034	.034

MOMENTS

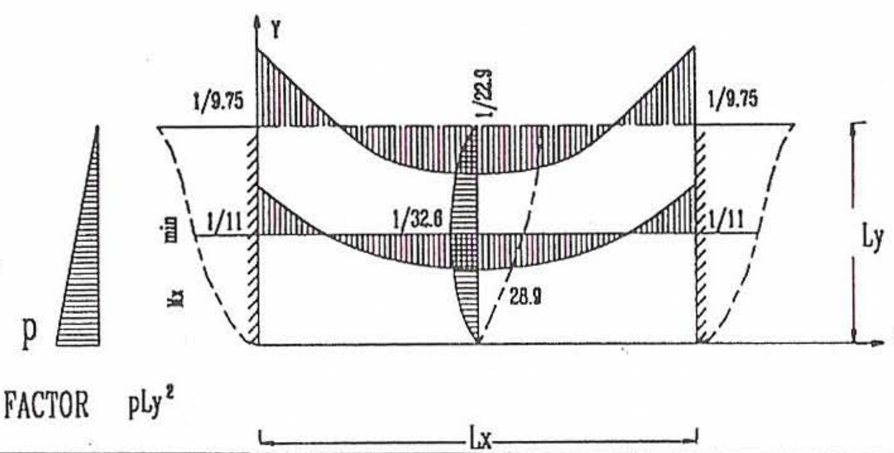


Ly/Lx = 0.50

C-6) SLAB FIXED ON TWO SIDES, SIMPLY SUPPORTED ON ONE SIDE AND SUBJECT TO TRIANGULAR LOAD

Position	L_y / L_x	0.25	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
$y = L_y$	$M_x min = -PL_y^2$:	3.2	4.1	6.3	9.8	14.8	21.8	32.3	46.1	64.6	88.0	117	150	187	230
	$M_x m = PL_y^2$:	13.0	13.8	17.6	22.9	30.5	40.7	55.2	74.2	98.0	129	166	210	262	324
$y = 0.5L_y$	$M_x min = -PL_y^2$:	5.7	6.7	8.8	11.0	13.3	16.0	18.9	22.3	26.0	29.9	34.6	39.5	45.1	51.3
	$M_x m = PL_y^2$:	24.3	25.3	28.3	32.6	37.7	43.4	49.6	56.3	63.6	72.2	81.4	91.5	102	115
$x = 0$ & $x = L_x$	$M_x min = -PL_y^2$:	5.7	6.7	8.8	11.0	13.2	16.0	18.6	21.6	24.6	27.9	31.4	34.7	39.2	43.5
	at y/L_y =	1.0	1.0	1.0	1.0	0.57	0.5	0.46	0.42	0.39	0.36	0.34	0.33	0.32	0.31
$x = 0.5L_x$	$M_y m = PL_y^2$:	18.3	20.2	25.7	33.6	44.6	60.1	81.6	111	152	209	288	390	532	750
	$M_y max = PL_y^2$:	17.7	19.1	23.5	28.9	35.9	44.4	54.2	65.5	78.4	93.5	133	132	153	175
	at y/L_y =	0.4	0.38	0.35	0.33	0.29	0.28	0.25	0.23	0.21	0.2	0.19	0.18	0.16	0.15
$x = 0.5L_y$	$M_y min$ (at y/L_y=.88) $= -PL_y^2$:	-	-	-	-	-	-	-	-1386	-1060	-957	-936	-1001	-1148	-1350
$x = 0.5L_x$ $y = L_y$	$\delta_m = pL_x^4 / E t^3$	.005	.005	.008	.009	.009	.009	.009	.008	.007	.007	.006	.006	.005	.005

MOMENTS

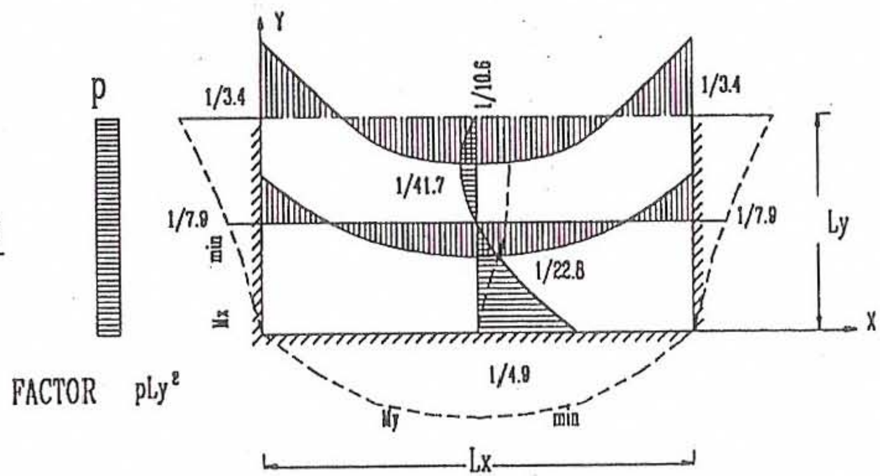


Ly/Lx = 0.50

C-7) SLAB FIXED ON THREE SIDES AND SUBJECT TO UNIFORM LOAD

Position	L_y / L_x	0.25	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
$x = 0.5L_x$	$M_y min = -PL_y^2$:	2.4	2.7	3.6	4.9	6.7	8.8	11.3	14.3	17.6	21.3	25.3	29.7	34.4	39.5
$y = 0.5L_y$	$M_x min = -PL_y^2$:	6.8	6.9	7.3	7.9	8.8	10.0	11.4	13.1	15.0	17.1	19.5	22.1	25.0	28.1
$y = L_y$	$M_x min(max) = -PL_y^2$:	2.2	2.3	2.7	3.4	4.5	5.8	7.4	9.4	11.6	14.1	16.9	19.9	23.2	26.7
$x = 0.5L_x$	$M_y m = PL_y^2$:	-16.1	-26.7	-26.8	88.1	62.9	63.8	73.4	89.6	113	145	190	250	333	446
$x = 0.5L_x$	$M_y max = PL_y^2$:	104	62.5	43.2	41.7	48.0	58.5	72.7	89.6	109	132	157	184	213	245
	at y / L_y =	0.88	0.8	0.75	0.7	0.65	0.6	0.55	0.5	0.45	0.4	0.35	0.32	0.3	0.28
$y = 0.5L_y$	$M_x m = PL_y^2$:	57.1	37.9	25.4	22.8	23.2	24.9	27.5	30.7	34.3	38.4	43.0	48.1	53.6	59.7
$y = L_y$	$M_x m = M_x max = PL_y^2$:	19.2	13.9	10.7	10.6	11.9	13.9	16.7	20.2	24.3	29.0	34.4	40.3	46.7	53.7
$x = 0.5L_x$, $y = L_y$	$\delta_m = pL_x^4 / Et^3$	.004	.007	.015	.021	.027	.030	0.32	0.33	0.34	0.35	0.35	0.35	0.34	0.34

MOMENTS

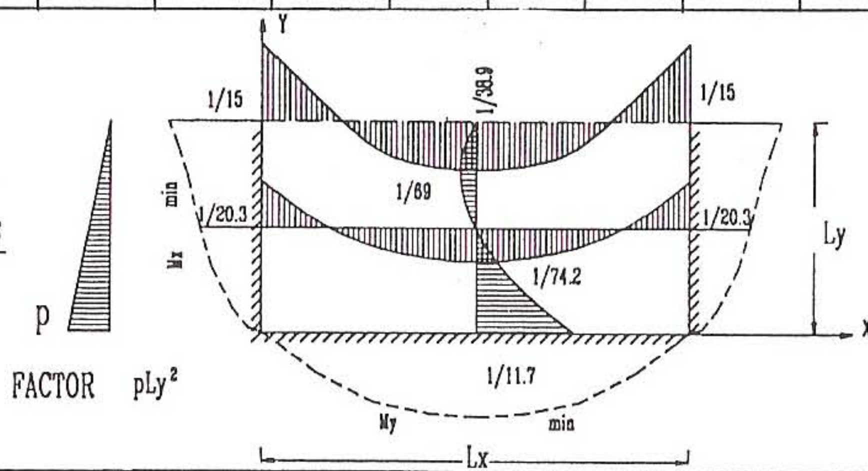


Ly/Lx = 0.50

C-8) SLAB FIXED ON THREE SIDES AND SUBJECT TO TRIANGULAR LOAD.

Position	L_y / L_x	0.25	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
$x = 0.5L_x$ $y = 0$	$M_y min = -PL_y^2$:	6.9	7.4	9.4	11.7	14.4	17.5	20.9	24.3	28.5	32.8	37.4	42.4	47.7	53.3
$y = 0.5L_y$	$M_x min = -PL_y^2$:	18.2	18.4	19.1	20.3	22.0	24.1	26.7	29.8	33.2	37.1	41.5	46.3	51.7	57.6
$y = L_y$	$M_x min = -PL_y^2$:	8.8	9.4	11.4	15.0	20.4	28.4	39.4	54.4	74.2	99.2	130	167	209	257
$x = 0.5L_x$	$M_x min = -PL_y^2$: at y/L_y =	8.8 1.0	9.4 1.0	11.4 1.0	15.0 1.0	20.4 1.0	23.6 0.6	26.6 0.55	29.8 0.5	33.1 0.47	37.0 0.45	41.1 0.43	45.4 0.4	49.5 0.38	54.2 0.36
$x = 0.5L_x$	$M_y m = PL_y^2$:	-243	419	91.3	70.8	70.9	79.2	93.7	115	144	184	238	310	408	541
	$M_y max = -PL_y^2$: at y/L_y =	156 0.75	103 0.7	72.7 0.65	69.0 0.6	70.9 0.5	79.0 0.47	86.5 0.45	103 0.4	118 0.35	137 0.32	160 0.31	186 0.3	213 0.3	242 0.28
$y = 0.5L_y$	$M_x m = PL_y^2$:	216	138	87.7	74.2	71.0	71.9	74.8	79.1	84.4	90.8	98.3	107	116	127
$y = L_y$	$M_x m = PL_y^2$:	71.5	51.3	39.0	38.9	43.9	59.1	66.4	84.5	108	139	177	224	281	347
$x = 0.5L_x$	$M_x max = PL_y^2$: at y/L_y =	71.5 1.0	51.3 1.0	39.0 1.0	38.9 1.0	43.9 1.0	53.1 1.0	64.1 0.8	73.7 0.7	82.1 0.6	90.3 0.55	98.3 0.5	106 0.47	115 0.45	125 0.42

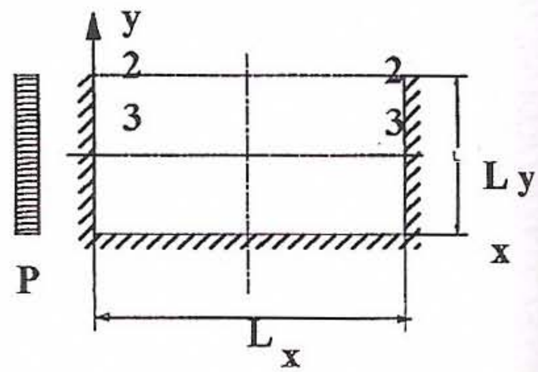
MOMENTS



$Ly/Lx = 0.50$

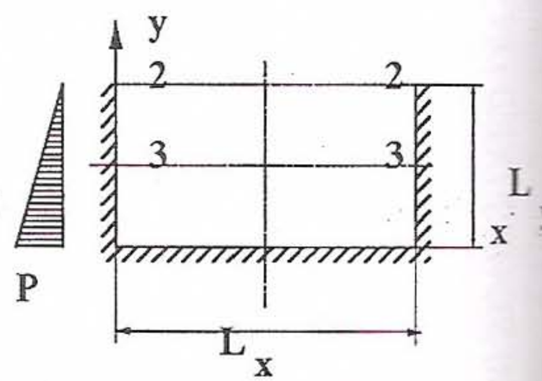
REACTIONS OF SLABS FIXED ON THREE SIDES AND SUBJECT TO UNIFORM AND TRIANGULAR LOADS.*

a) UNIFORM LOAD.



Position									
x/L_x	y/L_y	L_y/L_x	0.60	0.70	0.80	0.90	1.00	1.25	1.50
0.5	0	$R_1 = pL_y$:	1.44	1.70	1.95	2.20	2.50	3.22	4.02
0.1	1	$R_2 = pL_y$:	0.80	0.98	1.17	1.37	1.59	2.20	2.85
0.1	0.5	$R_3 = pL_y$:	2.00	2.00	2.07	2.17	2.30	2.53	3.05

b) TRIANGULAR LOAD.



Position									
x/L_x	y/L_y	L_y/L_x	0.60	0.70	0.80	0.90	1.00	1.25	1.50
0.5	0	$R_1 = pL_y$:	2.40	2.70	2.90	3.14	3.39	4.05	4.82
0.1	1	$R_2 = pL_y$:	6.50	8.70	11.6	15.8	22.2	69.5	-250
0.1	0.5	$R_3 = pL_y$:	4.40	4.40	4.50	4.60	4.80	5.35	6.13

* TIMOSHENKO : " PLATES AND SHELLS " FOR $\gamma = 1/m = 1/6$

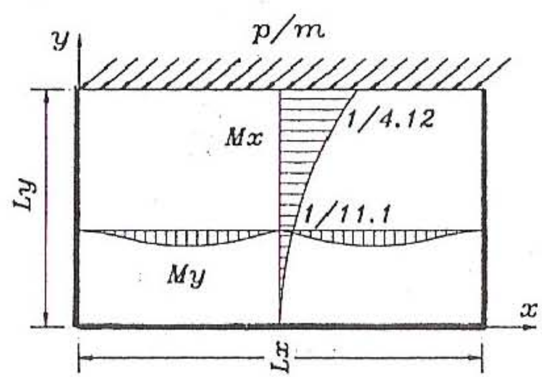
D-1) Slab Simply Supported on Three Sides and Subjected to Uniform Edge Load t/m`

$\frac{x}{L_x}$	$\frac{y}{L_y}$	L_y/L_x	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00	1.10	1.20	1.30	1.40	1.50
.5	1	M_x m = p L_x :	8.16	6.89	5.60	4.91	4.55	4.35	4.22	4.17	4.12	4.11	4.10	4.09	4.08	4.08
.5	.5	M_x m = p L_x :	16.1	13.7	11.0	9.85	9.42	9.45	9.75	10.3	11.1	12.1	13.3	14.7	16.4	18.4
.5	.5	M_y m = - p L_x :	465	200	89.2	52.9	38.7	32.3	28.8	27.5	27.2	27.7	28.8	30.4	32.5	35.1
.5		M_y min = -p L_x :	460	195	87.8	51.5	37.3	30.2	26.3	24.3	23.4	22.7	22.3	22.0	21.8	21.7
		at y/Ly =	.55	.55	.56	.58	.60	.62	.64	.66	.68	.70	.72	.74	.76	.78
.3&.7		M_y min =-p L_x : at y/Ly =	245 .57	145 .58	78.1 .59	52.5 .60	40.2 .62	34.0 .64	30.3 .66	28.4 .69	27.2 .71	26.3 .73	26.0 .75	25.9 .77	25.8 .79	25.8 .81

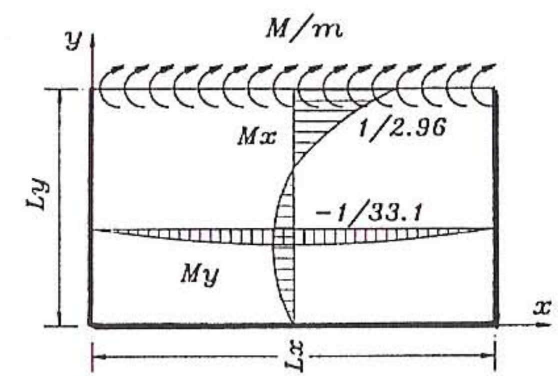
D-1`) Slab Simply Supported on Three Sides and Subjected to Uniform Edge Moment M m.t/m`

.5	1.	M_x m = M :	2.07	2.14	2.30	2.47	2.63	2.74	2.84	2.91	2.94	2.96	2.97	2.98	2.99	3.00
.5	.5	M_x m = M :	4.27	4.67	5.77	8.10	12.9	25.0	110	-71.5	-33.1	-24.5	-21.1	19-4	-18.9	-18.9
.5	.5	M_y m =-M :	2.02	2.07	2.24	2.55	3.01	3.65	4.52	5.68	7.28	9.52	12.5	16.3	22.2	32.2
0.3 0.7	.5	M_y 0.3 =-M :	2.14	2.27	2.57	3.03	3.65	4.45	5.62	7.15	9.16	11.8	15.6	21.2	29.4	41.3

(D-1)



(D-1`)



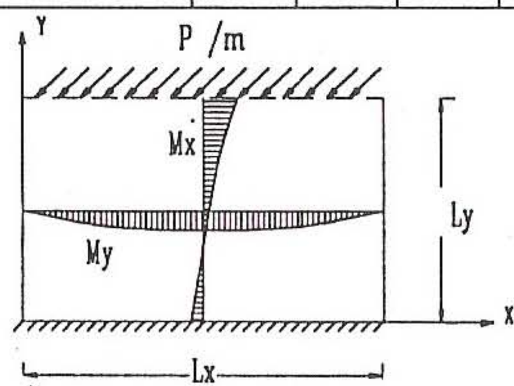
D-2) SLABS FIXED ON ONE SIDE AND SIMPLY SUPPORTED ON TWO SIDES AND SUBJECTED TO UNIFORM EDGE LOAD p/m AT THE FREE EDGE

$\frac{x}{L_x}$	$\frac{y}{L_y}$	L_y/L_x	0.25	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
.5	1.0	M_x m = p L_x :	38.9	22.6	11.4	7.71	6.06	5.21	4.75	4.48	4.31	4.22	4.16	4.13	4.12	4.11
.5	.5	M_x m = p L_x :	125	70.9	33.8	22.1	17.1	14.8	13.7	13.4	13.6	14.2	15.1	16.2	17.6	19.4
.5	0	M_y min =- p L_x :	4.7	4.34	4.16	4.38	4.90	5.76	6.95	8.48	10.5	13.1	16.8	21.7	28.3	36.2
.5	.5	M_y m =-p L_x :	10.8	10.6	11.5	13.3	15.7	18.2	20.6	22.9	24.6	26.4	28.2	30.3	32.6	35.6
.5		M_y min =-p L_x : at y/Ly =									23.6 0.66	23.1 0.68	22.7 0.70	22.4 0.72	22.3 0.74	22.3 0.75

D-2`) SLAB FIXED ON ONE SIDE AND SIMPLY SUPPORTED ON TWO SIDES AND SUBJECTED TO UNIFORM EDGE MOMNT M/m AT THE FREE EDGE

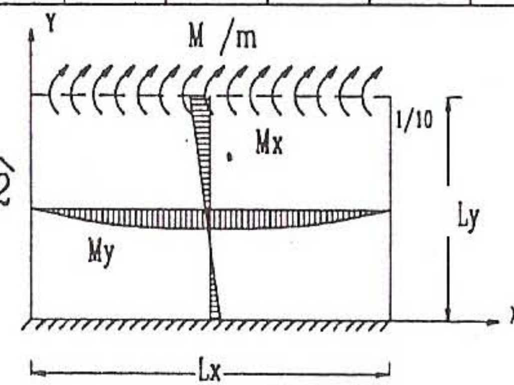
.5	1.0	M_x m = M :	6.50	4.67	3.30	2.86	2.73	2.73	2.76	2.81	2.86	2.90	2.94	2.97	2.99	3.00
.5	.5	M_x m = M :	21.9	15.8	12.0	12.3	15.2	23.2	48.8		61.4	35.5	29.5	22.9	21.0	20.6
.5	0	M_y min =M :	-1.38	-1.68	-2.83	-5.90	-20.7	33.9	15.7	12.9	12.0	12.5	13.5	15.4	18.2	23.5
.5	.5	M_y m =-M :	1.25	1.41	1.81	2.31	2.93	3.64	4.64	5.83	7.46	9.58	12.5	16.4	22.2	31.8

D-2



Ly/Lx = 1.00
FACTOR pLx^2

D-2`



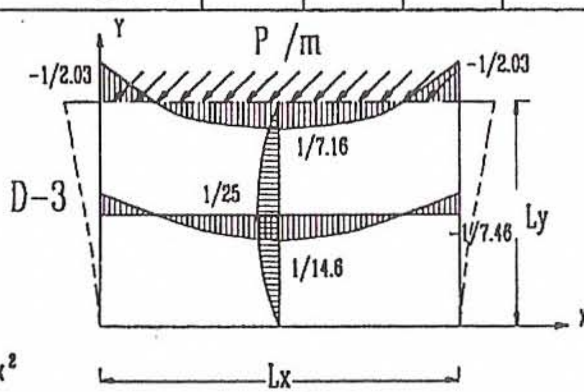
Ly/Lx = 1.00
FACTOR M

D-3) Slab Fixed on Two Sides, Simply Supported on one Side and Subjected to Uniform Edge Load p t/m'

$\frac{x}{L_x}$	$\frac{y}{L_y}$	L_y/L_x =	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.0	1.1	1.2	1.3	1.4	1.5
0.1	1	M_x min =-p L_x :	1.96	1.96	1.98	2.03	2.07	2.11	2.14	2.16	2.19	2.21	2.24	2.27	2.31	2.34
0.1	.5	M_x min =- p L_x :	4.67	5.05	5.99	7.46	9.61	12.6	16.9	23.2	32.1	43.5	66.7	100	165	260
.5	1	M_x m = p L_x :	8.78	7.87	7.19	7.05	7.05	7.10	7.14	7.17	7.18	7.19	7.20	7.21	7.23	7.25
.5	.5	M_x m =p L_x :	16.7	14.1	13.9	14.6	16.5	19.6	23.8	29.6	37.6	48.3	63.0	83.4	111	150
0.5	0.5	M_y m =- p L_x :	110	60.2	33.1	25.0	22.6	22.6	24.0	26.6	30.6	35.8	42.4	51.0	62.1	76.3
0.5	VARIAB	M_y min =-p L_x :	105	59.2	32.5	24.1	21.0	20.1	19.5	19.7	19.7	20.0	20.3	20.6	20.8	20.9
		at y/Ly =	.55	.56	.57	.59	.62	.66	.69	.72	.76	.78	.80	.82	.84	.85

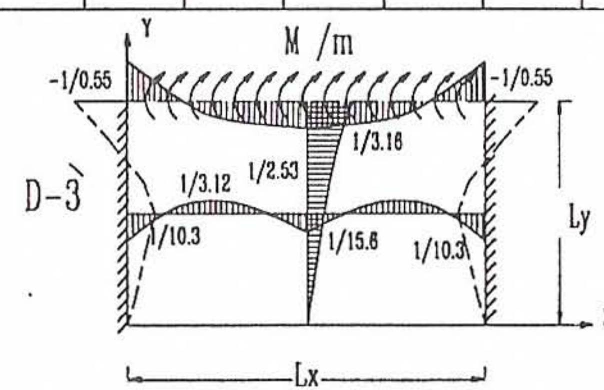
D-3`) Slab Fixed on Two Sides, Simply Supported on one Side and Subjected to Uniform Edge Moment M m.t./m'

$\frac{x}{L_x}$	$\frac{y}{L_y}$	L_y/L_x =	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.0	1.1	1.2	1.3	1.4	1.5
0.1	1	M_x min = -M :	0.42	0.46	0.52	0.55	0.57	0.59	0.61	0.62	0.64	0.67	0.69	0.72	0.74	0.77
0.1	.5	M_x min = M :	1.73	3.05	16.4	10.3	5.65	4.65	4.70	5.00	5.50	6.21	7.25	8.51	10.2	12.3
.5	1.0	M_x m = M :	2.21	2.41	2.82	3.16	3.37	3.46	3.49	3.49	3.50	3.50	3.51	3.51	3.52	3.54
.5	.5	M_x m = M :	4.44	5.15	7.74	15.6	75.7	45.5	22.5	17.8	16.3	16.3	16.9	18.3	20.4	23.2
.5	.5	M_y m = - M :	1.93	1.96	2.13	2.53	3.21	4.22	5.95	8.62	13.3	22.2	41.7	125	400	125
.3&.7	.5	M_y m = - M :	1.97	2.10	2.47	3.12	4.17	5.75	8.40	12.7	20.5	37.0	83.3	660	200	120



FACTOR pLx^2

Ly/Lx = 0.50



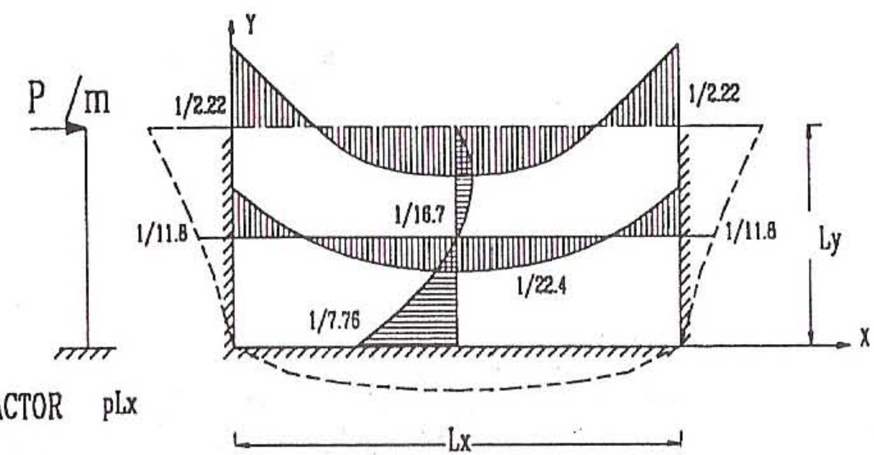
FACTOR M

Ly/Lx = 0.50

D-4) SLAB FIXED ON THERR SIDES AND SUBJECT TO UNIFORM EDGE LOAD p t/m.

Position x/L_x	Position y/L_y	L_y/L_x	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00	1.10	1.20	1.30	1.40	1.50
0;1	1	M_x min $= -p\,L_x$:	3.96	3.17	2.47	2.22	2.14	2.12	2.14	2.16	2.19	2.21	2.24	2.27	2.31	2.34
0;1	0.5	M_x min $= -p\,L_x$:	15.0	13.0	11.6	11.8	13.0	15.3	19.2	25.0	33.5	46.3	66.3	98.0	155	250
0.5	1	M_x m $= p\,L_x$:	31.2	18.8	10.8	8.45	7.62	7.35	7.21	7.19	7.18	7.19	7.20	7.21	7.23	7.25
0.5	0.5	M_x m $= p\,L_x$:	93.6	53.2	28.5	22.4	21.4	22.5	26.1	31.2	38.6	48.3	63.0	83.4	111	150
0.5	0	M_y min $= -p\,L_x$:	5.03	4.95	5.72	7.76	11.2	18.7	32.9	64.1	130	270	-	-	-	-
0.5	0.5	M_y m $= -p\,L_x$:	11.8	12.2	14.3	16.7	19.0	21.0	23.6	26.8	30.7	36.0	42.6	51.0	62.1	76.5
0.5	Variable	M_y min $= -p\,L_x$:	-	-	-	-	-	20.0	19.7	19.5	19.7	20.0	20.3	20.6	20.8	20.9
		at $y/L_y =$	-	-	-	-	-	0.63	0.68	0.73	0.76	0.78	0.80	0.82	0.84	0.85
0.5	1	$\delta_m = pL_x^3/Et^3$	.046	.067	.092	.109	.120	.126	.126	.133	.133	.126	.126	.126	.126	.126

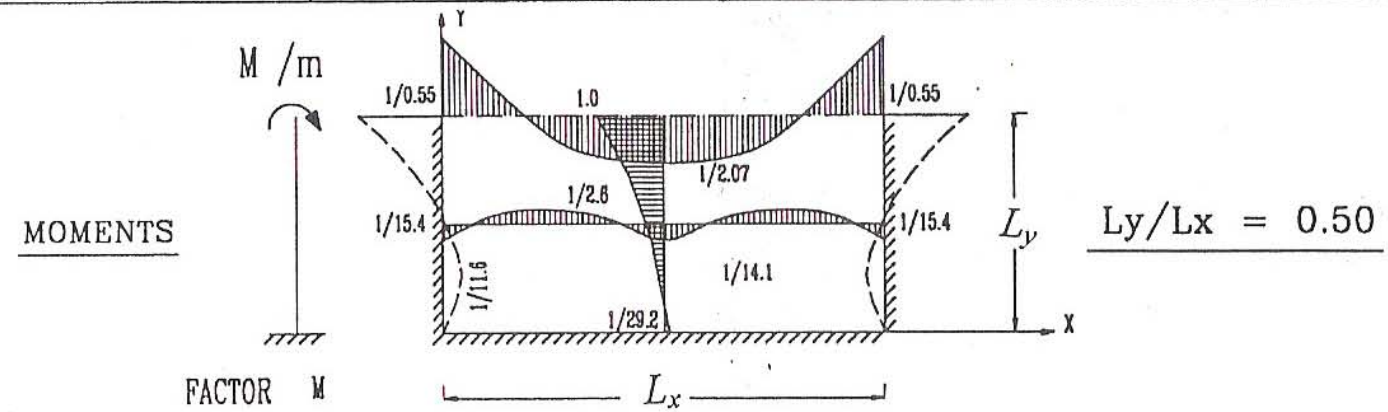
MOMENTS



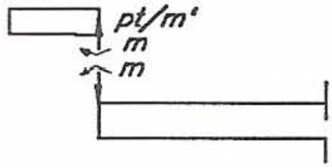
$Ly/Lx = 0.50$

D-4') SLAB FIXED ON THREE SIDES AND SUBJECT TO UNIFORM EDGE MOMENT M mt/m.

Position x/L_x	Position y/L_y	L_y/L_x	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00	1.10	1.20	1.30	1.40	1.50
0;1	1	M_x min. = -M :	0.61	0.57	0.54	0.55	0.56	0.57	0.60	0.62	0.64	0.67	0.69	0.72	0.74	0.77
0;1	0.5	M_x = M :	5.98	8.29	4.76	15.4	7.91	6.17	5.59	5.56	5.94	6.67	7.55	8.66	10.2	12.4
0;1	VAR.	M_x max. = M :	3.00	69.0	23.2	12.2	7.82	6.03	5.10	4.72	4.52	4.42	4.37	4.33	4.31	4.31
		at y/L_y =	0.14	0.19	0.29	0.39	0.47	0.54	0.59	0.63	0.66	0.69	0.72	0.74	0.76	0.77
0.5	1	M_x m = M :	5.40	4.00	3.14	3.07	3.18	3.29	3.40	3.44	3.48	3.49	3.50	3.52	3.53	3.54
0.5	0.5	M_x m = M :	16.90	12.3	10.6	14.1	29.4	∞	36.1	21.3	18.5	17.5	17.9	18.7	20.4	23.4
VAR	0.5	M_x min. = -M :		7.00	8.2	11.6	18.5	29.5	27.8	21.3	18.5	17.5	17.9	18.7	20.4	23.4
		at x/L_x =		.05	.08	.13	.15	.21	.27	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.5	0	M_y = M :	1.52	2.08	6.05	29.2	8.03	6.90	7.55	9.00	12.7	18.5	27.8	41.7	66.7	115
0.5	0.5	M_y m = -M :	1.32	1.52	2.01	2.60	3.36	4.39	6.07	8.62	13.1	21.7	40.0	100	∞	130
0.5	1	$\delta = ML_x^2/Et^3$	.273	.325	.363	.363	.353	.343	.333	.333	.325	.325	.316	.308	.308	.300

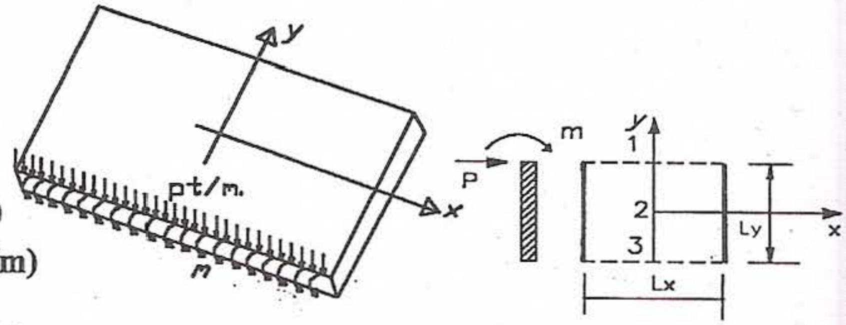


IV.2. BENDING MOMENTS IN RECTANGULAR PLATES SUPPORTED ON TWO OPPOSITE SIDES AND FREE AT THE OTHER TWO SIDES DUE TO LINE LOAD AND MOMENT AT THE FREE EDGE.



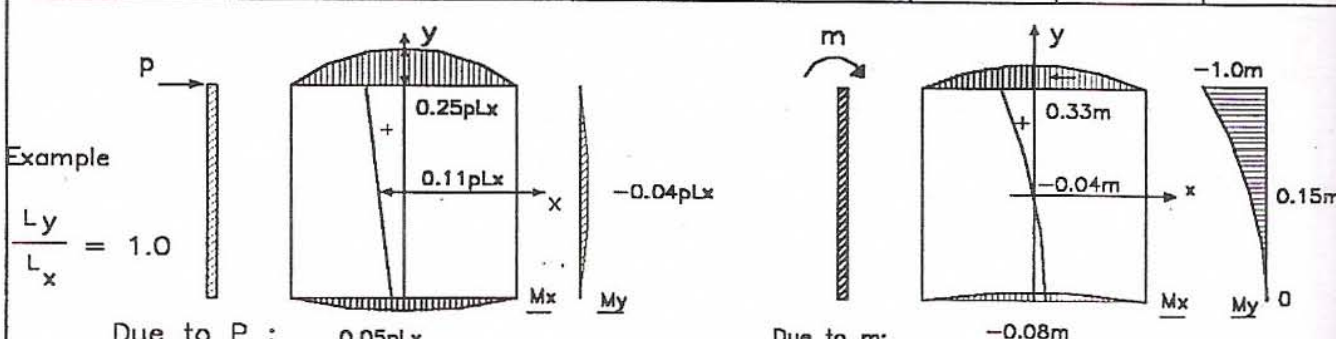
Line Load at free edge : p(t/m)

Moment at free edge :m(mt/m)

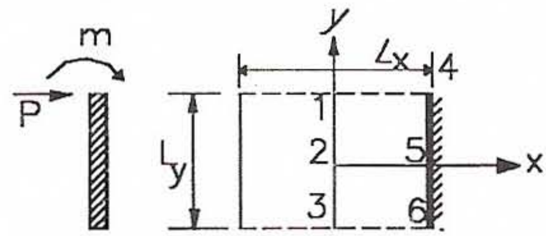


1. *Plate free at two opposite sides & simply supported at the other sides*

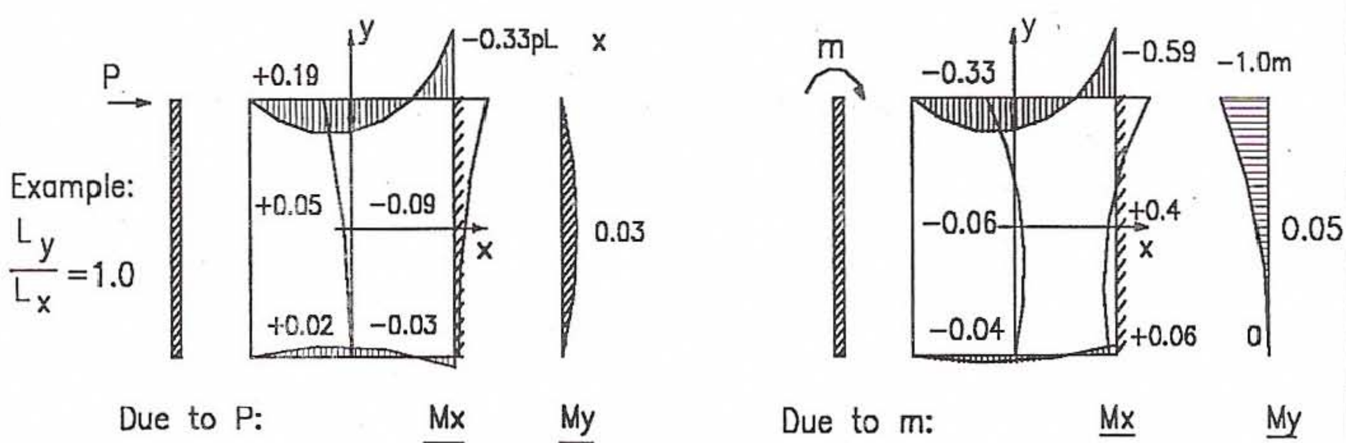
L_y/L_x	due to	M_x at middle of span			M_y at middle of span			Factor
		M_{x1}	M_{x2}	M_{x3}	M_{y1}	M_{y2}	M_{y3}	
3.0	p	+.25	+.01	0	0	-.01	0	pL_x
	m	+.33	-.01	0	-1.0	0	0	m
2.0	p	+.25	+.03	0	0	-.01	0	pL_x
	m	+.33	-.04	-.01	-1.0	-.01	0	m
1.6	p	+.25	+.05	+.01	0	-.02	0	pL_x
	m	+.33	-.05	-.02	-1.0	-.03	0	m
1.2	p	+.25	+.09	+.03	0	-.03	0	pL_x
	m	+.33	-.06	-.05	-1.0	-.10	0	m
1.0	p	+.25	+.11	+.05	0	-.04	0	pL_x
	m	+.33	-.04	-.08	-1.0	-.15	0	m
0.8	p	+.26	+.15	+.08	0	-.04	0	pL_x
	m	+.32	-.05	-.12	-1.0	-.22	0	m
0.5	p	+.32	+.25	+.19	0	-.04	0	pL_x
	m	+.29	-.03	-.20	-1.0	-.39	0	m
.25	p	+.53	+.50	+.47	0	-.03	0	pL_x
	m	+.25	0	-.25	-1.0	-.49	0	m



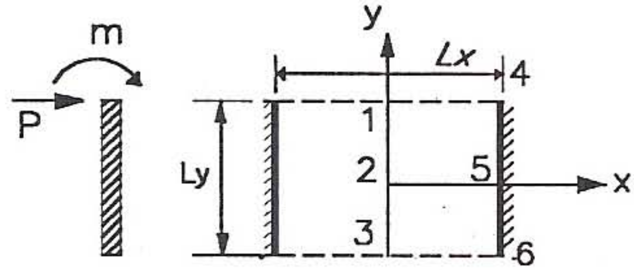
2. *PLATE FREE AT TWO SIDES , FIXED AT ONE SIDE AND SIMPLY SUPPORTED AT THE OTHER SIDE.*



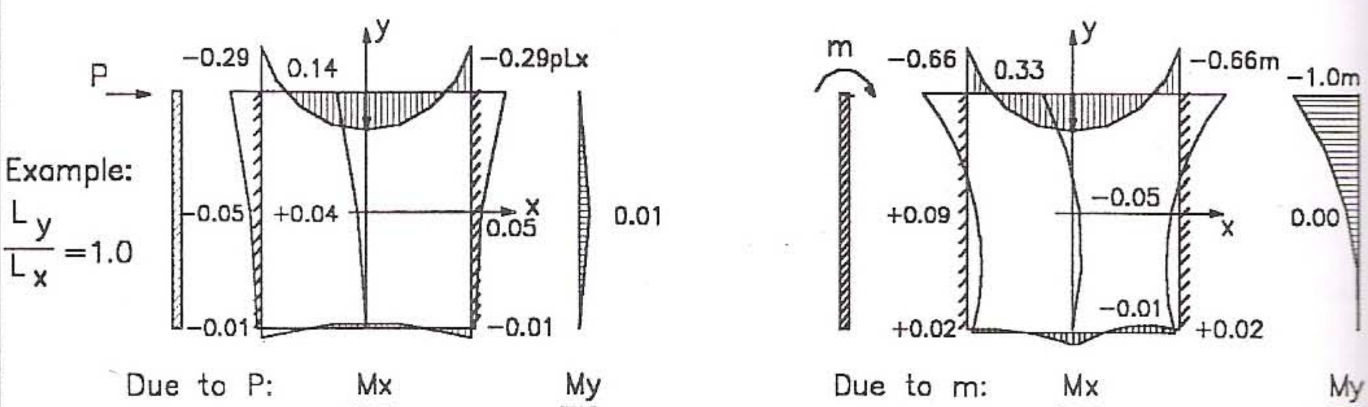
L_y/L_x	due to	M_x(mid.span)			M_y(mid.span)			M_x(at support)			Factor
		M_{x1}	M_{x2}	M_{x3}	M_{y1}	M_{y2}	M_{y3}	M_{x4}	M_{x5}	M_{x6}	
	p	+.19	0	0	0	0	0	-.33	0	0	pL_x
3.0	m	+.33	0	0	-1.0	0	0	-.59	+.01	0	m
	p	+.19	+.01	0	0	-.01	0	-.33	-.02	0	pL_x
2.0	m	+.33	-.02	0	-1.0	0	0	-.59	+.03	0	m
	p	+.19	+.02	0	0	-.01	0	-.33	-.04	0	pL_x
1.6	m	+.33	-.03	+.01	-1.0	0	0	-.59	+.06	+.01	m
	p	+.19	+.05	+.02	0	-.03	0	-.33	-.09	-.03	pL_x
1.0	m	+.33	-.06	-.04	-1.0	-.05	0	-.59	+.11	+.06	m
	p	+.19	+.08	+.03	0	-.02	0	-.33	-.13	-.05	pL_x
0.8	m	+.33	-.06	-.08	-1.0	-.10	0	-.58	+.11	+.14	m
	p	+.21	+.14	+.09	0	-.04	0	-.37	-.24	-.16	pL_x
0.5	m	+.31	-.04	-.15	-1.0	-.27	0	-.54	+.08	+.27	m
	p	+.31	+.28	+.25	0	-.03	0	-.56	-.50	-.45	pL_x
0.25	m	+.26	-.01	-.23	-1.0	-.45	0	-.47	+.01	+.42	m



3. PLATE FREE AT TWO SIDES AND FIXED AT THE OTHER TWO SIDES

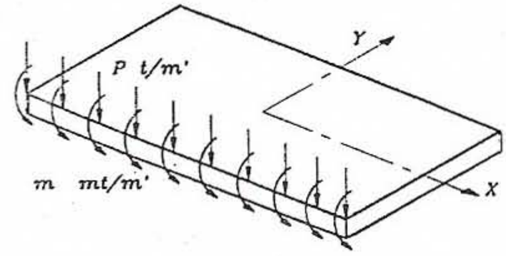


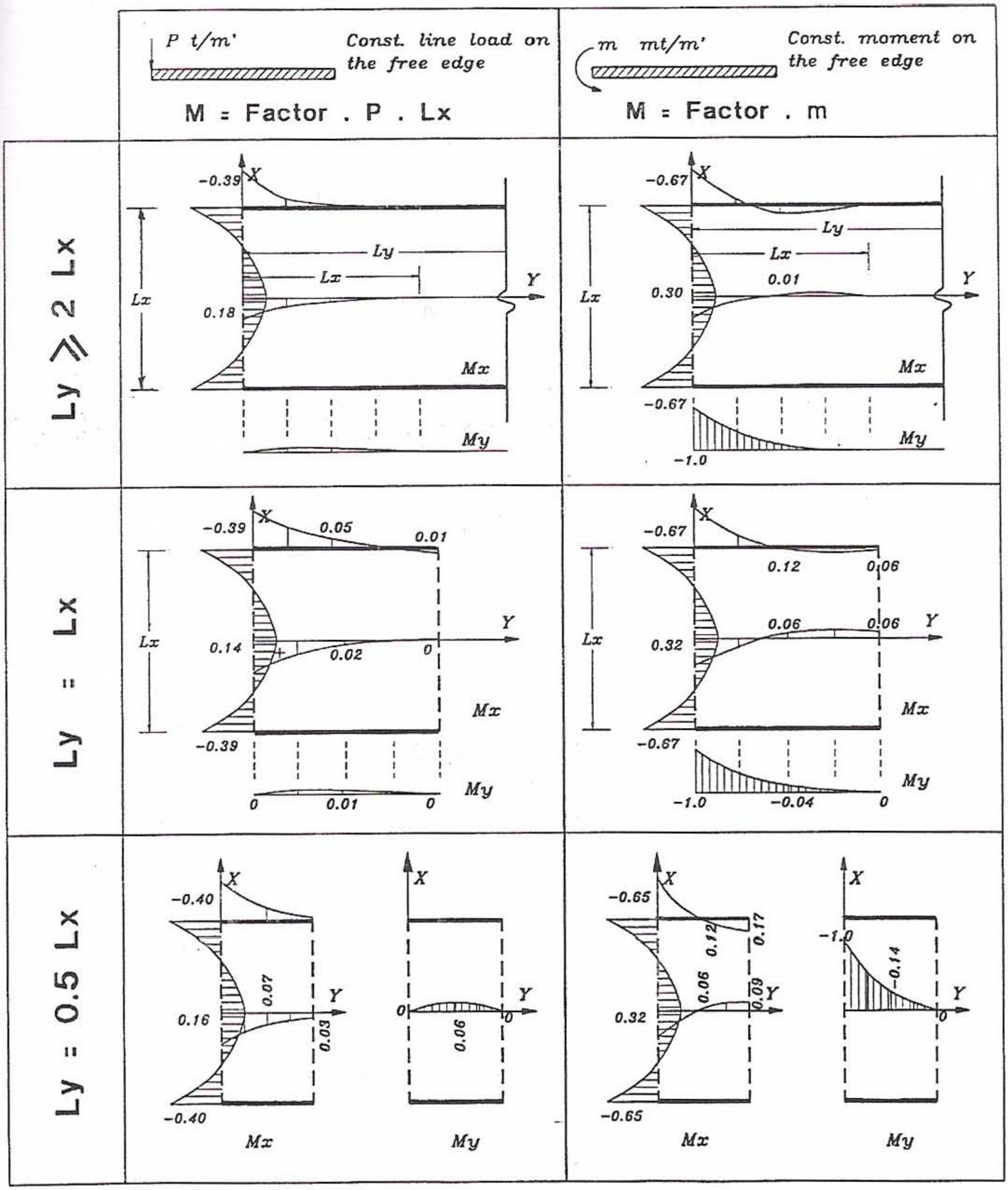
L_y/L_x	due to	M_x(mid.span)			M_y(mid.span)			M_x(at support)			Factor
		M_{x1}	M_{x2}	M_{x3}	M_{y1}	M_{y2}	M_{y3}	M_{x4}	M_{x5}	M_{x6}	
2.0	p	+.19	0	0	0	0	0	-.29	-.01	0	pL_x
	m	+.33	-.01	0	-1.0	0	0	-.67	+.01	0	m
1.6	p	+.14	+.01	0	0	-.01	0	-.29	-.01	0	pL_x
	m	+.33	-.02	0	-1.0	0	0	-.67	+.03	0	m
1.2	p	+.14	+.02	0	0		0	-.29	-.03	0	pL_x
	m	+.33	-.04	-.01	-1.0	0	0	-.67	+.07	+.01	m
1.0	p	+.14	+.02	0	0	-.01	0	-.29	-.05	-.01	pL_x
	m	+.33	-.05	-.01	-1.0	0	0	-.66	+.09	+.02	m
0.8	p	+.14	+.04	+.01	0	-.02	0	-.29	-.08	-.02	pL_x
	m	+.33	-.06	-.03	-1.0	-.04	0	-.67	+.12	+.06	m
0.5	p	+.15	+.07	+.03	0	-.03	0	-.30	-.14	-.06	pL_x
	m	+.32	-.06	-.09	-1.0	-.14	0	-.65	+.12	+.17	m
0.25	p	+.20	+.16	+.14	0	-.03	0	-.40	-.33	-.28	pL_x
	m	+.28	-.02	-.21	-1.0	-.33	0	-.56	+.04	+.42	m



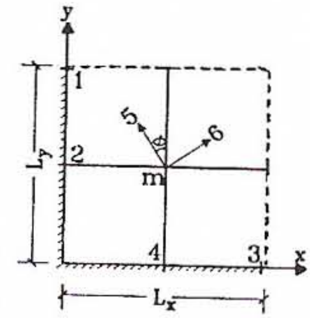
BENDING MOMENT DIAGRAMS

in rect. plate with two free sides and two fixed sides, due to line load & moment at free edge



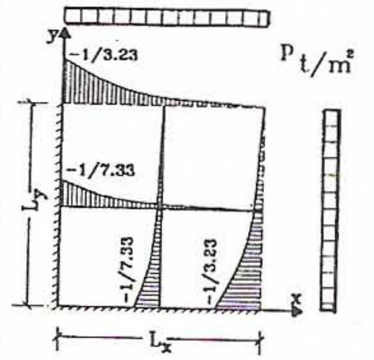


IV.3. Plate Fixed on two Adjacent Sides and Free on the Other two Sides :



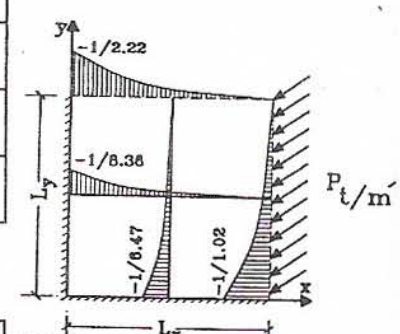
1) Uniform Load p t/m^2 :$M = p.L_x.L_y$ / factor :

L_y/L_x	M_{x1}	M_{x2}	M_{y3}	M_{y4}	M_{m5}	M_{m6}	φ°
1.00	-3.23	-7.33	-3.23	-7.33	16.7	-19.9	45
1.25	-3.29	-6.54	-3.41	-8.43	17.6	-20.4	41
1.50	-3.51	-6.11	-3.78	-9.60	20.2	-21.7	38
1.75	-3.81	-5.91	-4.27	-10.9	24.6	-23.4	35
2.00	-4.18	-5.90	-4.88	-12.4	31.0	-25.4	32



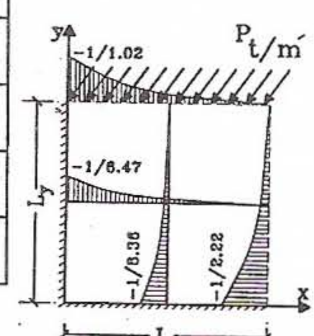
2) Line Load P t/m at edge $x=L_x$: $M = P.L_y$ / factor :

1.00	-2.22	-8.36	-1.02	-6.47	11.6	-5.24	36
1.25	-2.00	-5.70	-1.13	-6.33	11.0	-5.21	32
1.50	-2.01	-4.51	-1.30	-6.48	11.8	-5.40	29
1.75	-2.10	-3.95	-1.50	-6.85	13.7	-5.71	26
2.00	-2.26	-3.68	-1.74	-7.36	16.7	-6.08	23



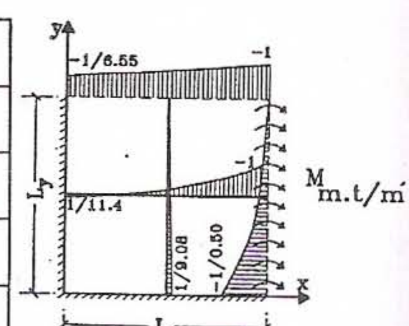
3) Line Load P t/m at edge $y=L_y$: $M = P.L_x$ / factor :

1.00	-1.02	-6.47	-2.22	-8.36	11.6	-5.24	54
1.25	-0.97	-6.90	-2.80	-13.1	14.0	-5.60	49
1.50	-0.95	-7.60	-3.76	-19.5	17.7	-6.22	46
1.75	-0.95	-8.51	-5.16	-27.8	22.9	-7.03	43
2.00	-0.97	-9.70	-7.25	-38.0	29.7	-8.05	41

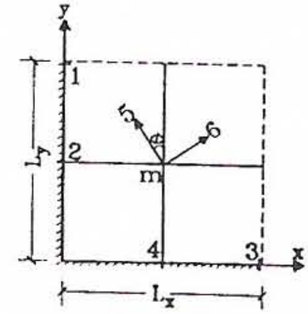


4) Moment M (m.t/m) at edge $x=L_x$: M = M / factor :

1.00	-6.55	11.4	-0.50	9.08	62.1	-2.93	27
1.25	-2.69	12.7	-0.48	12.5	16.8	-2.16	24
1.50	-1.81	13.1	-0.49	19.8	10.9	-1.79	21
1.75	-1.47	13.2	-0.50	35.8	9.30	-1.59	19
2.00	-1.30	15.5	-0.53	53.9	9.04	-1.46	17

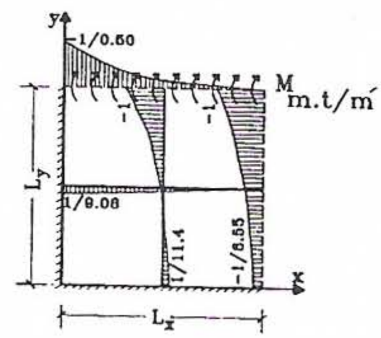


IV.3. Plate Fixed on two Adjacent Sides and Free on the Other two Sides :



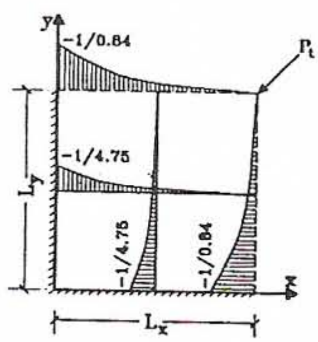
5) *Moment M (m.t/m) at edge $y=L_y$: $M = M / factor$:*

L_y/L_x	M_{x1}	M_{x2}	M_{y3}	M_{y4}	M_{m5}	M_{m6}	φ°
1.00	-0.50	9.08	-6.55	11.4	-62.1	-2.93	63
1.25	-0.53	6.35	10.1	10.8	-64.2	-4.47	60
1.50	-0.56	5.76	13.2	11.4	-36.3	-7.17	57
1.75	-0.61	4.75	11.2	19.3	-34.4	-12.3	55
2.00	-0.65	4.49	12.5	28.5	-38.2	-24.2	53



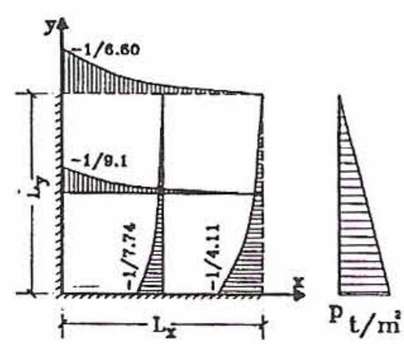
6) *Concentrated Load P (t) $x=L_x$, $y=L_y$: $M = P / factor$:*

1.00	-0.84	-4.75	-0.84	-4.75	6.16	-2.30	45
1.25	-0.73	-4.00	-1.06	-6.12	6.59	-2.38	42
1.50	-0.69	-3.72	-1.40	-7.92	7.66	-2.57	40
1.75	-0.68	-3.75	-1.91	-10.3	9.35	-2.87	38
2.00	-0.68	-3.96	-2.63	-13.5	11.6	-3.22	37



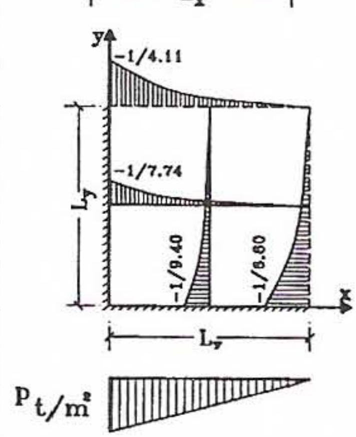
7) *Trinagular Load p t/m² : $M = pL_x.L_y /2\ factor$:*

1.00	-6.6	-9.1	-4.11	-7.74	-22.6	-54	39
1.25	-7.02	-8.08	-4.05	-8.18	23.7	-56	35
1.50	-7.85	-7.31	-4.19	-8.80	27.3	-60	31
1.75	-8.98	-6.92	-4.41	-9.51	33.7	-65	27
2.00	-1.03	-6.72	-4.15	-10.40	43.8	-68.4	23



7′) *Trinagular Load p t/m² : $M = pL_x.\ L_y /2\ factor$:*

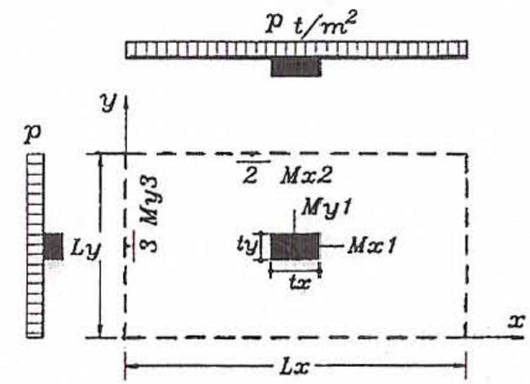
1.00	-4.11	-7.74	-6.6	-9.4	22.6	-54	51
1.25	-4.41	-7.50	-6.77	-11.3	24.5	-55.9	47
1.50	-4.85	-7.46	-7.34	-13.3	28.5	-60.4	45
1.75	-5.39	-7.56	-8.20	-15.6	34.8	-66.7	43
2.00	-5.95	-7.80	-9.29	-18.1	43.8	-74.1	41



IV.4. Plate Supported on a Central Support and Subjected to Uniform Load

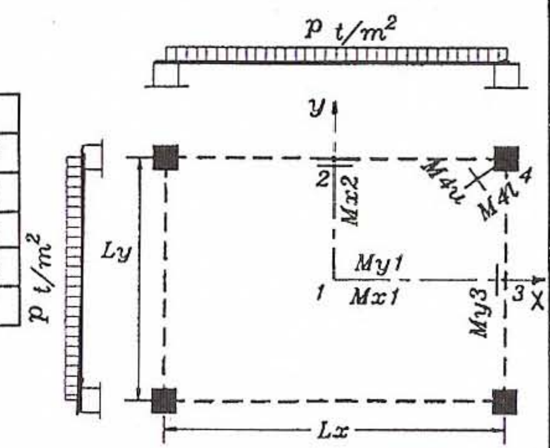
$M = p.L_x.L_y$ / factor $\lambda_x = t_x / L_x$, $\lambda_y = t_y / L_y$

Ly / Lx		1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
M_{x1}	$\lambda_x = 0.05$	3.73	3.85	3.94	3.98	4.02	4.05	4.07	4.09	4.11	4.13	4.15
	$\lambda_x = 0.10$	4.72	4.87	4.99	5.09	5.16	5.21	5.25	5.29	5.33	5.35	5.38
	$\lambda_x = 0.20$	6.37	6.68	6.98	7.20	7.30	7.38	7.46	7.52	7.58	7.64	7.70
M_{y1}	$\lambda_y = 0.05$	3.73	3.64	3.53	3.45	3.36	3.27	3.19	3.21	3.06	3.00	2.94
	$\lambda_y - 0.10$	4.72	4.55	4.38	4.24	4.11	4.00	3.89	3.78	3.69	3.60	3.51
	$\lambda_y = 0.20$	6.37	6.09	5.82	5.56	5.33	5.13	4.95	4.78	4.63	4.48	4.35
M_{x2}		13.2	14.7	16.7	18.9	21.8	24.4	27.8	32.3	37.0	43.5	47.6
M_{y3}		13.2	11.1	9.60	8.54	7.70	7.00	6.42	5.95	5.55	5.21	4.90



IV.4.1 PLATE SUPPORTED ON FOUR POINTS AND SUBJECTED TO UNIFORM LOAD p t/m² : $M = pL_xL_y$ / factor

L_y/L_x	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
M_{x1}	9.52	11.00	12.50	14.10	15.90	17.95	20.00	22.20	25.00	27.80	31.30
M_{y1}	9.52	8.33	7.46	6.75	6.16	5.67	5.27	4.92	4.61	4.34	4.10
M_{x2}	6.33	6.67	6.95	7.20	7.35	7.53	7.64	7.70	7.75	7.80	7.86
M_{y3}	6.33	5.98	5.68	5.38	5.10	4.86	4.63	4.41	4.20	4.02	3.85
$M_{4l} = -M_{4u}$	8.0	8.0	8.0	8.0	8.0	8.0	8.0	8.0	8.0	8.0	8.0



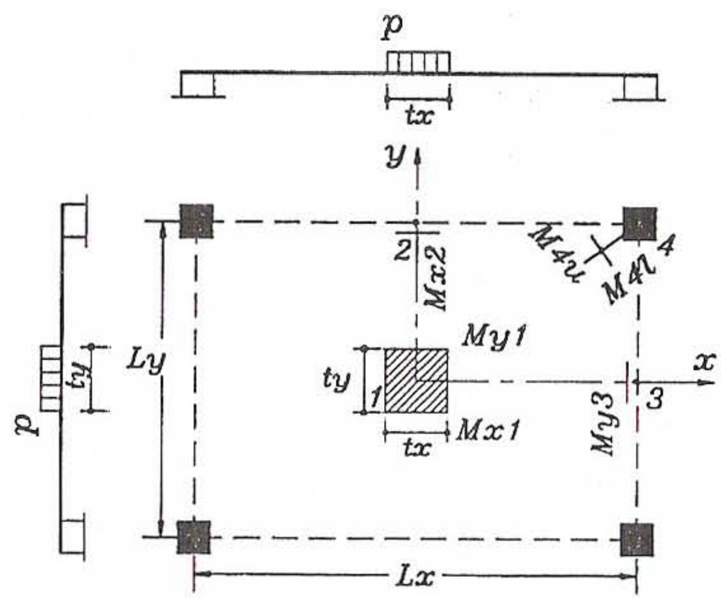
IV.4.2. PLATE SUPPORTED ON FOUR POINTS AND SUBJECTED TO CONCENTRATED LOAD P(t) :

M=P / factor

L_y/L_x		1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
M_{x1}	$\lambda_x = 0.05$	2.68	2.85	3.00	3.10	3.20	3.30	3.38	3.46	3.54	3.60	3.66
	$\lambda_x = 0.10$	3.15	3.37	3.56	3.75	3.90	4.03	4.15	4.27	4.38	4.49	4.59
	$\lambda_x = 0.20$	3.82	4.15	4.48	4.76	5.00	5.22	5.43	5.63	5.82	5.98	6.17
M_{y1}	$\lambda_y = 0.05$	2.68	2.53	2.40	2.28	2.17	2.07	1.99	1.91	1.84	1.77	1.71
	$\lambda_y = 0.10$	3.15	2.94	2.76	2.60	2.47	2.35	2.23	2.18	2.05	1.96	1.89
	$\lambda_y = 0.20$	3.82	3.52	3.27	3.05	2.86	2.69	2.55	2.42	2.31	2.20	2.11
M_{x2}		4.27	4.59	4.90	5.21	5.50	5.75	5.98	6.21	6.41	6.63	6.78
M_{y3}		4.27	3.89	3.57	3.30	3.07	2.87	2.69	2.53	2.39	2.27	2.16
$M_{4l}=M_{4u}$												

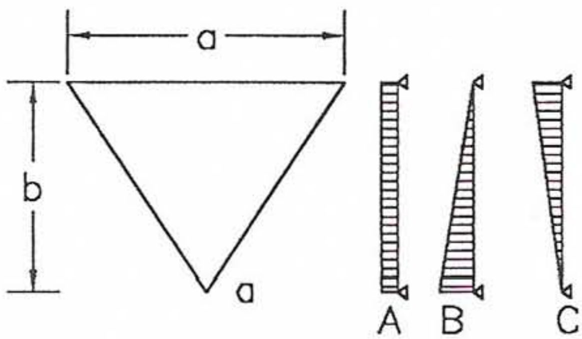
$$\lambda_x = t_x / L_x$$

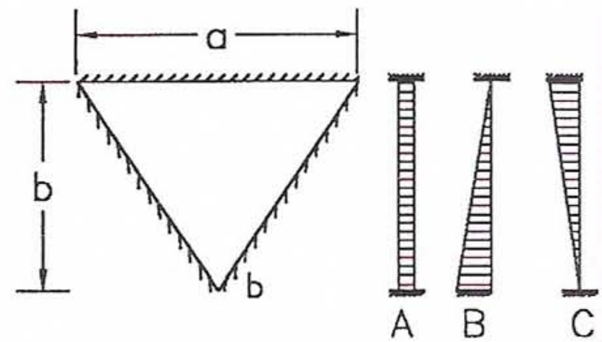
$$\lambda_y = t_y / L_y$$



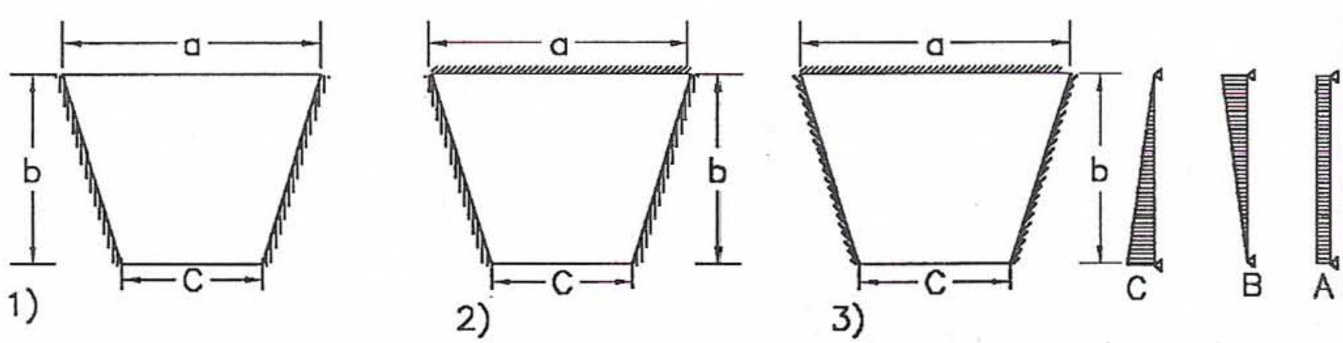
IV.5. Internal Forces In Triangular and Trapezoidal Plates

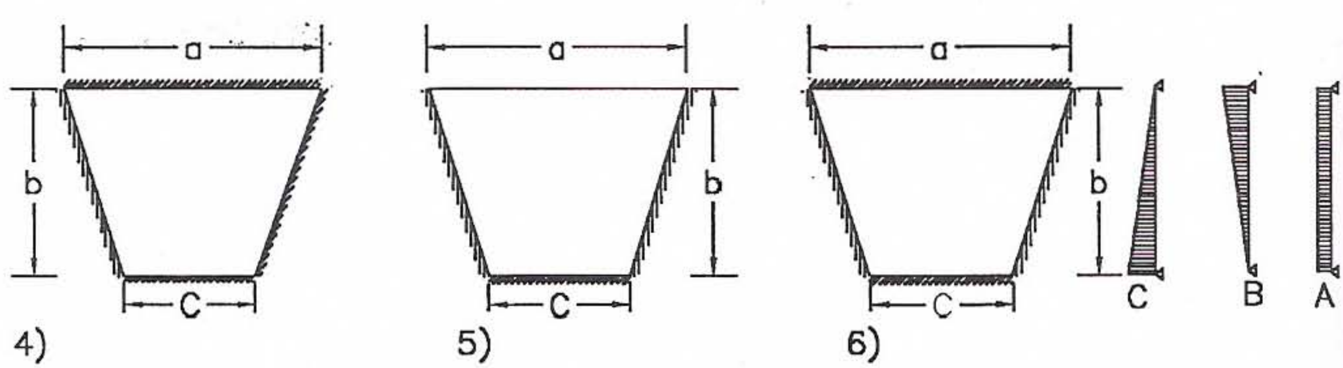
A) Internal forces in triangular plates (simply supported or totally fixed) due to rectangular or triangular loads



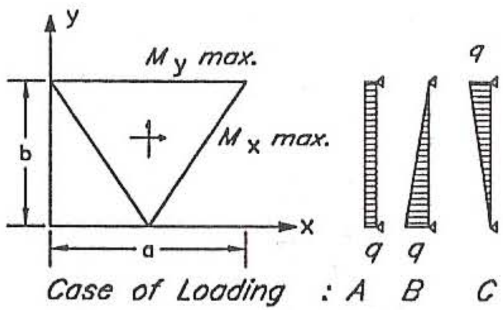


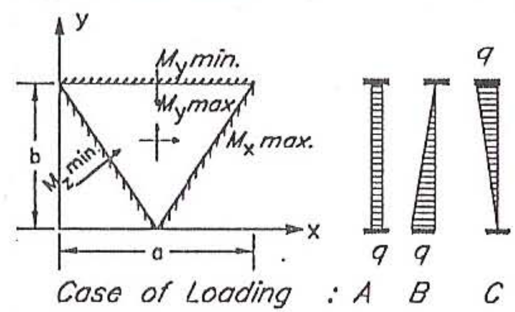
B. Internal forces in trapezoidal plates (simply supported, or totally fixed) due to rectangular or triangular loads





IV.5.A Bending Moments in Simply Supported and Totally Fixed Triangular Plates due to rectangular and triangular Loads.





Bending Moments = coeff. $q\ a^2$

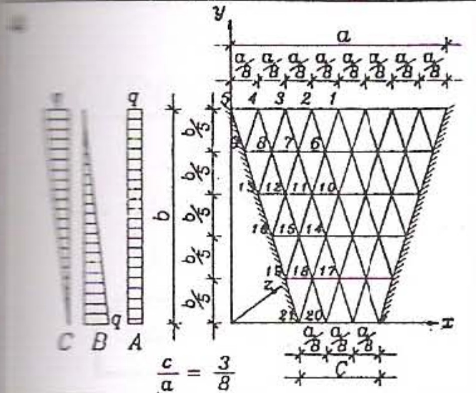
a) Simply Supported Triangular Plate

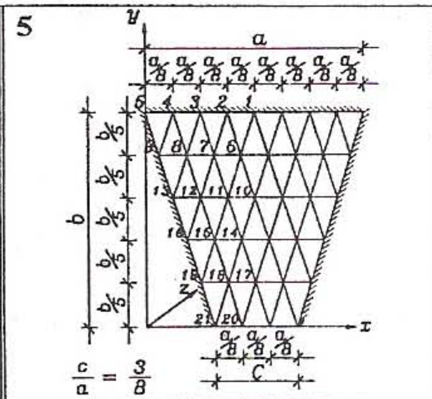
a/b	Case A		Case B		Case C	
	$M_x$max	$M_y$max	$M_x$max	$M_y$max	$M_x$max	$M_y$max
				+ ve	+ ve	+ ve
0.5	.0396	.0209	.0125	.0027	.0271	.0182
0.6	.0343	.0197	.0117	.0030	.0226	.0167
0.7	.0303	.0186	.0108	.0032	.0195	.0154
0.8	.0270	.0178	.0100	.0035	.0170	.0143
0.9	.0241	.0172	.0091	.0038	.0150	.0133
1.0	.0214	.0166	.0084	.0041	.0130	.0125
1.1	.0192	.0161	.0078	.0044	.0114	.0117
1.2	.0172	.0154	.0072	.0044	.0100	.0110
1.3	.0155	.0145	.0067	.0041	.0088	.0104
1.4	.0141	.0135	.0063	.0038	.0078	.0097
1.5	.0128	.0126	.0058	.0035	.0070	.0091
1.6	.0118	.0118	.0055	.0033	.0063	.0085
1.7	.0108	.0111	.0051	.0031	.0057	.0080
1.8	.0099	.0105	.0047	.0030	.0052	.0075
1.9	.0090	.0099	.0042	.0029	.0048	.0070
2.0	.0081	.0094	.0036	.0028	.0045	.0066

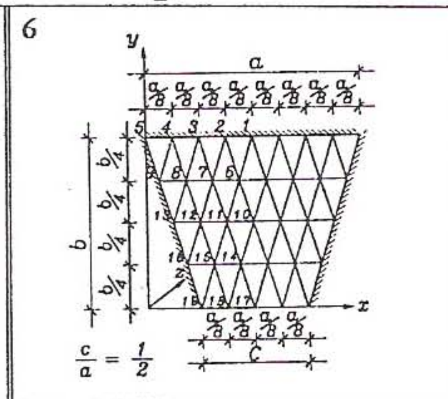
b) Totally Fixed Triangular Plate :

a/b	Case A				Case B				Case C			
	$M_x$max	$M_y$max	$M_y$min	$M_z$min	$M_x$max	$M_y$max	$M_y$min	$M_z$min	$M_x$max	$M_y$max	$M_y$min	$M_z$min
	+ ve	+ ve	- ve	- ve	+ ve	+ ve	- ve	- ve	+ ve	+ ve	- ve	- ve
0.5	.0187	.0089	.0210	.0378	.0046	.0007	.0036	.0085	.0141	.0082	.0174	.0293
0.6	.0169	.0098	.0219	.0335	.0053	.0022	.0042	.0087	.0116	.0076	.0177	.0248
0.7	.0150	.0100	.0220	.0297	.0053	.0028	.0044	.0087	.0097	.0072	.0176	.0210
0.8	.0131	.0096	.0215	.0264	.0048	.0028	.0046	.0086	.0083	.0068	.0169	.0178
0.9	.0114	.0089	.0206	.0234	.0043	.0024	.0046	.0081	.0071	.0065	.0160	.0153
1.0	.0098	.0082	.0196	.0207	.0034	.0020	.0046	.0071	.0061	.0062	.0150	.0136
1.1	.0087	.0076	.0184	.0183	.0034	.0018	.0044	.0063	.0053	.0058	.0140	.0120
1.2	.0077	.0071	.0173	.0162	.0031	.0017	.0043	.0056	.0046	.0054	.0130	.0106
1.3	.0069	.0067	.0163	.0145	.0029	.0017	.0042	.0051	.0040	.0050	.0121	.0094
1.4	.0061	.0063	.0153	.0131	.0026	.0017	.0041	.0048	.0035	.0046	.0112	.0083
1.5	.0055	.0059	.0144	.0120	.0024	.0017	.0041	.0046	.0031	.0042	.0103	.0074
1.6	.0049	.0056	.0136	.0111	.0021	.0017	.0041	.0044	.0028	.0038	.0095	.0067
1.7	.0044	.0053	.0128	.0103	.0019	.0017	.0040	.0042	.0025	.0035	.0088	.0061
1.8	.0039	.0050	.0120	.0096	.0017	.0017	.0038	.0040	.0022	.0033	.0082	.0056
1.9	.0034	.0048	.0112	.0091	.0015	.0017	.0035	.0039	.0019	.0031	.0077	.0052
2.0	.0029	.0045	.0103	.0087	.0013	.0016	.0025	.0038	.0016	.0029	.0072	.0049

Bending moments = coeff. q .a^2 /64 , $\nu = 0.16$, $b = (a-c)\dfrac{\sqrt{3}}{2}$ (continu)

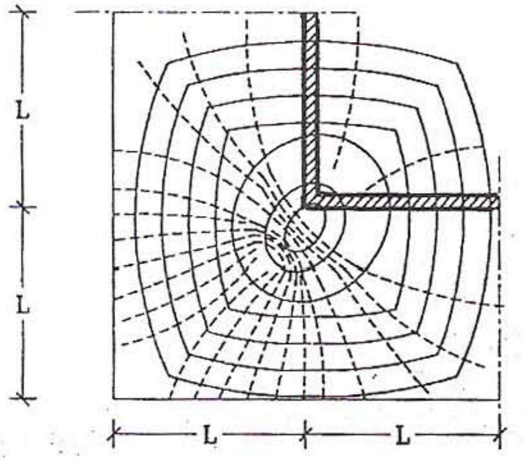




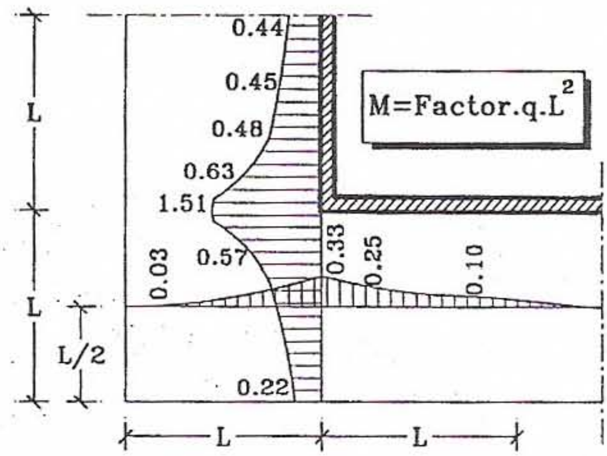


M at point	Shape of loading			Shape of loading			Shape of loading		
	A	B	C	A	B	C	A	B	C
M_{x6}	.1318	.0488	.0830	.1038	.0415	.0623	.3174	.1195	.1979
M_{x7}	.0791	.0161	.0630	.0915	.0301	.0614	.1830	.0539	.1291
M_{x8}	-.0303	-.0318	.0015	.0039	.0162	.0201	-.0596	-.0455	.0141
M_{x10}	.3567	.1679	.1888	.2145	.1100	.0145	.5045	.2272	.2773
M_{x11}	.2738	.1201	.1537	.2077	.1047	.1030	.3853	.1618	.2239
M_{x12}	-.0469	-.0604	.0135	.0617	.0219	-.0398	-.0332	-.0332	0
M_{x14}	.3777	.2041	.1736	.1325	.0815	.0510	.4658	.2367	.2291
M_{x15}	.0921	.0411	.0510	.1040	.0699	.0341	.0796	.0399	.0397
M_{x17}	.2272	.1418	.0854	.	.	.	.2596	.1538	.1058
M_{x18}	.1321	.0935	.0386	.	.	.	.1347	.0945	.0402
M_{y6}	.1285	-.0105	.1390	.2022	.0397	.1625	.6200	.1728	.4472
M_{y7}	.1466	.0022	.1444	.1957	.0408	.1549	.5169	.1805	.3764
M_{y8}	.1313	.0167	.1146	.1472	.0321	.1151	.2998	.0755	.2243
M_{y10}	.5381	.2171	.3210	.4942	.245	.249	.6688	.2841	.3847
M_{y11}	.4697	.1892	.2805	.4316	.2133	.2183	.6168	.2437	.3731
M_{y12}	.2679	.1180	.1499	.2390	.1211	.1179	.3028	.1225	.1809
M_{y14}	.4266	.2521	.1745	.1730	.1465	.0265	.4276	.2510	.1757
M_{y15}	.2533	.1296	.1237	.0743	.0848	-.0105	.1525	.1148	.0377
M_{y17}	.0388	.0931	-.0543	.	.	.	-.0937	.0618	-.1055
M_{y18}	-.0397	.0376	-.0773	.	.	.	-.1218	.0073	-.1291
M_{z9}	-.1561	-.0443	-.1118	-.1486	-.0504	-.0982	-.2762	-.0866	-.1896
M_{z13}	-.5359	-.2003	-.3356	-.4553	-.1949	-.2604	-.8190	-.3020	-.5170
M_{z16}	-.8687	-.4049	-.4638	-.5744	-.2996	-.2748	-.1583	-.5111	-.6472
M_{z19}	-.7973	-.4219	-.3754	-.2712	-.1356	-.1356	-.1110	-.5220	-.4890
M_{z21}	-.2989	-.1905	-.1184	.	.	.	-.3460	-.1980	-.1480
M_{x9}	-.1236	-.0351	-.0885	-.1176	-.0399	-.777	-.2187	-.0686	-.1501
M_{x13}	-.4240	-.1584	-.2656	-.3605	-.1543	-.2062	-.7590	-.3497	-.4093
M_{x16}	-.6876	-.3205	-.3671	-.4546	-.2371	-.2175	-.9170	-.8046	-.5124
M_{x19}	-.6575	-.3604	-.2971	-.2712	-.1356	-.1356	-.8003	-.4132	-.3871
M_{x21}	-.2989	-.1805	-.1184	.	.	.	-.3860	-.1980	-.1480
M_{y9}	-.0585	-.0166	-.0419	-.0557	-.0189	-.0368	-.1032	-.0325	-.0707
M_{y13}	-.2009	-.0751	-.1258	-.1708	-.0731	-.0977	-.3595	-.1656	-.1939
M_{y16}	-.3257	-.1518	-.1739	-.2153	-.1123	-.1030	-.4344	-.1917	-.2427
M_{y19}	-.3115	-.1707	-.1408	.	.	.	-.3794	-.1960	-.1834
M_{y20}	-.8347	-.4993	-.3354	.	.	.	-.9740	-.5510	-.4230
M_{y21}	-.2989	-.1805	-.1184	.	.	.	-.3460	-.1980	-.1418
M_{y1}	-1.070	-.411	-.6588	-.8431	-.3562	-.4869			
M_{y2}	-.9184	-.3417	-.5767	-.7470	-.3083	-.4387			
M_{y3}	-.5397	-.1806	-.3591	-.4740	-.1806	-.2934			
M_{y4}	-.1561	-.0443	-.1118	-.1486	-.0504	-.0982			
M_{y5}	0	0	0	0	0	0			

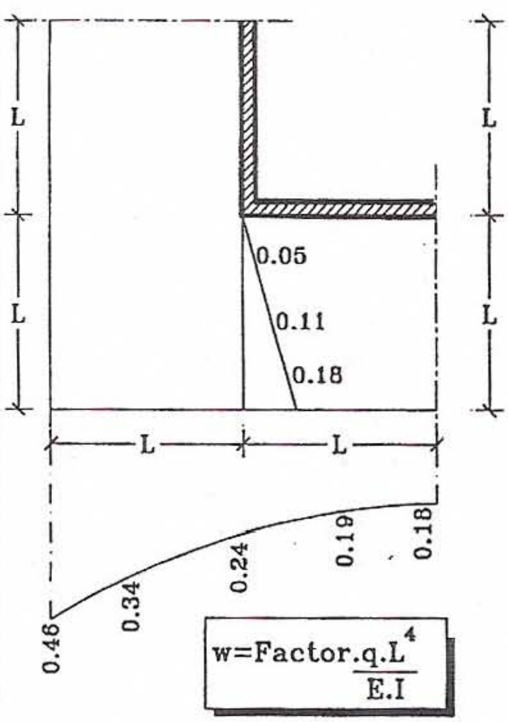
CORNER CANTILEVER SLABS



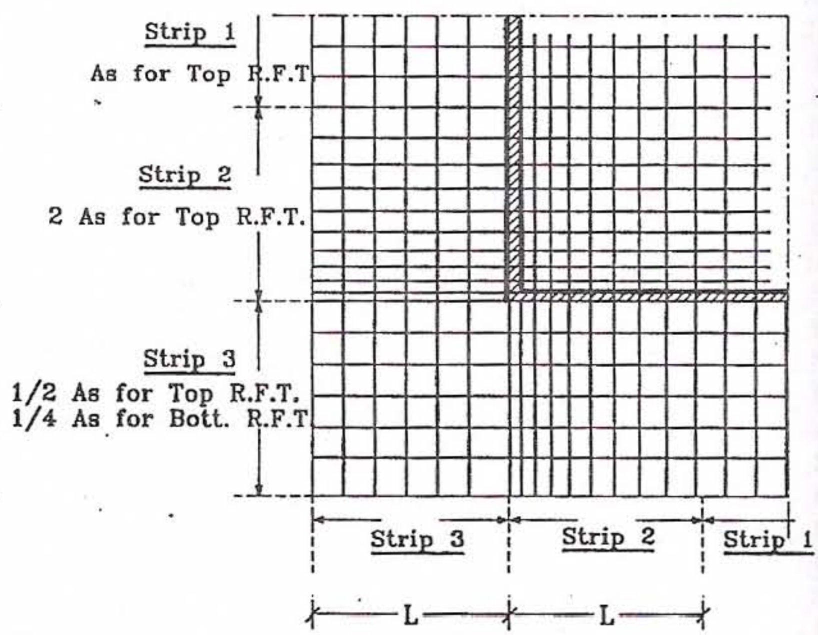
Stress Trajectories



B.M.D Under Uniform Load



Deflection Values



Details Of R.F.T.

CHAPTER V.

CIRCULAR BEAMS, CIRCULAR PLATES AND SURFACES OF REVOLUTIONS

V.I. *CIRCULAR BEAMS*

a) *Circular beam subject to uniform Load p and supported symmetrically on n supports:*

$$\varphi_O = \frac{\pi}{n}, \quad \text{Reaction} = \frac{2\pi r}{n} p$$

Max. shearing force to the right or left of any support :

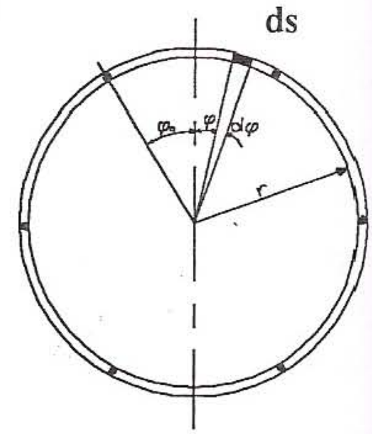
$$Q_{max} = \pm \frac{\pi r p}{n}$$

The bending moment M, the torsional moment M_t and the shearing force Q in any section at an angle φ from the center Line between two successive supports, are given by:

$$M = r^2 p \left(\frac{\pi \cos \varphi}{n \sin \varphi_0} - 1 \right)$$

$$M_t = -r^2 p \left(\frac{\pi \sin \varphi}{n \sin \varphi_0} - \varphi \right)$$

$$Q = -r p \varphi$$



The following table gives the reactions, the maximum shearing forces, bending moments and torsional moments in a circular beam of radius r supported symmetrically on n supports and subject to a total, uniformly distributed , Load P:

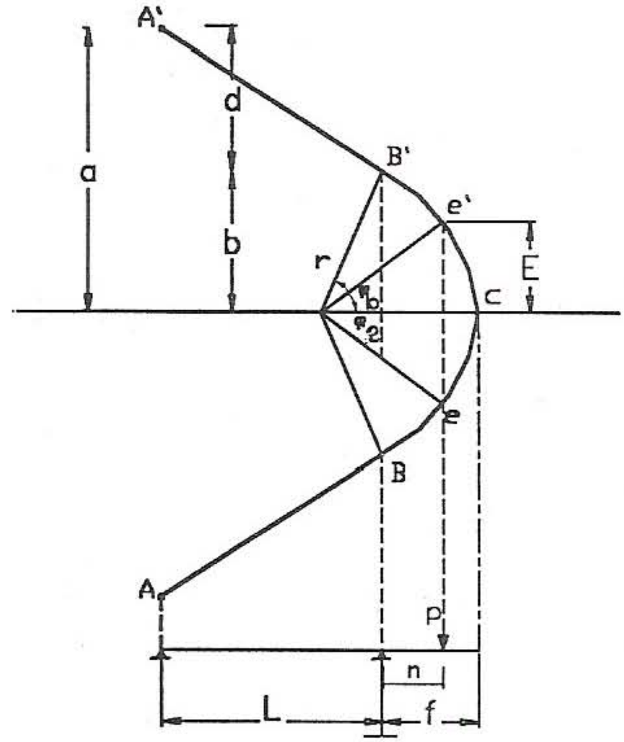
$$P = 2\pi r p$$

Number of supp. n	Load on each column R	Max. Shearing force $Q_{max.}$	Max. Bending Moment at center of each span M(+)	Max. Bending Moment Over center line of support M(-)	Max. torsional moment M_t	Central angle between axis of support & sec. of max. M_t
4	P/4	P/8	.0176 Pr	-.0322 Pr	.0053 Pr	19° 21'
6	P/6	P/12	.0075 Pr	-.0148 Pr	.0015 Pr	12° 44'
8	P/8	P/16	.0042 Pr	-.0083 Pr	.0006 Pr	9° 33'
10	P/10	P/20	.0032 Pr	-.0052 Pr	.0004 Pr	7° 38'
12	P/12	P/24	.0019 Pr	-.0037 Pr	.0002 Pr	6° 21'

Cantilever Circular Beam Symmetrically Loaded.

Due to symmetry, the shearing force Q and the torsional moment M_t at c are equal to zero, thus :

$Q_c = 0$ and $M_{tc} = 0$



For a single concentrated Load

P acting at c

Reactions :

$$R_A = -\frac{Pf}{2L} \quad \text{and} \quad R_B = \frac{P}{2}\left(1+\frac{f}{L}\right)$$

Bending moment at c

$$M_c = \frac{Pb}{2}\left(1 - \frac{df}{bL}\right)$$

For two equal concentrated load P acting at e and e`

$$R_A = -\frac{Pn}{L} \quad \text{and} \quad R_B = P\left(1 + \frac{n}{L}\right)$$

$$M_c = Pb\left(1 - \frac{m}{b} - \frac{nd}{bL}\right)$$

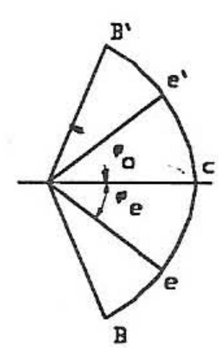
For a uniform load p/m on circular part B B`

$$R_A = -p\,r\,\varphi_0 \frac{s}{L} \quad \text{and} \quad R_B = p\,r\,\varphi_0\left(1 + \frac{s}{L}\right)$$

$$M_c = p\,b\,r\left[\varphi_0\left(1 - \frac{sd}{Lb}\right) - \frac{f}{b}\right]$$

in which s = distance of center of gravity of arch from B B`

$$\text{and } s = (\sin\varphi_0 - \varphi_0\cos\varphi_0) \cdot \frac{r}{\varphi_0}$$



C) Totally Fixed Cantilever Circular Beam Symmctrically Loaded

Due to symmetry $Q_c = 0$ and $M_{tc} = 0$ This beam is once statically indeterminate , the statically indeterminate value M_c can be determined as follows :

$$M_c = - \frac{\alpha_0}{\alpha_1} \text{ in which}$$

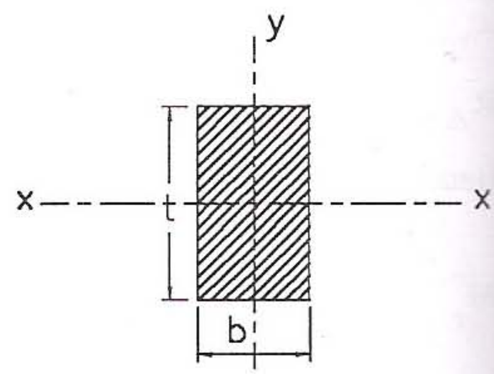
$$\alpha_0 = 2r \int_0^{\varphi_0} (M_0 \cos\varphi - M_{t0} \, x \sin\varphi) \, d\varphi \text{ and}$$

$$\alpha_1 = 2r \int_0^{\varphi_0} (\cos^2\varphi + x \sin^2\varphi) \, d\varphi =$$

$$\frac{r}{2} \left[2(x+1)\varphi_0 - (x-1)\sin 2\varphi_0 \right]$$

For half a circle $\varphi_0 = \frac{\pi}{2}$ and $\alpha_1 = \frac{r\pi}{2}(x+1)$ in which

$x = \frac{E}{G} \cdot \frac{I_y}{I'}$ and



E = modulus of elasticity

G = modulus of rigidity $\frac{E}{2(1+\nu)}$

ν = poissons ratio = $\frac{1}{6} \div \frac{1}{5}$ i.e.

E/G = 2.4

I_y = moment of inertia about y - y = $\frac{t\,b^3}{12}$

I'. = torsion modulus

For a rectangular section with $\frac{t}{b} = n > 1$ $\;I' = n \, \psi \, b^4$

In the following table, the values of ψ are given as a factor of $n = \frac{t}{b}$

n	1	1.5	2	3	4	6	8	10	∞
ψ	0.140	0.196	0.229	0.263	0.281	0.298	0.307	0.312	0.333

Therefore $\chi = 2.4 \, \frac{t\,b^3}{12} \cdot \frac{b}{t\,\psi\,b^4} = \frac{1}{5\,\psi}$ and can be extracted from the following table :

n	1	1.5	2	3	4	6	8	10	∞
χ	1.425	1.020	0.875	0.760	0.711	0.670	0.651	0.640	0.600

For a single concentrated Load P at c we get :

$$M_c = -P\,r\,.\,\frac{2\,\frac{\chi}{\chi-1}\,(\cos\varphi_0 - 1) + \sin^2\varphi_0}{2\,\frac{\chi+1}{\chi-1}\,\varphi_0 - \sin 2\varphi_0}$$

$$M_a = -\frac{P}{2}\,r\sin\varphi_0 + M_c\cos\varphi_0$$

$$M_t = \frac{p}{2}\,r\,(1 - \cos\varphi_0) - M_c\sin\varphi_0$$

For a uniform Load p/m`

$$M_c = +p\,r^2\,\frac{\frac{\chi+1}{\chi-1}(4\sin\varphi_0 - 2\varphi_0) - \frac{4\chi}{\chi-1}.\,\varphi_0.\,\cos\varphi_0 + \sin 2\varphi_0}{2\,\frac{\chi+1}{\chi-1}.\varphi_0 - \sin 2\varphi_0}$$

$$M_a = -p\,r^2\,(1 - \cos\varphi_0) + M_c\cos\varphi_0$$

$$M_t = p\,r^2.\,(\varphi_0 - \sin\varphi_0) - M_c\sin\varphi_0$$

If the beam is a fixed cantilever half circle, we get :

For a single concentrated Load P at c :

$$M_c = \frac{P\,r}{\pi} \qquad M_a = -\frac{p\,r}{2} \qquad M_{ta} = \frac{p\,r}{\pi}\left(\frac{\pi}{2} - 1\right)$$

For two equal concentrated Loads P at e and e`

$$M_c = \frac{-2\,P\,r}{\pi}\left[\cos\varphi_e - \left(\frac{\pi}{2} - \varphi_e\right)\sin\varphi_e\right]$$

$$M_a = -p\,r\sin\left(\frac{\pi}{2} - \varphi_e\right)$$

$$M_{ta} = \frac{2\,p\,r}{\pi}\left(\frac{\pi}{2} - \cos\varphi_e - \varphi_e\sin\varphi_e\right)$$

For a uniform Load p/m`

$$M_c = p\,\frac{r^2}{\pi}\,(4 - \pi) = 0.274\,p\,r^2$$

$$M_a = -p\,r^2$$

$$M_{ta} = p\,r^2\left(\frac{\pi}{2} - \frac{4}{\pi}\right) = 0.3\,p\,r^2$$

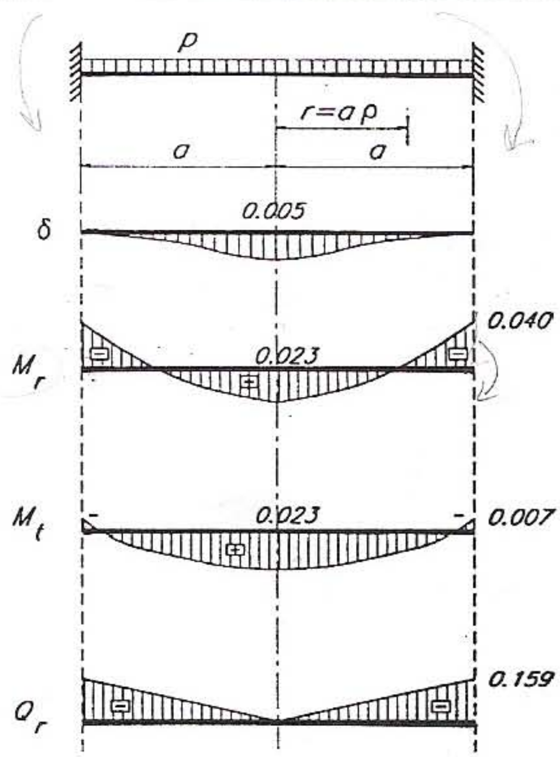
V.2. 1. *Solid Circular Plates Subjecteed To Uniform Load :-*

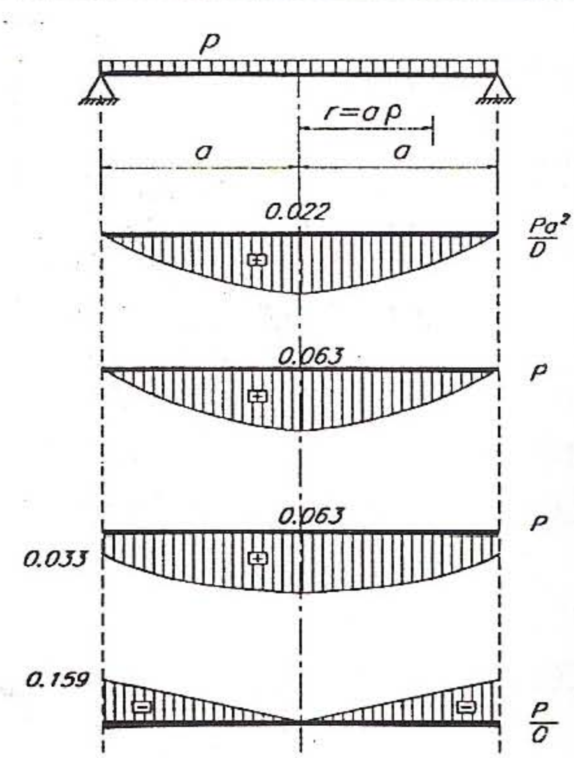
Moments in circular slab with fixed edge:-

$M = \text{coeff.} \times pR^2$

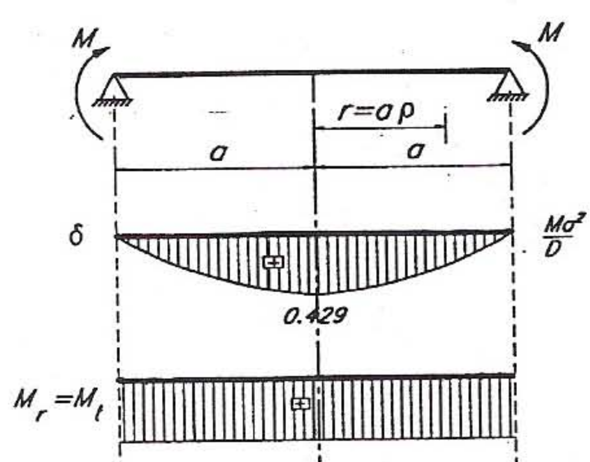
Positive sign indicates compression surface loaded

at point	0.00R	0.10R	0.20R	0.30R	0.40R	0.50R	0.60R	0.70R	0.80R	0.90R	1.00R
Mr	+.075	+.073	+.067	+.057	+.043	+.003	+.003	-.023	-.053	-.087	-.125
Mt	+.075	+.074	+.071	+.066	+.059	+.050	+.039	+.026	+.011	-.006	-.025





V.2. 2. *Solid Circular Plates Subjecteed To Single Edge Moment:-*



V.2.2 SOLID CIRCULAR PLATES SUBJECTED TO UNIFORM PARTIAL LOADS

Bending Moment = Coeff. x pb^2

r/a \ b/a	Radial Moment M_r								Tangential Moment M_φ							
	0	0.1	0.2	0.4	0.5	0.6	0.8	1.0	0	0.1	0.2	0.4	0.5	0.6	0.8	1.0
0	00	.825	.650	.469	0.408	.355	.266	.188	∞	.825	.650	.469	0.408	.355	.266	.188
.1	.576	.637	.603	.457	0.400	.350	.263	.186	.826	.762	.634	.465	0.405	.353	.265	.187
.2	.402	.417	.462	.422	0.378	.334	.254	.180	.652	.636	.588	.453	0.348	.348	.262	.185
.4	.229	.232	.242	.282	0.288	.272	.219	.158	.479	.475	.461	.407	0.368	.327	.250	.178
.6	.127	.129	.132	.145	0.156	.168	.160	.120	.377	.375	.368	.340	0.319	.293	.231	.165
.8	.056	.056	.057	.061	0.065	.068	.078	.068	.306	.304	.299	.280	0.266	.248	.203	.148
1.	0	0	0	0	0	0	0	0	.250	.249	.245	.230	0.218	.205	.170	.125

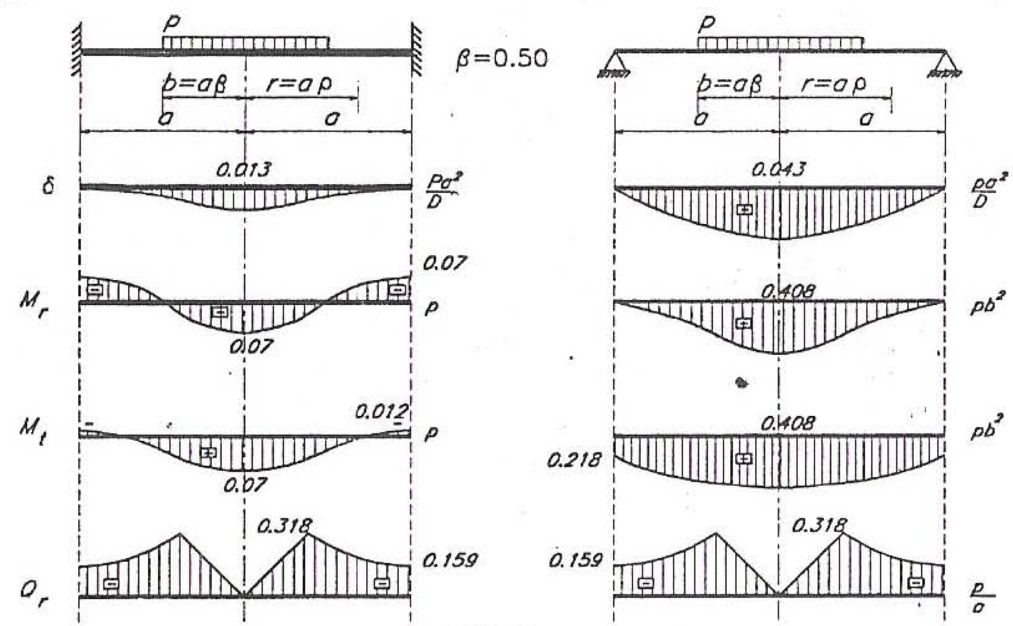
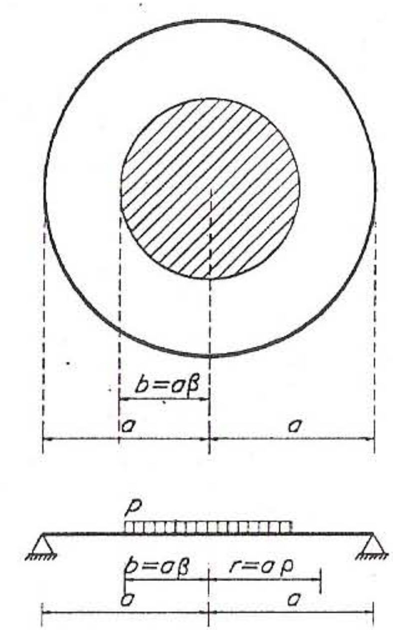


Fig.3



V.2.3. SOLID CIRCULAR PLATES SUBJECTED TO RING LOAD

b) Bending Moment = Coeff. x pb

b/a → / r/a ↓	Radial Moment M_r								Targenlial Moment M_φ							
	0	0.1	0.2	0.4	0.5	0.6	0.8	1.0	0	0.1	.2	0.4	0.5	0.6	0.8	1.0
0	∞	1.40	1.05	.668	0.534	.415	.202	.0	∞	1.40	1.05	.668	0.534	.415	.202	0
.1	1.15	1.40	1.05	.668	0.534	.415	.202	0	1.65	1.40	1.05	.668	0.534	.415	.202	0
.2	.805	.865	1.05	.668	0.534	.415	.202	0	1.31	1.24	1.05	.668	0.534	.415	.202	0
.4	.458	.471	.511	.668	0.534	.415	.202	0	.958	.940	.886	.668	0.534	.415	.202	0
.6	.255	.260	.273	.326	0.366	.415	.202	0	.755	.746	.718	.604	0.519	.415	.202	0
.8	.112	.113	.117	.134	0.147	.162	.202	0	.612	.605	.656	.509	0.451	.387	.202	0
.1	0	0	0	0	0	0	0	0	.500	.495	.480	.420	0.375	.320	.180	0

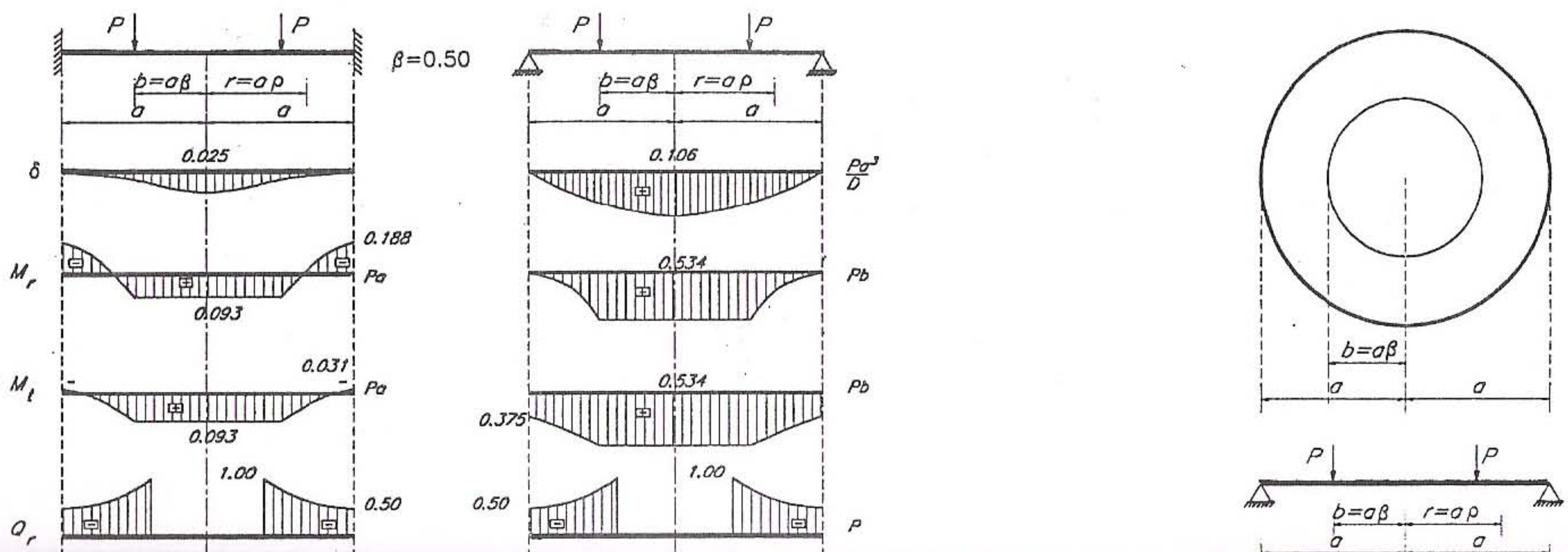
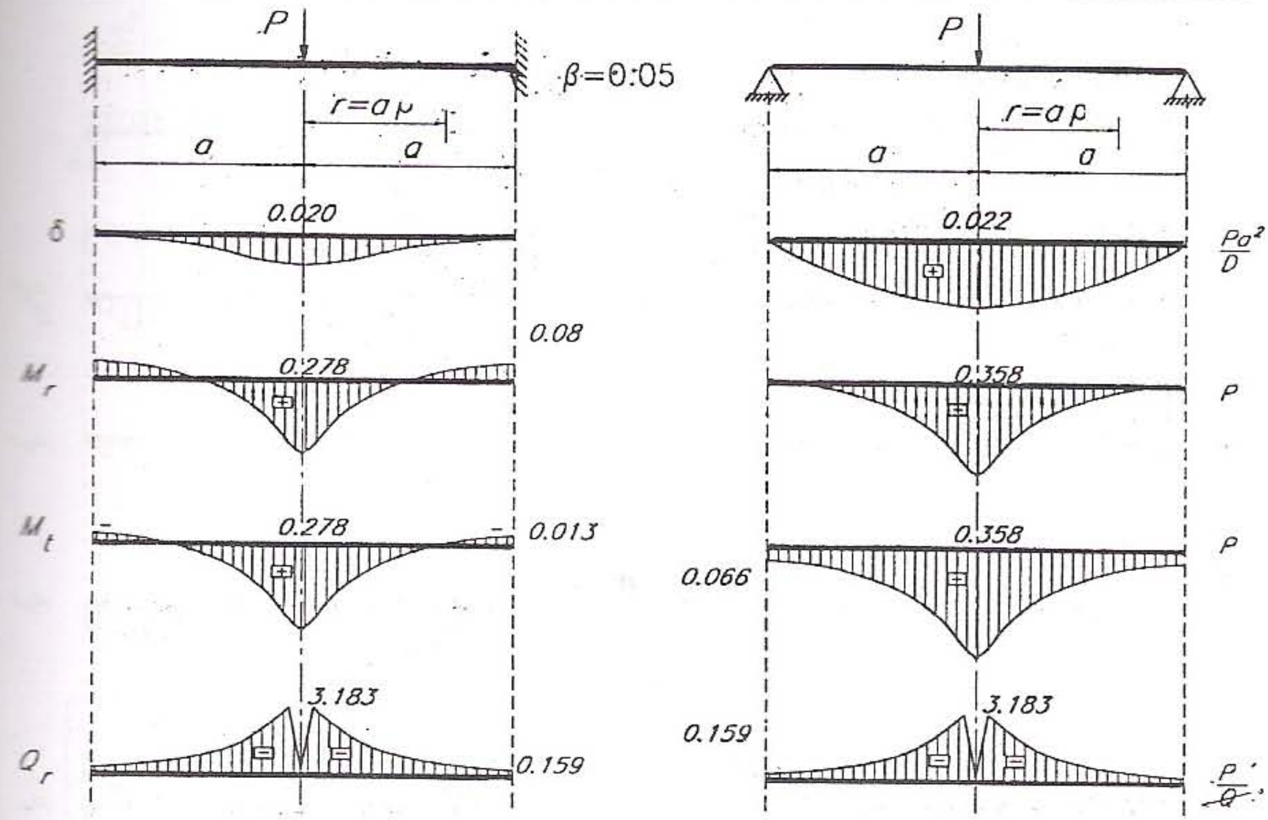
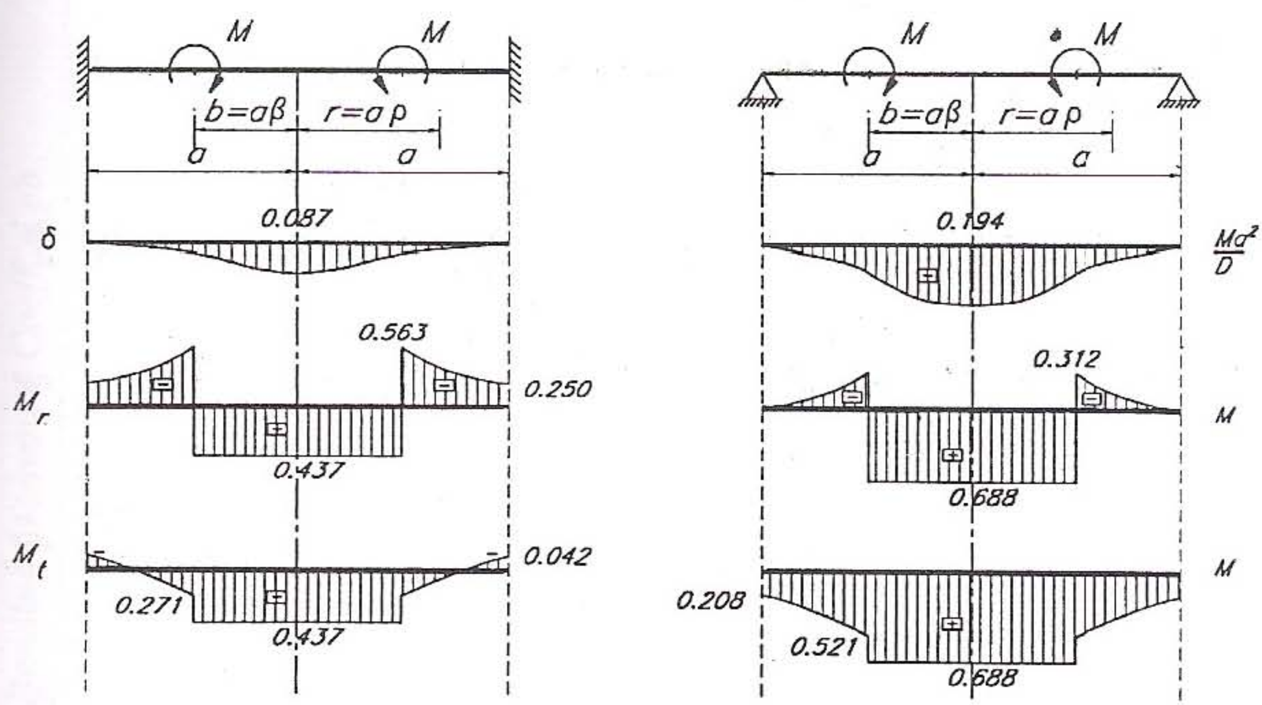


Fig.4

V.2.4. Solid Circular Plates Subjected Single concentrated load :-



V.2.5. Solid Circular Plates Subjected to edg moment :-



V.2.6. Circular Plates with circular hole subjecteed to uniform load :-

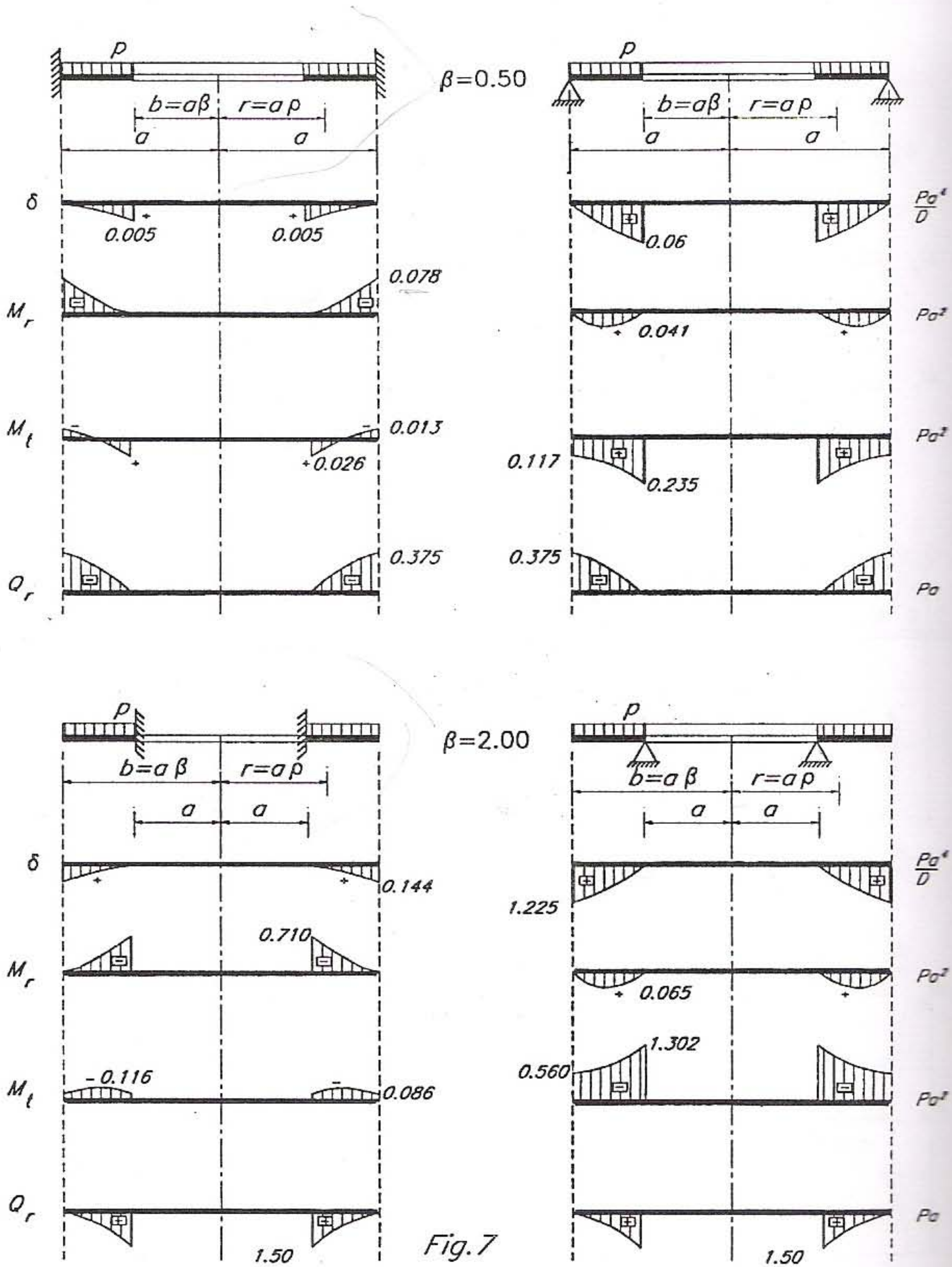
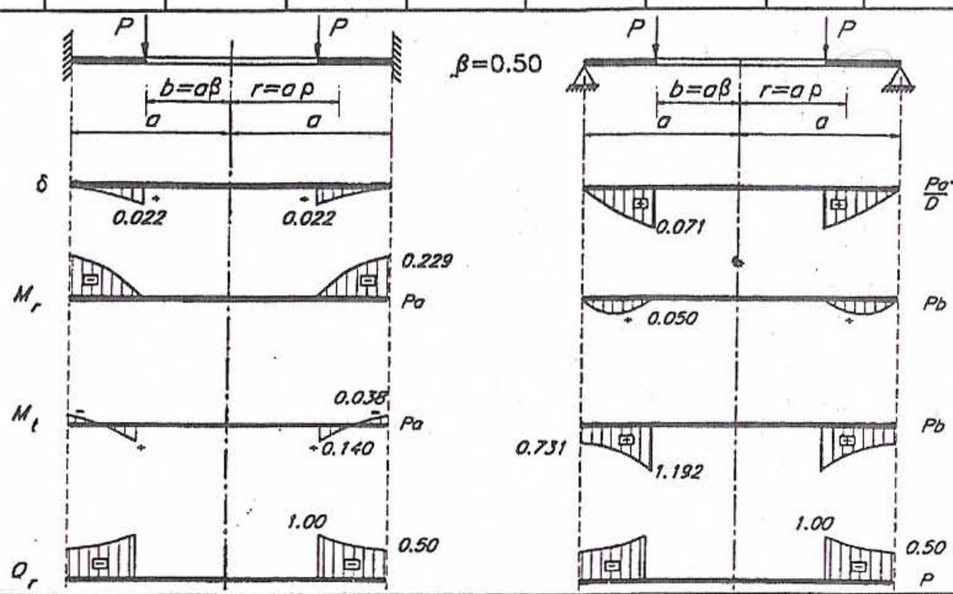


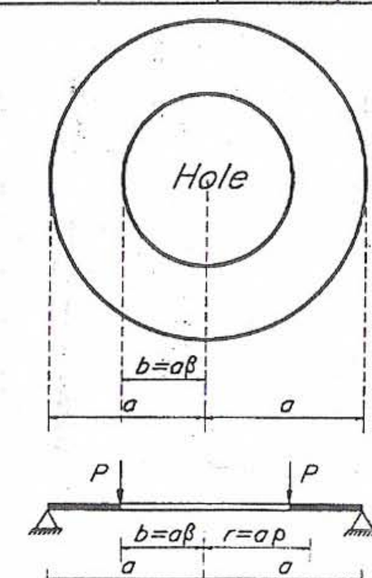
Fig.7

V.2.7 SIMPLY SUPPORTED CIRCULAR PLATE WITH CIRCULAR HOLE SUBJECTED TO EDGELOAD

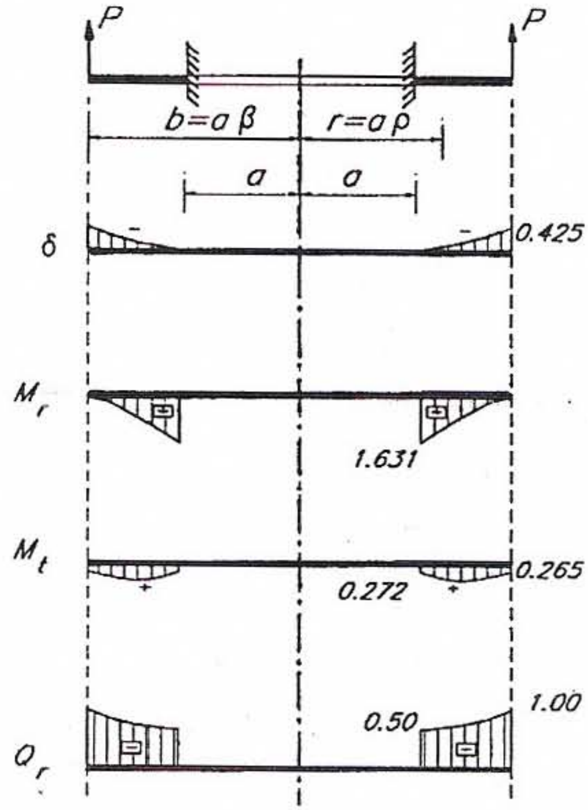
Bending Moment = Coeff. x pb

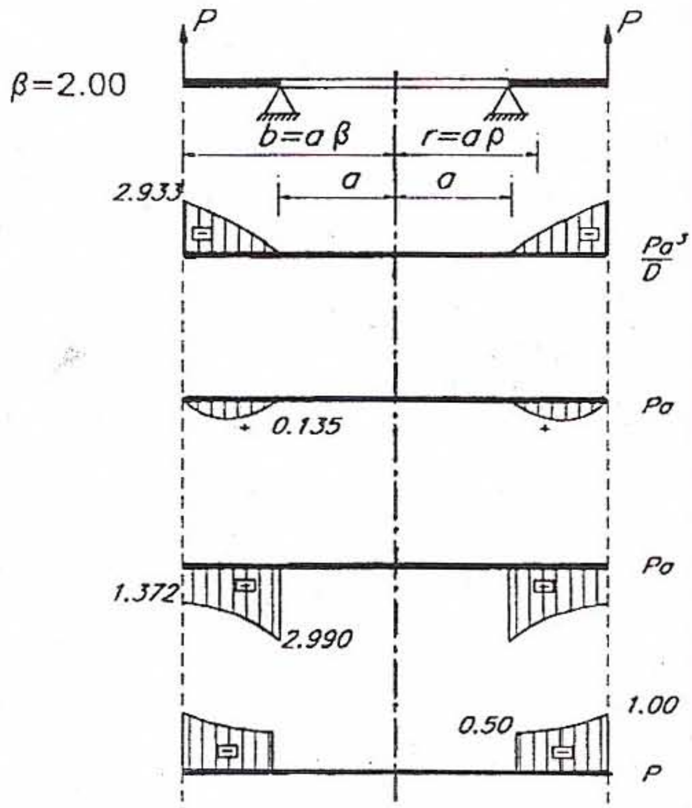
b/a \ r/a	M_r 0	M_r 0.1	M_r 0.2	M_r 0.4	M_r 0.5	M_r 0.6	M_r 0.8	M_r 1.0	M_φ 0	M_φ 0.1	M_φ 0.2	M_φ 0.4	M_φ 0.5	M_φ 0.6	M_φ 0.8	M_φ 1.0
0	∞								∞							
0.1	1.15	0							1.65	2.83						
0.2	.805	.256	0				.p.b		1.31	1.61	.218					
0.4	.458	.397	.282	0					.958	1.04	1.20	1.59				
0.6	.255	.235	.196	.100	0.050	0			.755	.780	.882	.108	1.192	1.30		
0.8	.112	.105	.093	.063	0.046	.031	0		.612	.641	.698	.835	0.908	.980	1.12	
1.0	0	0	0	0	0	0	0	0	.500	.523	.567	.675	0.731	.787	.897	-



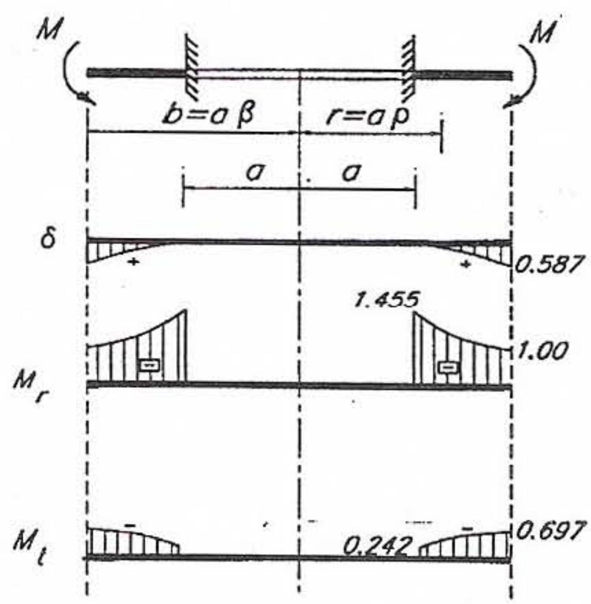


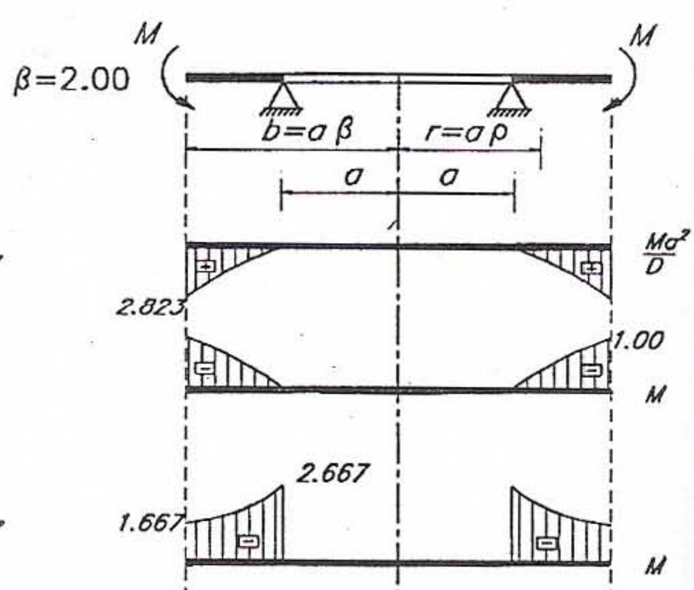
V.2.8. *Circular Plates with circular hole subjecteed to ring load :-*





V.2.9. *Circular Plates with circular hole subjected to edge moment :-*

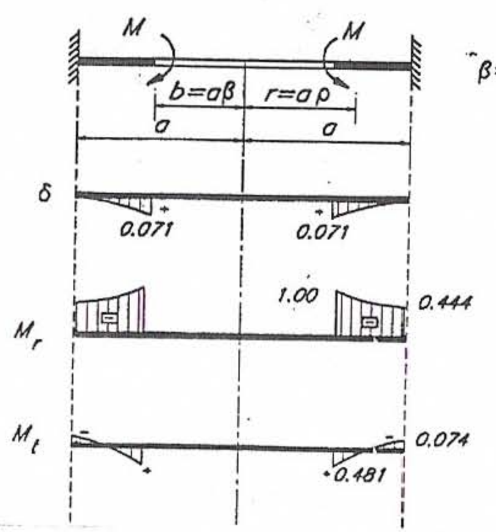




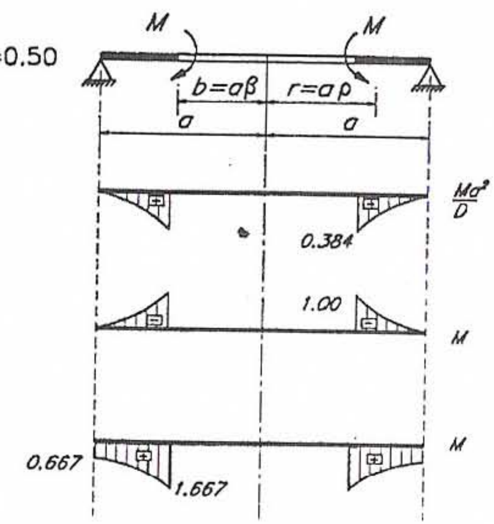
V.2.10 SIMPLY SUPPORTED CIRCULAR PLATE WITH CIRCULAR HOLE SUBJECTED TO EDGE MOMENT

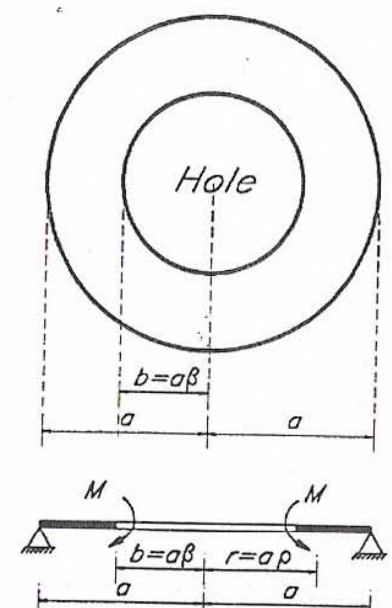
Bending Moment = Coeff. x M

r/a \ b/a	M_r								M_φ							
	0	0.1	0.2	0.4	0.5	0.6	0.8	1.0	0	0.1	0.2	0.4	0.5	0.6	0.8	1.0
0	0	∞							0	∞						
0.1	0	1.00							0	.102						
0.2	0	.242	1.00						0	.263	.108					
0.4	0	.053	.219	1.00					0	.073	.302	.138				
0.6	0	.018	.074	.338	0.593	1.00			0	.038	.157	.719	1.259	2.13		
0.8	0	.006	.023	.107	0.188	.316	1.00		0	.026	.107	.488	0.854	1.44	4.55	
1.0	0	0	0	0	0	0	0		0	.020	.083	.381	0.667	1.12	3.55	-



β=0.50

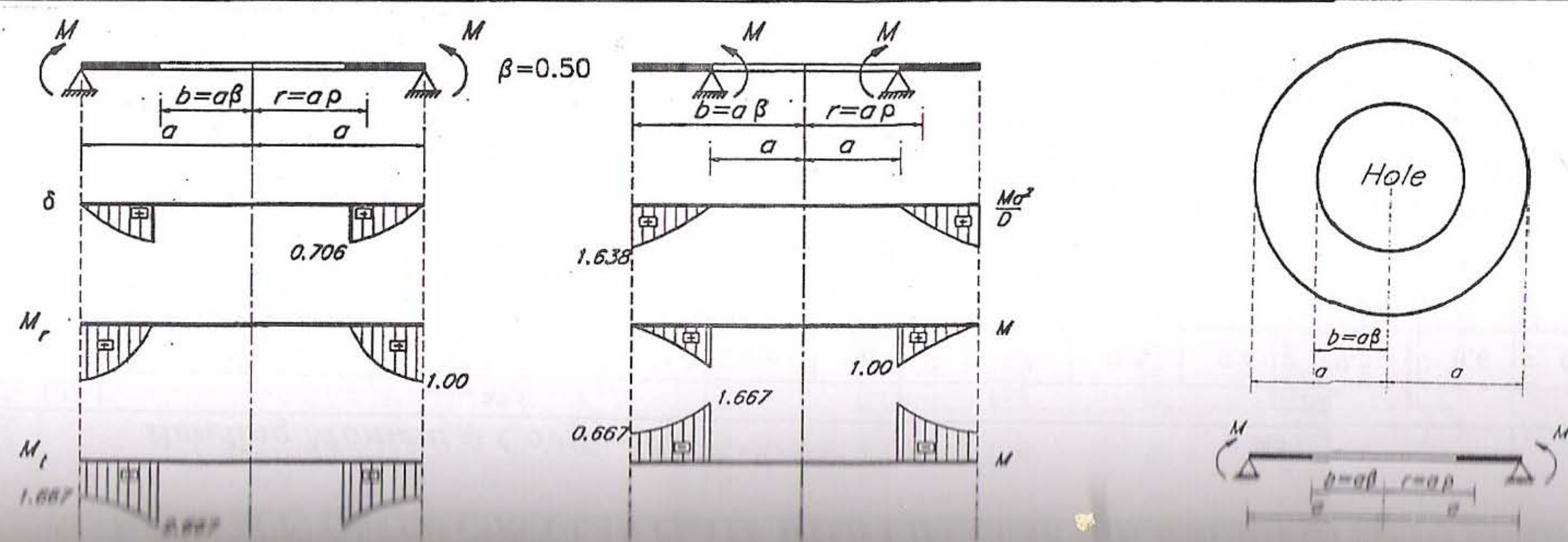




V.2.11 SIMPLY SUPPORTED CIRCULAR PLATE WITH CIRCULAR HOLE SUBJECTED TO EDGE MOMENT

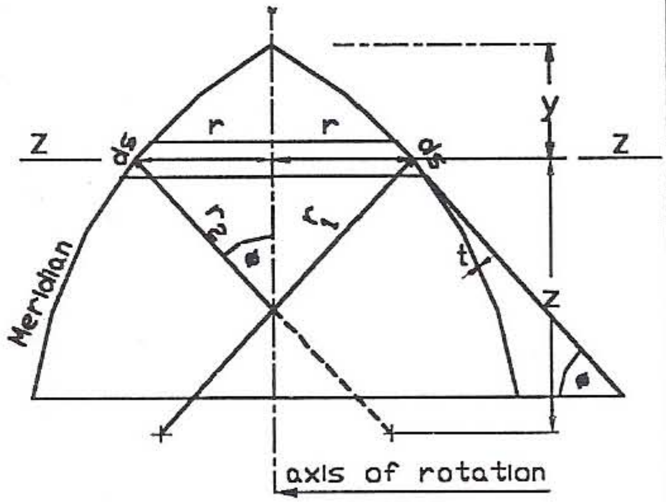
Bending Moment = Coeff. x M

r/a \ b/a	M_r 0	M_r 0.1	M_r 0.2	M_r 0.4	M_r 0.5	M_r 0.6	M_r 0.8	M_r 1.0	M_φ 0	M_φ 0.1	M_φ 0.2	M_φ 0.4	M_φ 0.5	M_φ 0.6	M_φ 0.8	M_φ 1.0
0	1.00								1.00							
0.1	1.00	0							1.00	2.02						
0.2	1.00	.757	0						1.00	1.26	2.08					
0.4	1.00	.947	.781	0					1.00	1.07	1.30	2.38				
0.6	1.00	.982	.926	.661	0.407				1.00	1.04	1.16	1.72	2.259	3.13		
0.8	1.00	.994	.977	.893	0.813	.684	0		1.00	1.03	1.11	1.49	1.854	2.44	5.55	
1.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.02	1.08	1.38	1.667	2.13	4.55	-



V.3.1 Internal Forces According to Membrane Theory

Notations :



r = radius normal to axis of revolution any Circular ring at any plane z - z

r_1 = radius of curvature of meridian

r_2 = cross radius of curvature along the normal to axis of rotation.

N_φ = resultant meridian force per unit length of circumference = $f_1.t$ where f_1 = meridian stress and t = thickness of shell

N_θ = resultant ring force per unit length of meridian. = $f_2.t$ where f_2 = ring stress.

H = Horizontal thrust of shell per unit length of circumference

W_φ = Sum of vertical forces above z - z (expressed through the angle φ).

P_r = Component of external distributed load normal to surface of revolution.

The meridian force N_φ can be given by $N_\varphi = W_\varphi / 2\pi r \sin\varphi$

And the ring force N_θ can be calculated from the relation $\dfrac{N_\varphi}{r_1} + \dfrac{N_\theta}{r_2} = p_r$

For spherical surface $r_1 = r_2 = a$ and then : $N_\varphi + N_\theta = a\,p_r$

For conical surface $r_1 = \infty$ and then : $N_\theta' = a.p_r$

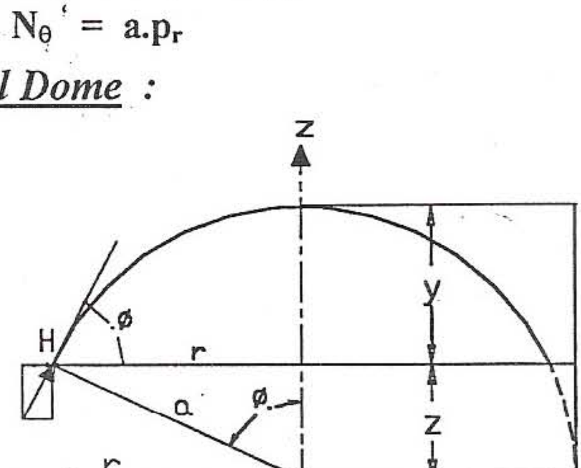
V.3.2. The Spherical Dome :

The relation between a , r & y is given by :

$$a = (r^2 + y^2)/2y$$

The surface area of a spherical dome is :

$$A = 2\pi\, a\, y$$ i.e. it is equal to the surface area of a cylinder having the same radius R and height y.



a) Internal Forces and Reactions due to Dead Load g/m2

The dead weight of a dome, height y, and included in a central angle φ is given by :

$$W_\varphi = g.A = g.2\pi\ a\ \ y = g.\ 2\pi\ a^2\ (1 - \cos\varphi)$$

The horizontal thrust $H = \dfrac{W\varphi}{2\pi r\ \tan\varphi} = \dfrac{ga\cos\varphi}{1+\cos\varphi} = ga\dfrac{z}{a+z}$

The meridian force $N_\varphi = \dfrac{H}{\cos\varphi} = g.\dfrac{a}{1+\cos\varphi} = g.\dfrac{a^2}{a+z}$

he ring force $N_\theta = a.p_r - N_\varphi = g.a\ (\cos\varphi - \dfrac{1}{1+\cos\varphi}) = g\ (z - \dfrac{a^2}{a+z})$

at the crown where : $\varphi = 0$, $\cos\varphi = 1$ and $z = a$, we get :

$$N_\varphi = H = g\frac{a}{2}\ \text{compression , and}$$

$$N_\theta = H = g\frac{a}{2}\ \text{compression}$$

at the foot of half spherical domes where $\varphi = 90°$, $\cos\varphi = o$ & $z = o$

we get : $N_\varphi = g\ a$ compression , $H = o$ and $N_\theta = - g\ a$ tension

$N_\theta = o$ when $\varphi = 51°\ 49'$ or $z = 0.618a$

$,y = 0.382\ a$, $r = 0.787\ a$

For $N_\theta = o$; we get the max. horizontal thrust

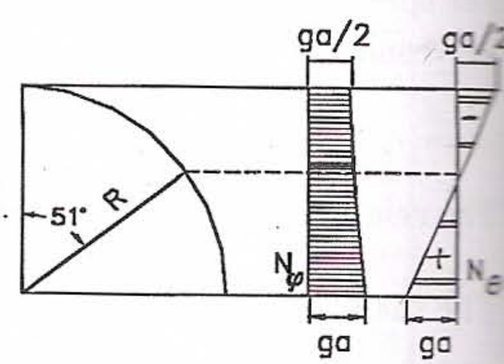
$H_{max} = 0.382\ g\ a$

The max. total horizontal thrust $\mathcal{H}_{max} = 2\pi r\ H_{max}$.

$= 2\pi\ 0.787\ a \times 0.382\ g\ a = 0.3\ (2\pi a^2 g)$, i.e.

is equal to 0.3 the total dead load on half THE he sphere.

The meridian and ring forces of spherical domes subjected to dead loads g/m^2 is shown in fig. and given in the following table :



φ	0	10	20	30	40	50	60	70	80	90	factors
N_φ	.500	.505	.516	.537	.566	.608	.667	.747	.838	1.0	-g a
N_θ	.500	.480	.425	.330	.200	.034	-.167	-.402	-.62	-1.0	-g a

Internal Forces and Reactions due to Live Load p/m2 On (Horizontal Projection)

The total load $W_\varphi = p.\ \pi.\ r^2 = p.\ \pi\ a^2 \sin^2 \varphi$

The horizontal thrust $H = \dfrac{W\varphi}{2\ \pi\ r\ \tan\varphi} = p\ a\ \dfrac{\cos\varphi}{2} = p.z/2$

The meridian force $N_\varphi = \dfrac{H}{\cos\varphi} = \dfrac{W\varphi}{2\ \pi\ r\ \sin\varphi} = p\ a\ /\ 2 = \text{const. (compr.)}$

i.e. the meridian forces due to p are constant in the dome.

The ring force $N_\theta = \dfrac{d(Hr)}{ds} = \dfrac{p\ a \cos 2\varphi}{2} = \dfrac{p}{2R}(2z^2 - a^2)$

at the crown : $z = a$ and then we get $N_\varphi = N_\theta = p\ a/2$ compression

at the foot of half spherical domes where $z = o$, we have :

$N_\theta = -\ p\ a\ /2$ tension and $N_\varphi = p\ a\ /2$ Compression

$N_\theta = 0$ where $2\ z^2 = a^2$ or $z = 0.707\ a$ which corresponds to

$\varphi = 45°$. i.e. $r = z = 0.707\ a$

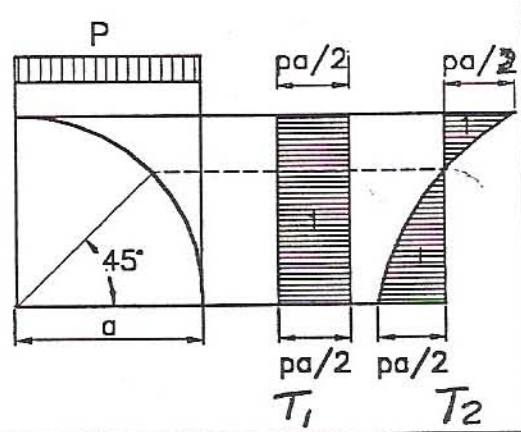
Introducing this value in equation of H, we get :

$H_{max} = p\ z/2 \qquad = 0.3535\ p\ a$

The max. total horizontal thrust is therefore given by :

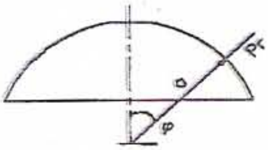
$\mathcal{H}_{max} = 2\ \pi \times 0.707\ a \times 0.3535\ p\ a = 0.5\ \pi\ a^2\ p$

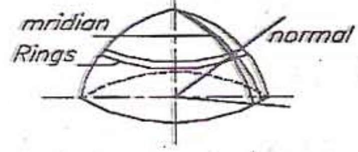
i.e. the max. total horizontal thrust of a spherical dome is equal to half the total live loads on half the sphere.



The meridian and ring forces, can accordingly be illustrated as shown in the above figure.

c) *General Cases of Spherical Domes* *

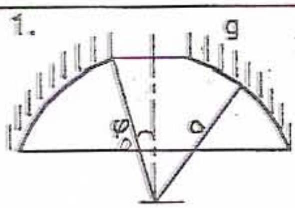
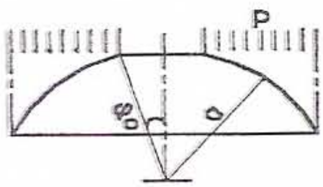
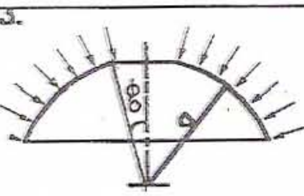
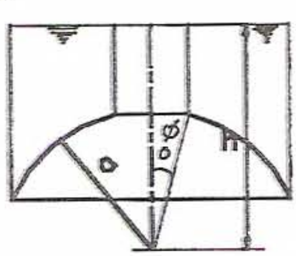




N_φ = Meridian force/m`

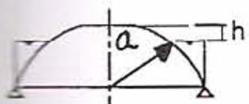
N_θ = Ring force /m`

T = Unit central shear

1.  D.L. g/m^2	Dead load g/m^2 surface : $p_r = g\cos\varphi$: $T = 0$ $N_\varphi = -g\,a\,\dfrac{\cos\varphi_0 - \cos\varphi}{\sin^2\varphi}$ $N_\theta = g\,a\left(\dfrac{\cos\varphi_0 - \cos\varphi}{\sin^2\varphi} - \cos\varphi\right)$ For $\varphi_o = 0$ (no vertex opening) : $N_\varphi = -g\,a\,\dfrac{1}{1+\cos\varphi}$; $N_\theta = g\,a\left(\dfrac{1}{1+\cos\varphi} - \cos\varphi\right)$
2.  L.L. p/m^2	Live Load p/m^2 horizontal $p_r = p\cos^2\varphi$; $T = 0$ $N_\varphi = -p\,\dfrac{a}{2}\left(1 - \dfrac{\sin^2\varphi_0}{\sin^2\varphi}\right)$; $N_\theta = p\,\dfrac{a}{2}\left(1 - \dfrac{\sin^2\varphi_0}{\sin^2\varphi} - 2\cos^2\varphi\right)$ For $\varphi_0 = 0$: $N_\varphi = -p\,\dfrac{a}{2}$; $N_\theta = -p\,\dfrac{a}{2}.\cos 2\varphi$
3.  $p_r = p$	$N_\varphi = -p\,\dfrac{a}{2}\left(1 - \dfrac{\sin^2\varphi_0}{\sin^2\varphi}\right)$; $N_\theta = -p\,\dfrac{a}{2}\left(4\ \dfrac{\sin^2\varphi_0}{\sin^2\varphi}\right)$ For $\varphi_0 = 0$: $N_\varphi = N_\theta = -p\,a/2$
4.  **Liquied press** $p_r = \gamma\,(h - a.\cos\varphi)$	$N_\varphi = -\gamma\,a^2\left[\dfrac{h}{2a}\left(1 - \dfrac{\sin^2\varphi_0}{\sin^2\varphi}\right) - \dfrac{\cos^3\varphi_0 - \cos^3\varphi}{3\sin^2\varphi}\right]$; $T = 0$ $N_\theta = -\gamma\,a\left[\dfrac{h}{2}\left(1 + \dfrac{\sin^2\varphi_0}{\sin^2\varphi}\right) + \dfrac{a}{3}\left(\dfrac{\cos^3\varphi_0 - \cos^3\varphi}{\sin^2\varphi} - 3\cos\varphi\right)\right]$ For $\varphi_o = 0$ (no vertex opening) : $N_\varphi = -\gamma\,a^2\left[\dfrac{h}{2a} - \dfrac{1}{3}\left(1 + \dfrac{\cos^2\varphi}{1+\cos\varphi}\right)\right]$ $N_\theta = -\gamma\,a^2\left[\dfrac{h}{2a} - \cos\varphi + \dfrac{1}{3}\left(1 + \dfrac{\cos^2\varphi}{1+\cos\varphi}\right)\right]$

c) General Cases of Spherical Domes* (continu.)

5



Liquid Pressure
$p_r = \gamma\,(a-h-a\cos\varphi)$

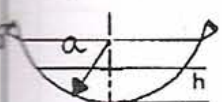
For points above the liquid level : $N_\varphi = N_\theta = 0$

For points below the liquid level :

$$N_\varphi = -\gamma\,\frac{a^2}{6}\left[\frac{h}{a}\left\{\frac{h}{a\sin^2\varphi}\left(3-\frac{h}{a}\right)-3\right\}+1-\frac{2\cos^2\varphi}{1+\cos\varphi}\right]$$

$$N_\theta = -\,a^2.\gamma\left(1-s\,\varphi-\frac{h}{a}\right)-N_\varphi$$

6



Liquid Pressure
$p_r = \gamma\,(a-h-a\cos\varphi)$

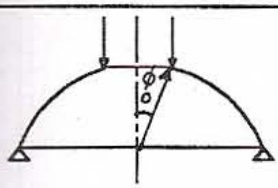
For points above the liquid level :

$$N_\varphi = \gamma.\frac{h^2}{6}\left(3-\frac{h}{a}\right)\frac{1}{\sin\varphi'}\;;\quad N_\theta = -\gamma\,\frac{h^2}{6}\left(3-\frac{h}{a}\right)\frac{1}{\sin^2\varphi'}$$

For points below the liquid level :

$$N_\varphi = \gamma\,\frac{a^2}{6}\left[\frac{3h}{a}-1+\frac{2\cos^2\varphi'}{1+\cos\varphi'}\right];\quad N_\theta = \gamma\,a^2\left[\frac{h}{a}-1+\cos\varphi'\right]-N_\varphi$$

7



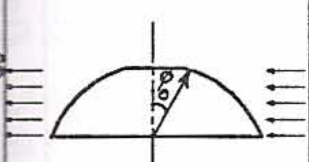
Edge load
p / m

$$N_\varphi = -\,p\,\frac{\sin\varphi_0}{\sin\varphi}\quad;\quad N_\theta = p\,\frac{\sin\varphi_0}{\sin^2\varphi}$$

For $\varphi_o = 0$ (no vertex opening; Single load P at the vertex):

$$N_\varphi = -\,P/2\pi a\sin^2\varphi\quad;\quad N_\theta = P/2\pi a\sin^2\varphi$$

8



Wind pressure.
$P_r = w.\sin\varphi\cos\varphi$

$$N_\varphi = -\,w.\frac{a\cos\theta\cos\varphi}{3\sin^3\varphi}.\left[3\left(\cos\varphi_0-\cos\varphi\right)-\left(\cos^3\varphi_0-\cos^3\varphi\right)\right]$$

$$N_\theta = w.\frac{3\cos\theta}{\sin^3\varphi}\left[\cos\varphi\left(3\cos\varphi_0-\cos^3\varphi_0\right)-3\sin^2\varphi-2\cos^4\varphi\right]$$

$$T = w\,\frac{a\sin\theta}{3\sin^3\varphi}\left[3\left(\cos\varphi_0-\cos\varphi\right)-\cos^3\varphi_0-\cos^3\varphi\right]$$

For $\varphi_o = 0$ (no vertex opening) :

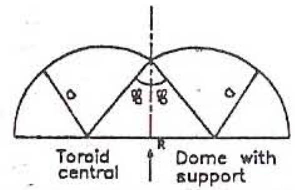
$$N_\varphi = w.\,\frac{a\cos\theta\cos\varphi}{\sin^3\varphi}\left(2-3\cos\varphi+\cos^3\varphi\right)$$

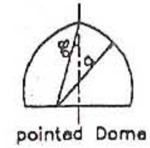
$$N_\theta = w.\,\frac{a\cos\varphi}{\sin^3\varphi}\left(2\cos\varphi-3\sin^2\varphi-2\cos^4\varphi\right)$$

$$T = w.\,\frac{a\sin\theta}{3\sin^3\varphi}\left(2-3\cos\varphi+\cos^3\varphi\right)$$

V.3.3. **TOROID SHELL**

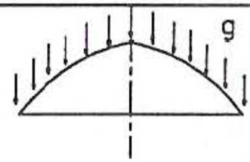
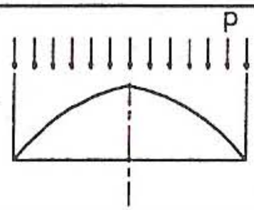
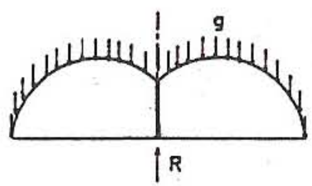
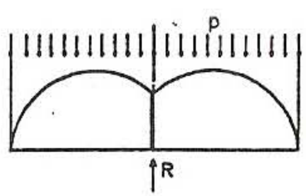
a) Ring axis bisects the cross-section :





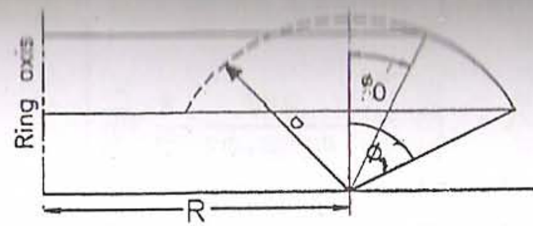
N_φ MERIDIAN FORCE/m´

N_θ RING FORCE/m´

System	*Loading*	N_φ	N_θ
	$p_x = g \sin\varphi$ $p_r = g \cos\varphi$	$-g.a \dfrac{\cos\varphi_o - \cos\varphi - (\varphi-\varphi_o)\sin\varphi_o}{(\sin\varphi - \sin\varphi_o)\sin\varphi}$	$-g.\dfrac{a}{\sin^2\varphi}.[(\varphi-\varphi_o)\sin\varphi_o - (\cos\varphi_o - \cos\varphi) + (\sin\varphi - \sin\varphi_o)\sin\varphi\cos\varphi]$
	$p_x = p \sin\varphi\cos\varphi$ $p_r = p. \cos^2\varphi$	$-p.\dfrac{a}{2}(1 - \dfrac{\sin\varphi_o}{\sin\varphi})$	$-p.\dfrac{a}{2}.[\cos 2\varphi + 2\sin\varphi\sin\varphi_o - \dfrac{\sin^2\varphi_o}{\sin^2\varphi}]$
	$p_x = g \sin\varphi$ $p_r = g \cos\varphi$ $R = 2g\pi (a\, \varphi_o \sin\varphi_o - 2\sin^2.\dfrac{\varphi_o}{2})$	$-g.a.\dfrac{1 - \cos\varphi + \varphi\sin\varphi_o}{\sin\varphi(\sin\varphi + \sin\varphi_o)}$	$-g.a[\cos\varphi - \dfrac{1-\cos\varphi}{\sin^2\varphi} + \sin\varphi_o(ctg\varphi - \dfrac{\varphi}{\sin^2\varphi})]$
	$p_x = p \sin\varphi \cos\varphi$ $p_r = p \cos^2\varphi$ $R = p\,\pi\, a^2 . \sin^2\varphi_o$	$-p.\dfrac{a}{2}. \dfrac{\sin\varphi + 2\sin\varphi_o}{2\sin\varphi + \sin\varphi_o}$	$-p.\dfrac{a}{2}. (\cos 2\varphi - 2\sin\varphi\sin\varphi_o)$

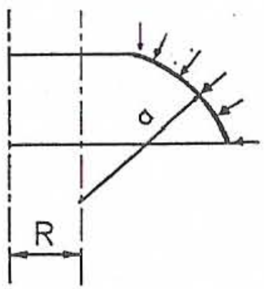
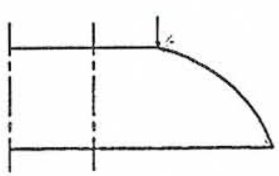
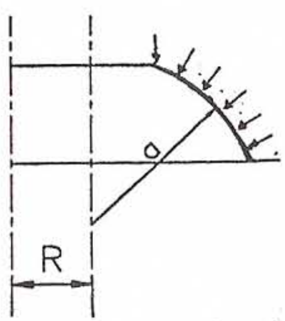
V.3.3. TOROID SHELL

b) Ring axis does not bisect the cross-section :

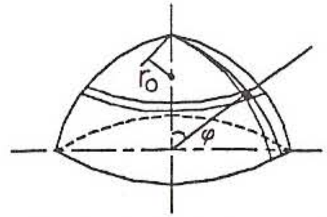


N_φ MERIDIAN FORCE / m'

N_θ RING FORCE / m'

System	Loading	N_φ	N_θ
	$p_x = g \sin\varphi$ $p_r = g \cos\varphi$	$-g \dfrac{Ra(\varphi-\varphi_o)+a^2(\cos\varphi_o-\cos\varphi)}{(R+a\sin\varphi)\sin\varphi}$ For $\varphi_o = -\varphi_1$ (symmetrical sections) $-g \dfrac{Ra\varphi + a^2(1-\cos\varphi)}{(R+a\sin\varphi).\sin\varphi}$	$-\dfrac{g}{\sin\varphi}(R+a\sin\varphi)\cos\varphi$ $-R(\varphi-\varphi_o)-a(\cos\varphi_o-\cos\varphi)$ For $\varphi_o = -\varphi_1$ (symmetrical sections) $-\dfrac{g}{\sin\varphi}\left[(R+a\sin\varphi)\cos\varphi-R\varphi-a(1-\cos\varphi)\right]$
	Edge Load p_L	$-p_L \dfrac{R+\sin\varphi_o}{(R+a\sin\varphi)\sin\varphi}$	$\dfrac{p_L}{a} . \dfrac{R+a\sin\varphi_o}{\sin^2\varphi}$
	$p_r = p$	$-\dfrac{p}{2(R+a\sin\varphi)\sin\varphi}$ $\left[(R+a\sin\varphi)^2-(R+a\sin\varphi_o)^2\right]$ For $\varphi_o = -\varphi_1$ (symmetrical sections) $-\dfrac{p\,a}{2}\dfrac{2R+a\sin\varphi}{R+a\sin\varphi}$	$-\dfrac{p}{2\sin^2\varphi}\left[2R\sin\varphi_o + a(\sin^2\varphi_o+\sin^2\varphi)\right]$ For $\varphi_o = -\varphi_1$ (symmetrical sections) $-\dfrac{p\,a}{2}$

V.3.4. Other Shells of Revolution with Curved Meridian :

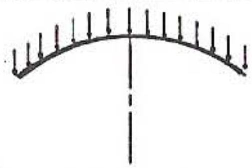
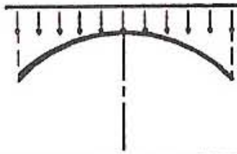
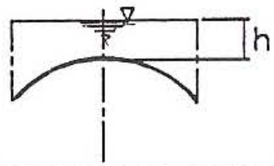
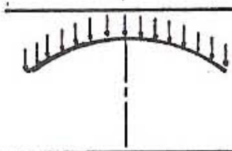
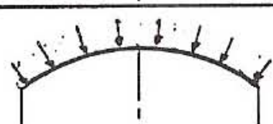
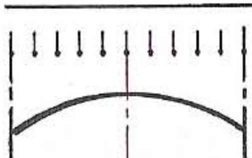


r_o *Radius of curvature at vertex*

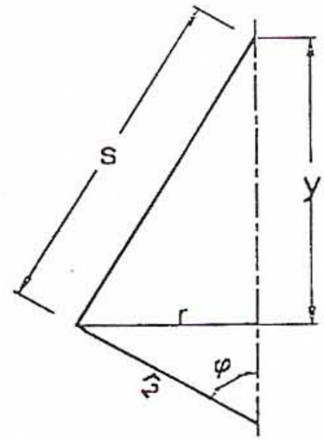
p_x, p_r *Surface load component*

N_φ *Meridian force /m*

N_θ *Ring force /m*

System	Loading	N_φ	N_θ
PARABOLA 	Dead load $p_x = g \sin\varphi$ $p_r = g \cos\varphi$	$-g.\frac{r_o}{3}.\frac{1-\cos^3\varphi}{\sin^2\varphi\cos^2\varphi}$	$-g\,\frac{r_o}{3}\,\frac{2-\cos^2\varphi+\cos^3\varphi}{\sin^2\varphi}$
	$p_x = p \sin\varphi\cos\varphi$ $p_r = p\ \cos^2\varphi$	$-p\,\frac{r_o}{2}\,\frac{1}{\cos\varphi}$	$-p\,\frac{r_o}{2}\cos\varphi$
	$p_r = \gamma\,(h+\frac{r_o}{2}tg^2\varphi)$	$-\gamma\frac{r_o}{2}(h+\frac{r_o}{4}tg^2\varphi)\frac{1}{\cos\varphi}$	$-\gamma\frac{r_o}{2}\,[h(2tg^2\varphi+1)$ $+r_o tg^2\varphi\ (tg^2\varphi+\frac{3}{4})]\cos\varphi$
	$p_r = p$	$-p.\frac{r_o}{2\cos\varphi}$	$-p\,\frac{r_o}{2}.\frac{1+\sin^2\varphi}{\cos\varphi}$
CYCLOID 	Dead load $p_x = g \sin\varphi$ $p_r = g \cos\varphi$	$-2g.r_o.\frac{\varphi\sin\varphi+\cos\varphi-1/3\cos^3\varphi-2/3}{(2\varphi+\sin 2\varphi)\sin\varphi}$	$-gr_o\,[\frac{1-\cos^3\varphi}{3\sin^2\varphi\cos\varphi}-\frac{\varphi}{2}\tan\varphi-\frac{1}{2}\sin^2\varphi]$
	Live load $p_x = p \sin\ \cos\varphi$ $p_r = p \cos^2\varphi$	$-p.\frac{r_o}{8}.\frac{2\varphi+\sin 2\varphi}{\sin\varphi}$	$-p.\frac{r_o}{16}.\frac{2\varphi+\sin 2\varphi}{\sin\varphi}.(4\cos^2\varphi-\frac{2\varphi}{\sin 2\varphi}-1)$

V.3.5. Conical Shells



The surface area of a conical surface, is given by :

$$A = 2\pi r \cdot \frac{s}{2} = \pi y^2 \frac{\cos\varphi}{\sin^2\varphi}$$

The meridian force $$N_\varphi = \frac{W_\varphi}{2\pi r \sin\varphi} = \frac{W\varphi}{2\pi y \cos\varphi}$$

The ring force $$N_\theta = r_2 \cdot p_r$$

in which $$r_2 = r \cdot \frac{s}{y} = y \frac{\cos\varphi}{\sin^2\varphi}$$

a) *Internal Forces due to Dead Load g/m²* :

The dead weight of a cone, height y, and included in a central angle φ is :

$$W_\varphi = gA = g \cdot \pi y^2 \cdot \frac{\cos\varphi}{\sin^2\varphi}$$

The meridian force $$N_\varphi = \frac{W\varphi}{2\pi y \cos\varphi} = \frac{g\pi y^2 \cdot \frac{\cos\varphi}{\sin^2\varphi}}{2\pi y \cos\varphi} = \frac{g\,y}{2\sin^2\varphi}$$

The ring force $$N_\varphi = r_2\, p_r = y \frac{\cos\varphi}{\sin^2\varphi} \cdot g\cos\varphi = g\,y\cot^2\varphi$$

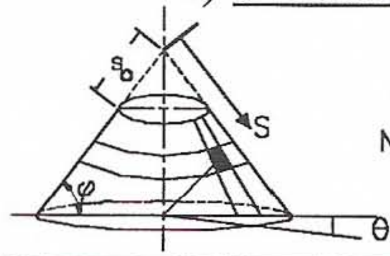
or $$N_\theta = g\frac{r^2}{y}$$ always compression

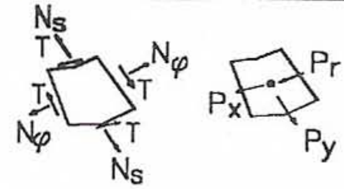
b) ***Internal Forces due to Live Load p/m² horizontal Projections :***

The live load p/m2 (horizontal)corresponds to (p cos φ) per meter square surface, so that we can determine N_φ & N_θ if we replace g by p cos φ in the previous equations, so that :

$$N_\varphi = p\,\frac{y}{2}\,\frac{\cos\varphi}{\sin^2\varphi} \quad \text{and} \quad N_\theta = p\,y\,\frac{\cos^3\varphi}{\sin^2\varphi}$$

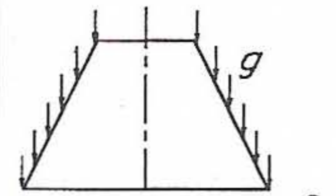
C) GENERAL CASES OF CONICAL SHELLS





N_s = Meridian force / m
N_θ = Ring force / m
T = Unit central shear

1



Dead load g/m²
$p_r = g.\cos\phi$

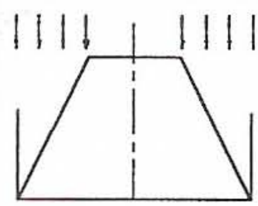
Dead load g/m² surface : $p_r = g.\cos\varphi$

$$N_s = -g\,\frac{s^2 - s_o^2}{2s}\cdot\frac{1}{\sin\phi}\;;\quad N_\theta = -gs\,\frac{\cos^2\phi}{\sin\phi}\;;\quad T = o$$

For $s_o = 0$ (complete cone) ;

$$N_s = -g\,\frac{s}{2\sin\phi}\;;\quad N_\theta = -g\,\frac{s.\cos^2\phi}{\sin\phi}$$

2



Live load p/m²
$p_r = p.\cos^2\phi$

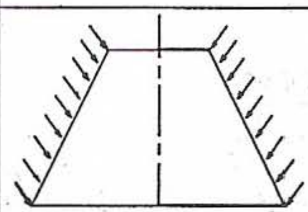
Live load p/m² horizontal : $p_r = p.\cos^2\varphi$

$$N_s = -p\,\frac{s^2 - s_o^2}{2s}\cot\phi\;;\quad N_\theta = -p\,s\,\frac{\cos^2\phi}{\sin\phi}\;;\quad T = o$$

For $s_o = 0$: $N_s = -p.\frac{s}{2}.\cot\phi$;

$$N_\theta = -p.s.\frac{\cos^3\phi}{\sin\phi}$$

3



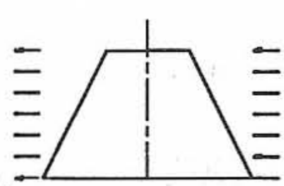
$p_r = p$

Normal pressure $p_r = p$

$$N_s = -p\,\frac{s^2 - s_o^2}{2s}\cot\phi\;;\quad N_\theta = -p\,s\cot\phi\;;\quad T = o$$

For $s_o = 0$: $N_s = -p.\frac{s}{2}.\cot\phi$; $N_\theta = -p\,s\cot\phi$

4



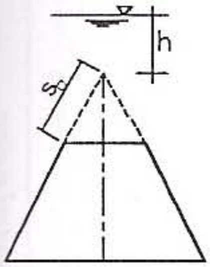
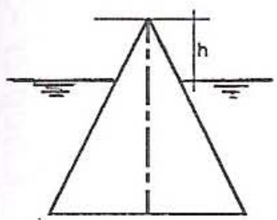
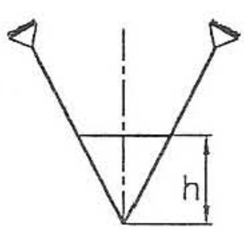
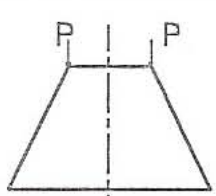
Wind press.
w/m²
$p_r = w.\sin\phi.\cos\theta$

$$N_s = -w.\frac{s}{2}\left[\cos\phi - \frac{1}{3\cos\phi} - \frac{s_o^2}{s^2}\left(\cos\phi - \frac{1}{\cos\phi}\right) - \frac{2\,s_o^3}{3s^2\cos\phi}\right].\cos\theta$$

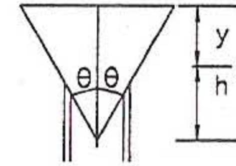
$$N_\theta = -w.s.\cos\phi.\cos\theta\;;\quad T = -w.\frac{s^3 - s_o^3}{3s^2}\sin\theta$$

For $s_o = 0$: $N_s = -\frac{w\,s}{2}\left(\cos\phi - \frac{1}{3\cos\phi}\right).\cos\theta$

$$N_\theta = -w.s.\cos\phi.\cos\theta\;;\quad T = -w.\frac{s}{3}.\sin\theta$$

5	 Liquid press. $p_r = \gamma(h + s.\sin\varphi)$	$N_s = -\frac{\gamma}{s}.(h\,\frac{s^2 - s_o^2}{2}\cot\phi + \frac{s^3 - s_o^3}{3}\cos\phi)$ $N_\theta = -\gamma.s.(h\cot\phi + s.\cos\phi)$ For $s_o = 0$ (complete cone) ; $N_s = -\gamma.s.(\frac{h}{2}\cot\phi + \frac{s}{3}\cos\phi)$; $N_\theta = -\gamma.s.(h\cot\phi + s.\cos\phi)$
6	 $p_r = \gamma\,(s.\sin\varphi - h)$	For points above the liquid level $N_s = N_\varphi = 0$ For points below the liquid level : $N_s = -\frac{\gamma}{6\,s}\left[\frac{\cos\phi}{\sin^3\phi}h^3 + s^2(2s\cos\phi - 3h\cot\phi)\right]$ $N_\theta = -\gamma\,s\,[s\cos\phi - h\cot\phi]$
7	 $p_r = \gamma(h - s.\sin\phi)$	For points above the liquid level : $N_s = \frac{\gamma\,h^3}{6\,s}.\frac{\cos\phi}{\sin^3\phi}$; $N_\theta = o$ For points below the liquid level : $N_s = \frac{\gamma s}{2}(3h\cot\phi - 2s.\cos\phi)$; $N_\theta = \gamma.s.(h\cot\phi - s.\cos\phi)$
8	 Edge load p/m	$N_s = -p.\frac{s_o}{s}.\frac{1}{\sin\phi}$; $N_\theta = 0$; $T = 0$ For $s_o = 0$: $N_s = -\frac{P}{2\,\pi\,s\sin\varphi\cos\phi}$; $N_\theta = 0$

4. INTERNAL FORCES IN PASTE CONTAINERS.



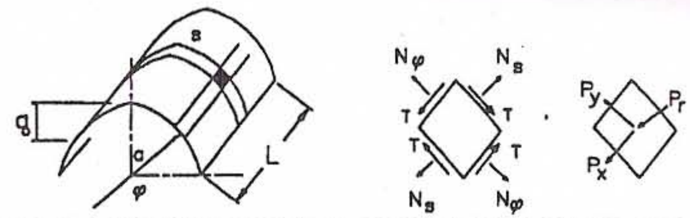
	Ring tensile force T_2		Meridian comp. force above ring beam		Meridian tensile force below ring beam	
	Due to paste w/m^2	Due to own weight g/m^2	Due to paste w/m^2	Due to own weight g/m^2	Due to paste w/m^2	Due to own weight g/m^2
GENERL	$\frac{w\,y\,h\tan\theta}{\cos\theta}$	$g\,h\tan^2\theta$ max: $g\,H\tan^2\theta$ at top	$\frac{w}{6}\cdot\frac{3H-2y}{h}\cdot y^2\,\frac{\tan\theta}{\cos\theta}$	$\frac{g}{2\cos^2\theta}\cdot y\cdot\frac{2H.y}{h}$	$\frac{wh}{6}(H+2y)\frac{\tan\theta}{\cos\theta}$ max: $\frac{3}{16}wH^2\,\frac{\tan\theta}{\cos\theta}$ at $y = H/4$	$\frac{gh}{2\cos^2\theta}$
θ = 30° at any depth	$\frac{2}{3}w\ y\ h$	$g\,\frac{h}{3}$	$\frac{w\,y^2}{g}\cdot\frac{3H-2y}{h}$	$\frac{2gy}{3}\cdot\frac{2H-y}{h}$	$\frac{wh}{g}(H+2y)$	$\frac{2gh}{3}$
θ = 30° at mid height $y = h = \frac{H}{2}$	$\frac{w\,H^2}{6}$	$g\,\frac{H}{6}$	$\frac{w\,H^2}{9}$	$g\ \ H$	$\frac{w\,H^2}{9}$	$\frac{g\,H}{3}$
θ = 30° $y = o$		1/3			1/9	2/3
$= H/4$	1/8	1/4	1/43.2	1/2.57	1/8	1/8
$= H/2$	1/6	1/6	1/9	1	1/9	1/3
$=3H/4$	1/8	1/12	3/8	2.5	1/144	1/6
$=\ \ H$						
Mul tiplicator	$w.H^2$	gH	$w.H^2$	gH	$w.H^2$	gH

V.3.6. BARREL SHELL

Boundry conditions :

$N_s = o$ *for :*

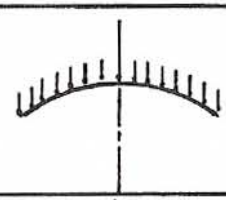
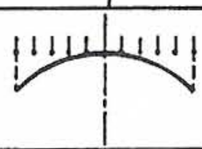
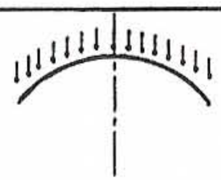
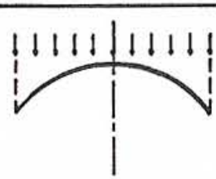
$s = o$ & $s = L$



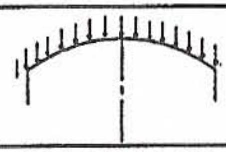
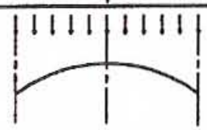
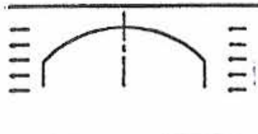
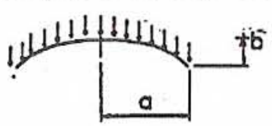
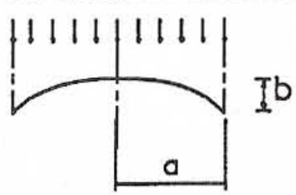
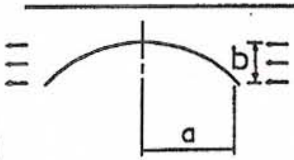
N_s, N_φ *unit normal forces*

T unit central shear

p_x, p_y, p_r *Surfaces load components*

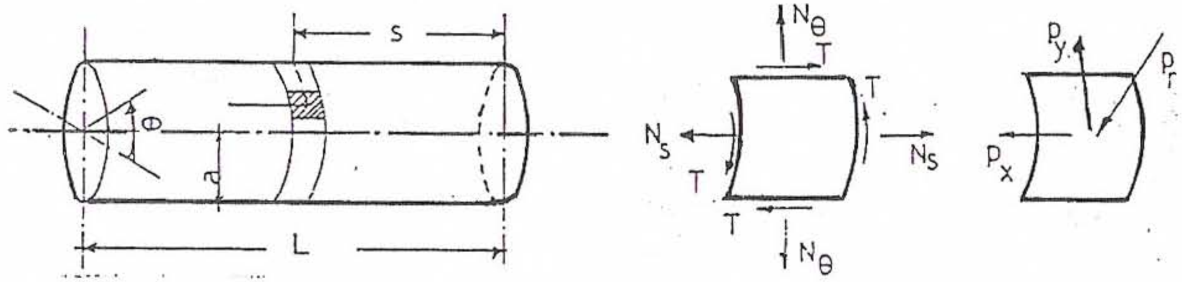
	System	Loading	N_s	N_φ	T
Circle		Dead Load $p_r = g \sin\varphi$	$-g.\frac{s}{a}(L-s)\sin\varphi$	$-g.\ a.\ \sin\varphi$	$-g.\ (L-2s).\cos\varphi$
Circle		Live Load $p_r = p \sin^2\varphi$	$p.\frac{3s}{2a}.(L-s)(1-2\sin^2\varphi)$	$-p\ .\ a\ .\ \sin^2\varphi$	$-p\,(\frac{L}{2}-s)\,3\sin\varphi.\cos\varphi$
Circle		Wind Load $p_r = w \cos\varphi$	$-w.\ \frac{s}{2a}(L-s)\cos\varphi$	$-w\ .\ a\ .\cos\varphi$	$w.(\frac{L}{2}-s).\sin\varphi$
Parabola		Dead Load $p_r = g \sin\varphi$	$g\,\frac{s}{2a_o}(L-s)\sin^4\varphi$	$-g\,\frac{a_o}{\sin^2\varphi}$	$g.(\frac{L}{2}-s).\cos\varphi$
Parabola		Live Load $p_r = p \sin^2\varphi$	o	$-p\,\frac{a_o}{\sin\varphi}$	o
Parabola		Wind Load $p_r = w \cos\varphi$	$w\,\frac{s}{2r_o}(L-s)\sin\varphi$ $\cos\varphi\ x(3+2\sin^2\varphi)$	$-w\,a_o\,\frac{\cos\varphi}{\sin^2\varphi}$	$w.(\frac{L}{2}-s)\,\frac{1+2\cos^2\varphi}{\sin\varphi}$

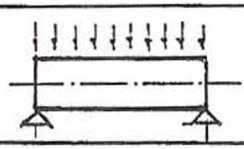
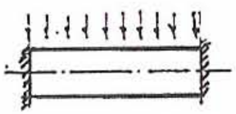
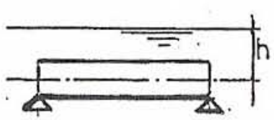
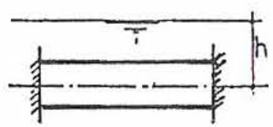
BARREL SHELL (continued) Cycloid & Ellipse

System	Loading	N_s	N_φ	T
CYCLOID 	Dead Load $p_r = g \sin\varphi$	$-g \frac{3s}{2a_o}(L-s)$	$-g\, a_o \sin^2\varphi$	$-g(\frac{L}{2}-s)\,3.\cos\varphi$
	Live Load $p_r = p \sin^2\varphi$	$p \frac{2s}{a_o}(L-s)\frac{1-2\sin^2\varphi}{\sin\varphi}$	$-p\; a_o\; \sin^3\varphi$	$-4p(\frac{L}{2}-s)\sin\varphi\cos\varphi$
	Wind Load $p_r = w \cos\varphi$	$-w \frac{s}{a_o}(L-s)(1-\cos\varphi)\frac{\cos j}{\sin^3\varphi}$	$-w\; a_o \sin\varphi\, .\cos\varphi$	$-w\,(\frac{L}{2}-s)\frac{1-2\sin^2\varphi}{\sin\varphi}$
	$p_y = -g\cos\varphi$	$-g\frac{s}{2}(L-s)\;(\frac{3a^2b^2-\eta^2}{a^2b^2.\eta^{1/2}})\sin\varphi$	$-g\,a^2b^2\frac{\sin\varphi}{\eta^{3/2}}$	$-g(\frac{L}{2}-s).\cos\varphi$ $.\left[2a^2+(a^2-b^2)\sin^2\varphi\right]/\eta$
$\eta = a^2\cos\varphi^2 + b^2\sin\varphi^2$ 	$p_r = p\sin^2\varphi$	$-p\frac{3s(L-s)}{2a^2b^2\eta^{1/2}}.[b^2(a^2\cos^2\varphi - b^2\sin^2\varphi$ $+2\eta^2(\sin^2\varphi-\cos^2\varphi)]$	$-p\,a^2b^2\frac{\sin^2\varphi}{\eta^{3/2}}$	$p(\frac{L}{2}-s)(b^2-a^2).$ $3\sin\varphi\cos\varphi/\eta$
ELLIPSE 	$p_r = w\cos\varphi$	$-w.\frac{s}{2}.\frac{(L-s)\cos\varphi}{a^2b^2\eta^{1/2}}[\eta^2$ $+3\eta(b^2-a^2)(1-3\sin^2\varphi)$ $-6(b^2-a^2)^2\sin^2\varphi\cos^2\varphi]$	$-w\,a^2b^2\frac{\cos\varphi}{\eta^{3/2}}$	$-w(\frac{L}{2}-s)(b^2-a^2).$ $3\sin\varphi(1+\cos^2\varphi)/\eta$

V.3.7. CIRCULAR CYLINDRICAL SHELL (CIRCULAR PIPE)

N_s = Longitudinal force / m
N = Ring force /m
T = Unite central shear.



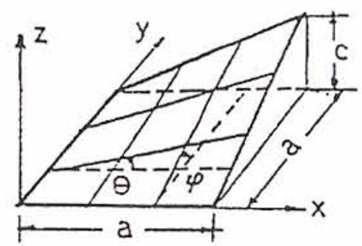
	Loads	N_s	N_θ	T
	Load p t/m^2	$-p.\frac{s}{a}(L-s).\sin\theta$	$-p.\ a.\ \sin\theta$	$-p.\ (L-2s).\ \cos\theta$
	$P_r = p\sin\theta$	$p[\frac{L^2}{6a}-\gamma a-\frac{s}{a}(L-s)]\sin\theta$ ν = Poisson's ratio	$-p.\ a.\ \sin\theta$	$-p.\ (L-2s).\ \cos\theta$
		$-\gamma.\frac{s}{2}(L-s).\sin\theta$	$\gamma.a^2(\frac{h}{a}-\sin\theta)$	$\gamma a.(\frac{L}{2}-s).\cos\theta$
	Liquid pressure $pr = -\gamma(h - a\sin\theta)$	$-\gamma\{[\frac{L^2}{12}-\gamma a^2-$ $\frac{s}{2}(L-s)]\sin\theta-\gamma ah\}$	$\gamma a^2(\frac{h}{a}-\sin\theta)$	$\gamma a\,(1/2-s)\cos\theta$

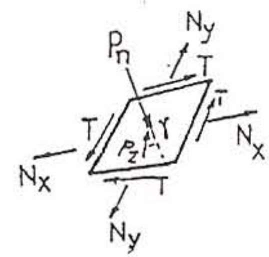
El-Behairy R.C. Design Handbook Chapter -5-

V.3.8. HYPERBOLIC PARABOLOID

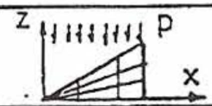
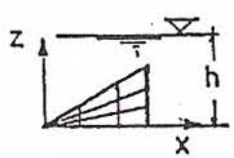
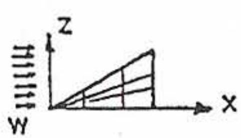
Equation of surface :

$$z = \frac{xy}{n} ; n = \frac{a^2}{n}$$





N_x , N_y = unit, normal forces
T = unit central shear
p_n, p_z = surface load component along the normal & z-axis
Boundary conditions :
(x o, Nx = o) & (y=o, Ny =o).

	Loads	N_s	N_θ	T
	$p_r = -g$	$-g.\frac{y}{2}\log\frac{x+\sqrt{x^2+y^2+n^2}}{y^2+n^2} . \frac{\cos\varphi}{\cos\theta}$	$-g.\frac{x}{2}\log\frac{y+\sqrt{x^2+y^2+n^2}}{x^2+n^2} . \frac{\cos\theta}{\cos\varphi}$	$\frac{g}{2}\sqrt{x^2+y^2+n^2}$
	$P_r = -p\cos\theta$	0	0	$p . \frac{a^2}{2c}$
	$P_n = \gamma(h - \frac{xy}{n})$	$\frac{\gamma}{2n^2}[\frac{x^4}{4}+\frac{x^2}{2}(5y^2+n^2) - 4nhxy]\frac{\cos\varphi}{\cos\theta}$	$\frac{\gamma}{2n^2}[\frac{y^4}{4}+\frac{y^2}{2}(5x^2+n^2) - 4nhxy]\frac{\cos\theta}{\cos\varphi}$	$\frac{\gamma}{2n}(h - \frac{xy}{n}) , (x^2+y^2+n^2)$
	$p_n = p$	$-p.\frac{2xy}{n} , \frac{\cos\varphi}{\cos\theta}$	$-p\frac{2xy}{n} , \frac{\cos\theta}{\cos\varphi}$	$p . \frac{x^2+y^2 n^2}{2n}$
	$P_n = w\frac{y^2}{x^2+y^2+n^2}$	$-w.\frac{y}{n}(x+\frac{y^2}{\sqrt{y^2+n^2}}$ $\text{arctg}\frac{x}{\sqrt{y^2+n}})\frac{\cos\varphi}{\cos\theta}$	$-w.\frac{y}{n}(y-\sqrt{x^2+n^2}$, $\text{arctg}\frac{y}{\sqrt{x^2+x^2}})\frac{\cos\theta}{\cos\varphi}$	$w . \frac{y^2}{2n}$

CHAPTER VI

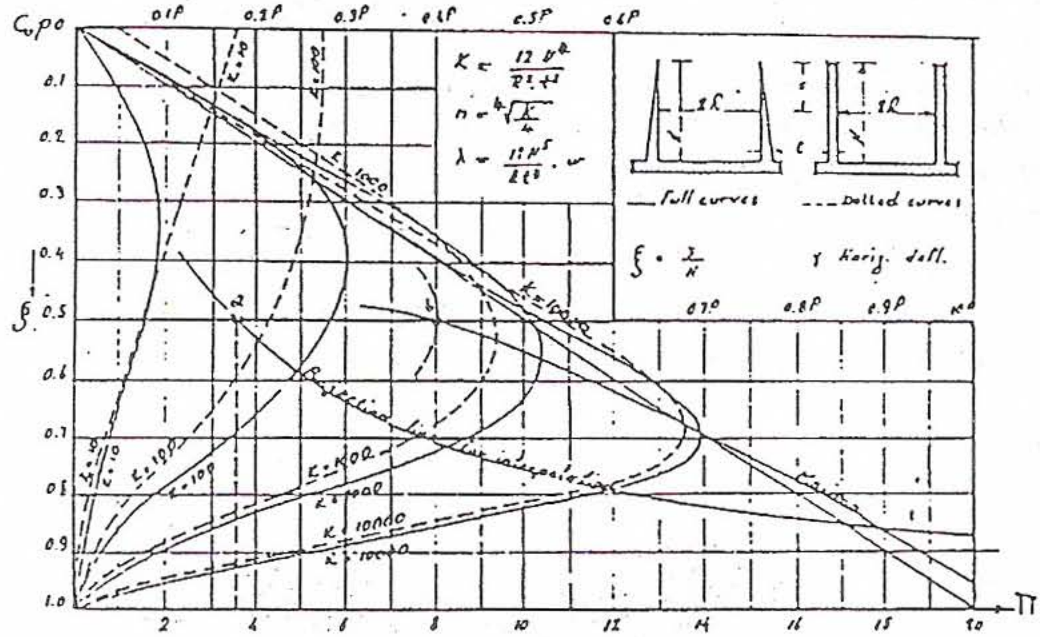
CIRCULAR TANKS

Page

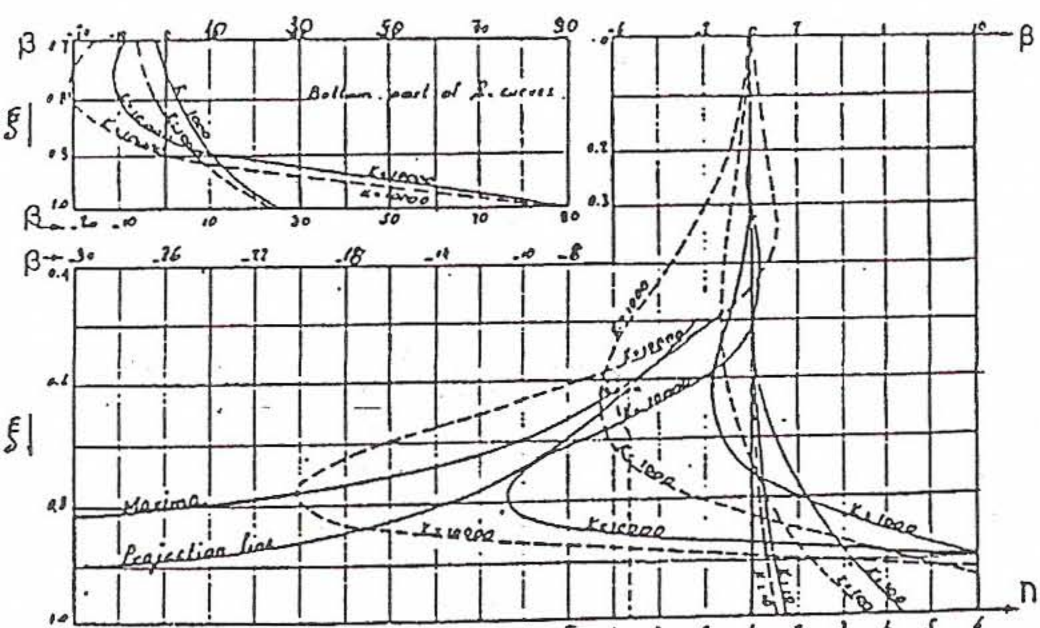
VI.1. *Ring Tension and Bending Moments in Open Circular Tanks With Walls Fixed at Bottom due To Triangular Load*

a. Ring Tension : T = Co.W.H.R; Where : $Co = y.\ \frac{K}{\gamma}.\frac{t_x}{t}$

$W = \text{Weight of Liquid} / m^3$

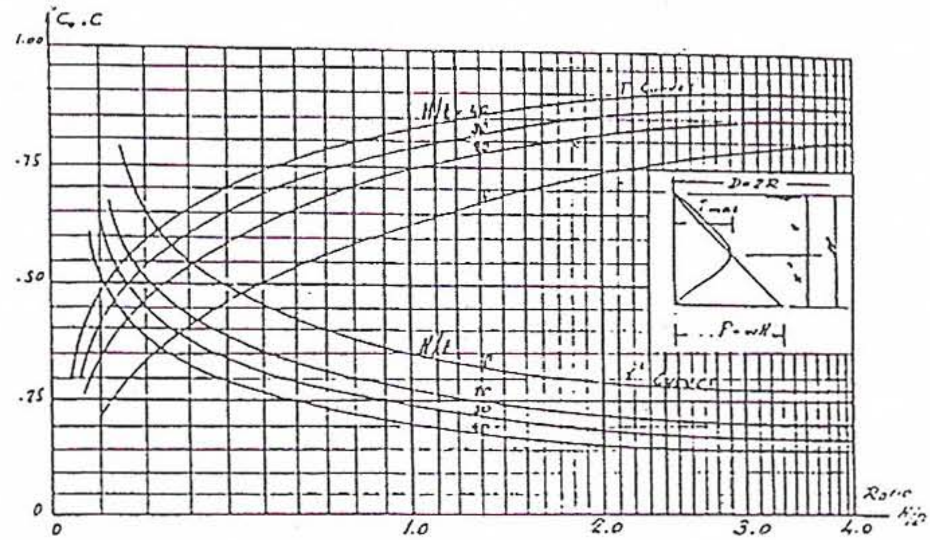


b. Bending Moment : $M = (\frac{d^2}{d^2}\frac{y}{\zeta}.\frac{k}{\lambda}\frac{t^3{}_x}{3}).\frac{R^2.t^2.w}{12H} = \beta.\frac{R^2.t^2.w}{12H}$



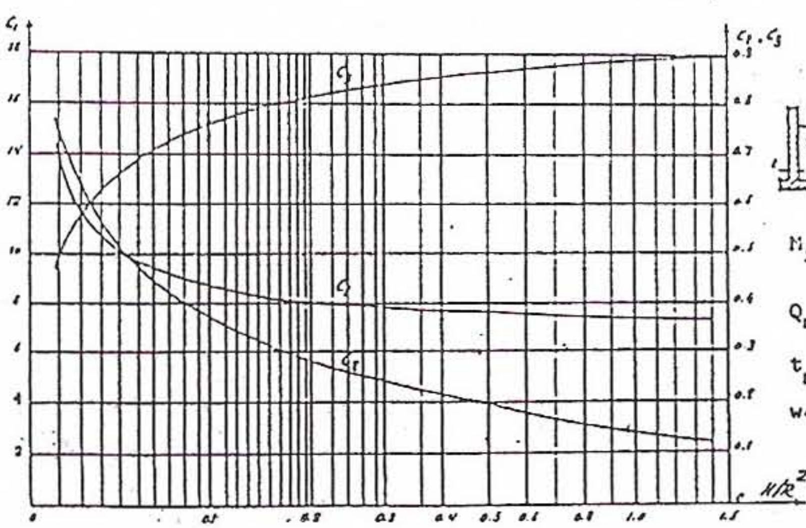
VI.2. Values Of Maximum Ring - Tension and its Position For Circular Tanks

$t_{max} = c_o . P. R kg / m, x' = C. H ms$ (by W . S . Gray)



VI.3. Simplified Method For Detrmining the Fixing Moment, the Shearing Force and the Thickness Of the Wall at Base

(by . Prof . Dr . M . Hilal)



H/R2	1.4	1.2	1.0	0.8	0.6	0.4	0.2	0.1	0.08	0.06	0.04	0.02	0.01
n	16.4	15.2	13.9	12.5	10.9	9.00	6..50	4.77	4.32	3.82	3.26	2.46	1.93
C1	7.40	7.44	7.49	7.55	7.65	7.82	8.22	8.80	9.04	9.4	10.1	11.7	14.4
C2	.118	.127	.138	.154	.175	.211	.284	.375	.409	.455	.525	.646	.769
C3	.800	.895	.890	.882	.870	.852	.812	.756	.736	.708	.663	.570	.462

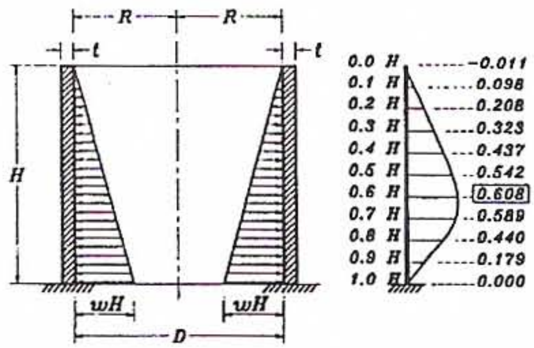
a- Tension in circular rings

Example :-

$H^2 / D\,t = 10$

Positive sign indicates tension

Fixed base , Free top — Triangular Load

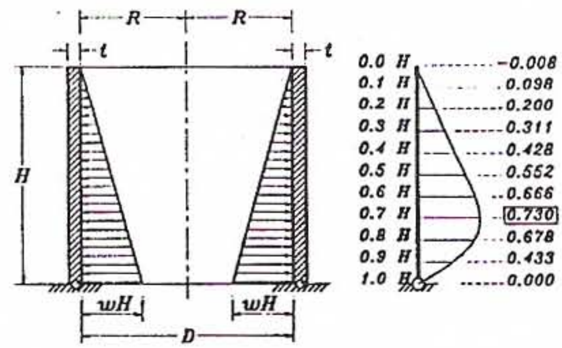


T = coeff. x w H R

Table I :-

Level	coeff.
0.0 H	-0.011
0.1 H	0.098
0.2 H	0.208
0.3 H	0.323
0.4 H	0.437
0.5 H	0.542
0.6 H	0.608
0.7 H	0.589
0.8 H	0.440
0.9 H	0.179
1.0 H	0.000

Hinged base , Free top — Triangular Load

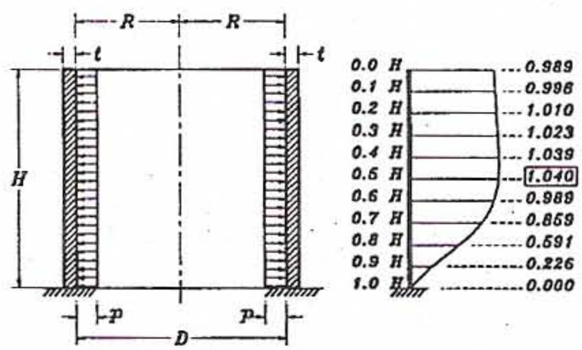


T = coeff. x w H R

Table II :-

Level	coeff.
0.0 H	-0.008
0.1 H	0.098
0.2 H	0.200
0.3 H	0.311
0.4 H	0.428
0.5 H	0.552
0.6 H	0.666
0.7 H	0.730
0.8 H	0.678
0.9 H	0.433
1.0 H	0.000

Fixed base , Free top — Rectangular Load

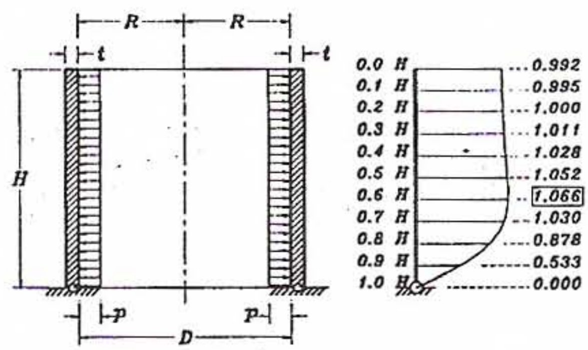


T = coeff. x p R

Table III :-

Level	coeff.
0.0 H	0.989
0.1 H	0.996
0.2 H	1.010
0.3 H	1.023
0.4 H	1.039
0.5 H	1.040
0.6 H	0.989
0.7 H	0.859
0.8 H	0.591
0.9 H	0.226
1.0 H	0.000

Hinged base , Free top — Rectangular Load

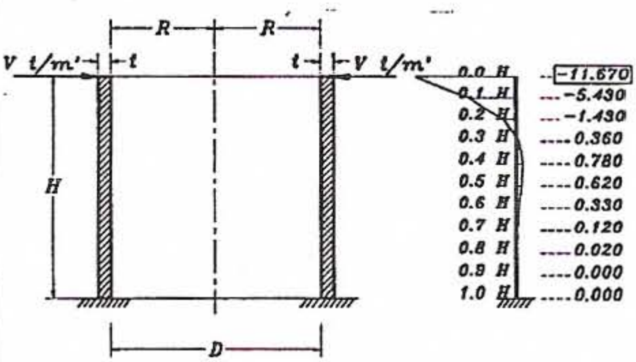


T = coeff. x p R

Table IV :-

Level	coeff.
0.0 H	0.992
0.1 H	0.995
0.2 H	1.000
0.3 H	1.011
0.4 H	1.028
0.5 H	1.052
0.6 H	1.066
0.7 H	1.030
0.8 H	0.878
0.9 H	0.533
1.0 H	0.000

Fixed base , Free top — Shear per m, V, applied at top



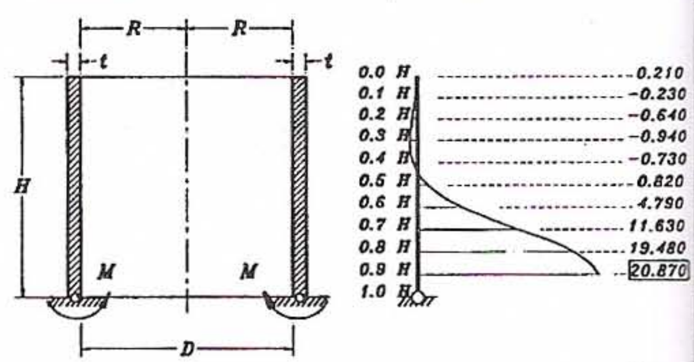
T = coeff. x V R/H

Table V :-

Level	coeff.
0.0 H	-11.670
0.1 H	-5.430
0.2 H	-1.430
0.3 H	0.360
0.4 H	0.780
0.5 H	0.620
0.6 H	0.330
0.7 H	0.120
0.8 H	0.020
0.9 H	0.000
1.0 H	0.000

Shear per m, V, applied at top

Hinged base , Free top — Moment per m, M, applied at base



$T = coeff. \times M\,R/H^2$

Table VI :-

Level	coeff.
0.0 H	0.210
0.1 H	-0.230
0.2 H	-0.640
0.3 H	-0.940
0.4 H	-0.730
0.5 H	0.820
0.6 H	4.790
0.7 H	11.630
0.8 H	19.480
0.9 H	20.870
1.0 H	

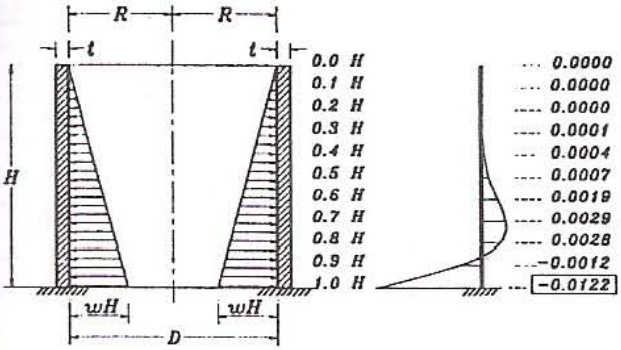
Moment per m, M, applied at base

b- Moment in cylindrical wall

Example :- $H^2 / D\,t = 10$

Positive sign indicates tension in the outside

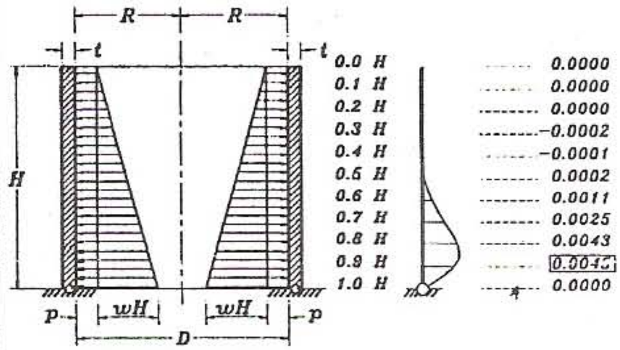
Fixed base , Free top — Triangular Load



Level	Coeff.
0.0 H	0.0000
0.1 H	0.0000
0.2 H	0.0000
0.3 H	0.0001
0.4 H	0.0004
0.5 H	0.0007
0.6 H	0.0019
0.7 H	0.0029
0.8 H	0.0028
0.9 H	−0.0012
1.0 H	−0.0122

Table VII :- $M = \text{coeff.} \times w H^3$

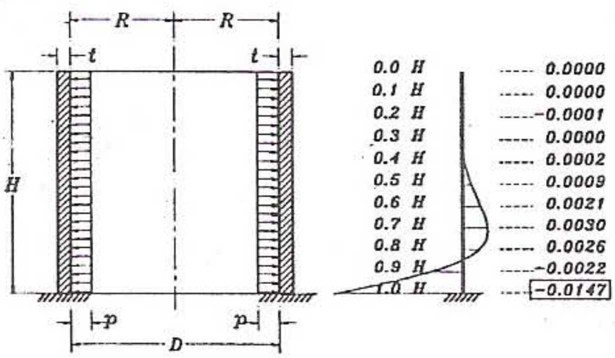
Hinged base , Free top — Trapezoidal Load



Level	Coeff.
0.0 H	0.0000
0.1 H	0.0000
0.2 H	0.0000
0.3 H	−0.0002
0.4 H	−0.0001
0.5 H	0.0002
0.6 H	0.0011
0.7 H	0.0025
0.8 H	0.0043
0.9 H	0.0045
1.0 H	0.0000

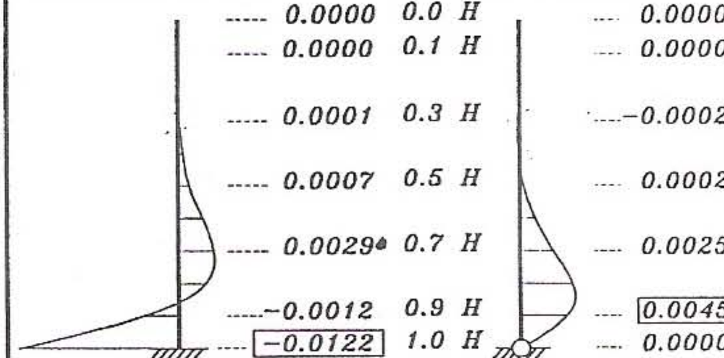
Table VIII :- $M = \text{coeff.} \times (w H^3 + p H^2)$

Fixed base , Free top — Rectangular Load



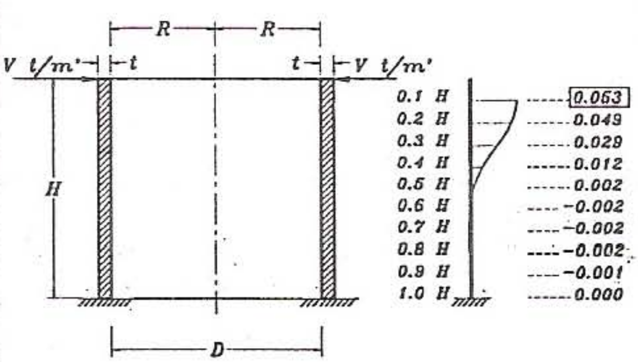
Level	Coeff.
0.0 H	0.0000
0.1 H	0.0000
0.2 H	−0.0001
0.3 H	0.0000
0.4 H	0.0002
0.5 H	0.0009
0.6 H	0.0021
0.7 H	0.0030
0.8 H	0.0026
0.9 H	−0.0022
1.0 H	−0.0147

Table IX :- $M = \text{coeff.} \times p H^2$



Fixed base , Free top (Triangular load)	Level	Hinged base , Free top (Trapezoidal load)
0.0000	0.0 H	0.0000
0.0000	0.1 H	0.0000
0.0001	0.3 H	−0.0002
0.0007	0.5 H	0.0002
0.0029	0.7 H	0.0025
−0.0012	0.9 H	0.0045
−0.0122	1.0 H	0.0000

Shear per m, V, applied at top

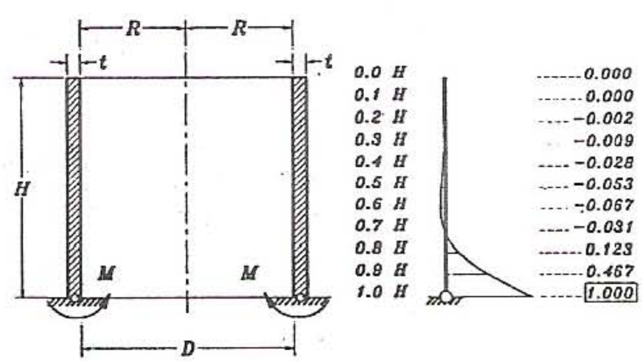


Level	Coeff.
0.1 H	0.063
0.2 H	0.049
0.3 H	0.029
0.4 H	0.012
0.5 H	0.002
0.6 H	−0.002
0.7 H	−0.002
0.8 H	−0.002
0.9 H	−0.001
1.0 H	0.000

Table X :- $M = \text{coeff.} \times V H$

Shear per m, V, applied at top

Moment per m, M, applied at base



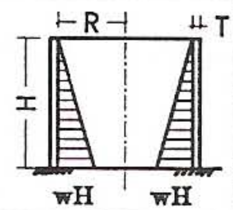
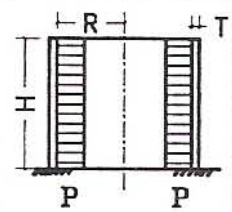
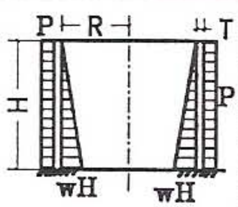
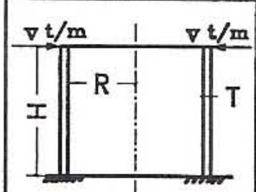
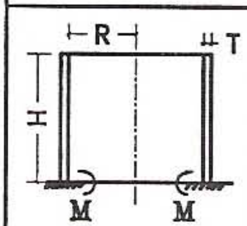
Level	Coeff.
0.0 H	0.000
0.1 H	0.000
0.2 H	−0.002
0.3 H	−0.009
0.4 H	−0.028
0.5 H	−0.053
0.6 H	−0.067
0.7 H	−0.031
0.8 H	0.123
0.9 H	0.467
1.0 H	1.000

Table XI :- $M = \text{coeff.} \times M$

Moment per m, M, applied at base

V.I Portland Association Tables :

RING TENSION & MOMENTS IN CIRCULAR TANKS :

LOADING	END COND.	RING TENSION		F. E. MOMENTS	
		TABLE	T=coeff.x	TABLE	T=coeff.x
	Fixed Base Hinged Base	I II	w.H.R w.H.R	VII ———	$w.H^3$ ———
	Fixed Base Hinged Base	III IV	p.R p.R	IX ———	$p.H^2$ ———
	Hinged Base	———	———	VIII	$w.H^3 + p.H^2$
	Fixed Base	V	v.R/H	X	v.H
	Hinged Base	VI	$M.R/H^2$	XI	M

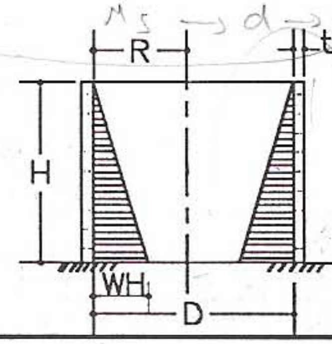
SHEAR AT BASE OF CYLINDERICAL WALL : [TABLE XII]

1- Triangular Load ------------------------ $w.H^2$

2- Rectangular Load ---------------------- p.H

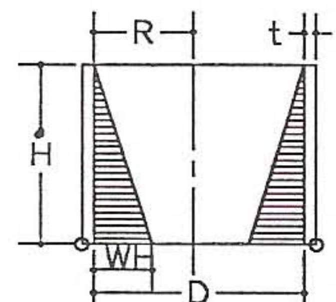
3- Moment at Base ----------------------- M/H

Table I : **Tension in circular rings**
Triangular Load
Fixed base, free top
T = coeff. * wHR
Positive sign indicates tension



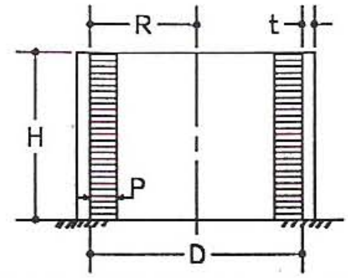
$\frac{H^2}{Dt}$	Coefficients at point									
	0.0H	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H
0.4	+ 0.149	+ 0.134	+ 0.120	+ 0.101	+ 0.082	+ 0.066	+ 0.049	+0.029	+0.014	+0.004
0.8	+ 0.263	+ 0.239	+ 0.215	+ 0.190	+ 0.160	+ 0.130	+ 0.096	+0.063	+0.034	+0.010
1.2	+ 0.283	+ 0.271	+ 0.254	+ 0.234	+ 0.209	+ 0.180	+ 0.142	+0.099	+0.054	+0.016
1.6	+ 0.265	+ 0.268	+ 0.268	+ 0.266	+ 0.250	+ 0.226	+ 0.185	+0.134	+0.075	+0.023
2.0	+ 0.234	+ 0.251	+ 0.273	+ 0.285	+ 0.285	+ 0.274	+ 0.232	+0.172	+0.104	+0.031
3.0	+ 0.134	+ 0.203	+ 0.267	+ 0.322	+ 0.357	+ 0.362	+ 0.330	+0.262	+0.157	+0.052
4.0	+ 0.067	+ 0.164	+ 0.256	+ 0.339	+ 0.403	+ 0.429	+ 0.409	+0.334	+0.210	+0.073
5.0	+ 0.025	+ 0.137	+ 0.245	+ 0.346	+ 0.428	+ 0.477	+ 0.469	+0.398	+0.259	+0.092
6.0	+ 0.018	+ 0.119	+ 0.234	+ 0.344	+ 0.441	+ 0.504	+ 0.514	+0.447	+0.301	+0.112
8.0	- 0.011	+ 0.104	+ 0.218	+ 0.335	+ 0.443	+ 0.534	+ 0.575	+0.530	+0.381	+0.151
10.	- 0.011	+ 0.098	+ 0.208	+ 0.323	+ 0.437	+ 0.542	+ 0.608	+0.589	+0.440	+0.179
12.	- 0.005	+ 0.097	+ 0.202	+ 0.312	+ 0.429	+ 0.543	+ 0.628	+0.633	+0.494	+0.211
14.	- 0.002	+ 0.098	+ 0.200	+ 0.306	+ 0.420	+ 0.539	+ 0.639	+0.666	+0.541	+0.241
16.	0.000	+ 0.099	+ 0.199	+ 0.304	+ 0.412	+ 0.531	+ 0.641	+0.687	+0.582	+0.265

Table II : **Tension in circular rings**
Triangular Load
Hinged base, free top
T = coeff. * wHR
Positive sign indicates tension



$\frac{H^2}{Dt}$	Coefficients at point									
	0.0H	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H
0.4	+ 0.474	+ 0.440	+ 0.395	+ 0.352	+ 0.308	+0.264	+0.215	+0.165	+0.111	+0.057
0.8	+ 0.423	+ 0.402	+ 0.381	+ 0.358	+ 0.330	+0.297	+0.249	+0.202	+0.145	+0.076
1.2	+ 0.350	+ 0.355	+ 0.361	+ 0.362	+ 0.358	+0.343	+0.309	+0.256	+0.186	+0.098
1.6	+ 0.271	+ 0.303	+ 0.341	+ 0.369	+ 0.385	+0.385	+0.362	+0.314	+0.233	+0.124
2.0	+ 0.205	+ 0.260	+ 0.321	+ 0.373	+ 0.411	+0.434	+0.419	+0.369	+0.280	+0.151
3.0	+ 0.074	+ 0.179	+ 0.281	+ 0.375	+ 0.449	+0.506	+0.519	+0.479	+0.375	+0.210
4.0	+ 0.017	+ 0.137	+ 0.253	+ 0.367	+ 0.469	+0.545	+0.579	+0.553	+0.447	+0.256
5.0	- 0.008	+ 0.114	+ 0.235	+ 0.356	+ 0.469	+0.562	+0.617	+0.606	+0.503	+0.294
6.0	- 0.011	+ 0.103	+ 0.223	+ 0.343	+ 0.463	+0.566	+0.639	+0.643	+0.547	+0.327
8.0	- 0.015	+ 0.096	+ 0.208	+ 0.324	+ 0.443	+0.564	+0.661	+0.697	+0.621	+0.386
10.	- 0.008	+ 0.095	+ 0.200	+ 0.311	+ 0.428	+0.552	+0.666	+0.730	+0.678	+0.433
12.	- 0.002	+ 0.097	+ 0.197	+ 0.302	+ 0.417	+0.541	+0.664	+0.750	+0.720	+0.477
14.	- 0.000	+ 0.098	+ 0.197	+ 0.299	+ 0.408	+0.531	+0.659	+0.761	+0.752	+0.513
16.	+ 0.002	+ 0.100	+ 0.198	+ 0.299	+ 0.403	+0.521	+0.650	+0.764	+0.776	+0.536

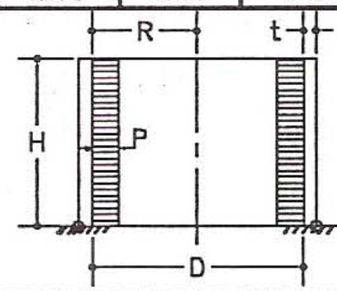
Table III : Tension in circular rings
Rectangular Load
Fixed base, free top



T = + coeff. * pR
Positive sign indicates tension

$\frac{H^2}{Dt}$	Coefficients at point									
	0.0H	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H
0.4	0.582	0.505	0.431	0.353	0.277	0.206	0.145	.092	.046	.013
0.8	1.052	0.921	0.796	0.669	0.542	0.415	0.289	.179	.089	.024
1.2	1.218	1.078	0.946	0.808	0.665	0.519	0.378	.246	.127	.034
1.6	1.257	1.141	1.009	0.881	0.742	0.600	0.449	.294	.153	.045
2.0	1.253	1.144	1.041	0.929	0.806	0.667	0.514	.345	.186	.055
3.0	1.160	1.112	1.061	0.998	0.912	0.796	0.646	.459	.258	.081
4.0	1.085	1.073	1.057	1.029	0.977	0.887	0.746	.553	.322	.105
5.0	1.037	1.044	1.047	1.042	1.015	0.949	0.825	.629	.379	.128
6.0	1.010	1.024	1.038	1.045	1.034	0.986	0.879	.694	.430	.149
8.0	0.989	1.005	1.022	1.036	1.044	1.026	0.953	.788	.519	.189
10.	0.989	0.998	1.010	1.023	1.039	1.040	0.996	.859	.591	.226
12.	0.994	0.997	1.003	1.014	1.031	1.043	1.022	.911	.652	.262
14.	0.997	0.998	1.000	1.007	1.022	1.040	1.035	.949	.705	.294
16.	1.000	0.999	0.999	1.003	1.015	1.032	1.040	.975	.750	.321

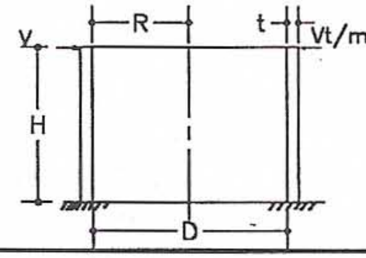
Table IV : Tension in circular rings
Rectangular Load
Hinged base, free top



T = + coeff. * pR
Positive sign indicates tension

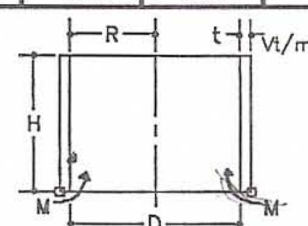
$\frac{H^2}{Dt}$	Coefficients at point									
	0.0H	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H
0.4	1.474	1.340	1.195	1.052	0.908	0.764	0.615	0.465	.311	.154
0.8	1.423	1.302	1.181	1.058	0.930	0.797	0.649	0.502	.345	.166
1.2	1.350	1.255	1.161	1.062	0.958	0.843	0.709	0.556	.386	.198
1.6	1.271	1.203	1.141	1.069	0.985	0.885	0.756	0.614	.433	.224
2.0	1.205	1.160	1.121	1.073	1.011	0.934	0.819	0.669	.480	.251
3.0	1.074	1.079	1.081	1.075	1.049	1.006	0.919	0.779	.575	.310
4.0	1.017	1.037	1.053	1.067	1.069	1.045	0.979	0.853	.647	.356
5.0	0.992	1.014	1.035	1.056	1.069	1.062	1.017	0.906	.703	.394
6.0	0.989	1.003	1.023	1.043	1.063	1.066	1.039	0.943	.747	.427
8.0	0.985	0.996	1.008	1.024	1.043	1.064	1.061	0.997	.821	.486
10.	0.992	0.995	1.000	1.011	1.028	1.052	1.066	1.030	.878	.533
12.	0.998	0.997	0.997	1.002	1.017	1.041	1.064	1.050	.920	.577
14.	1.000	0.998	0.997	0.999	1.008	1.031	1.059	1.061	.952	.613
16.	1.002	1.000	0.998	0.999	1.003	1.021	1.050	1.064	.976	.636

Table V : Tension in circular rings
Shear per m,V, applied at top
Fixed base, free top
T = coff . * VR/H
Positive sign indicates tension



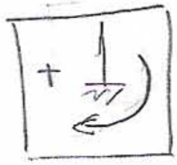
$\frac{H^2}{Dt}$	Coefficients at point									
	0.0H	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H
0.4	- 1.57	- 1.32	- 1.08	- 0.86	- 0.65	- 0.47	- 0.31	- 0.18	- 0.08	- 0.02
0.8	- 3.09	- 2.55	- 2.04	- 1.57	- 1.15	- 0.80	- 0.51	- 0.28	- 0.13	- 0.03
1.2	- 3.95	- 3.17	- 2.44	- 1.79	- 1.25	- 0.81	- 0.48	- 0.25	- 0.10	- 0.02
1.6	- 4.57	- 3.54	- 2.60	- 1.80	- 1.17	- 0.69	- 0.36	- 0.16	- 0.05	- 0.01
2.0	- 5.12	- 3.83	- 2.68	- 1.74	- 1.02	- 0.52	- 0.21	- 0.05	+ 0.01	+ 0.01
3.0	- 6.32	- 4.37	- 2.70	- 1.43	- 0.58	- 0.02	+ 0.15	+ 0.19	+ 0.13	+ 0.04
4.0	- 7.34	- 4.73	- 2.60	- 1.10	- 0.19	+ 0.26	+ 0.38	+ 0.33	+ 0.19	+ 0.06
5.0	- 8.22	- 4.99	- 2.45	- 0.79	+ 0.11	+ 0.47	+ 0.50	+ 0.37	+ 0.20	+ 0.06
6.0	- 9.02	- 5.17	- 2.27	- 0.50	+ 0.34	+ 0.59	+ 0.53	+ 0.35	+ 0.17	+ 0.01
8.0	- 10.42	- 5.36	- 1.85	- 0.02	+ 0.63	+ 0.66	+ 0.46	+ 0.24	+ 0.09	+ 0.01
10.	- 11.67	- 5.43	- 1.43	+ 0.36	+ 0.78	+ 0.62	+ 0.33	+ 0.12	+ 0.02	0.00
12.	- 12.76	- 5.41	- 1.03	+ 0.63	+ 0.83	+ 0.52	+ 0.21	+ 0.04	- 0.02	0.00
14.	-13.77	-5.34	- 0.68	+ 0.80	+ 0.81	+ 0.42	+ 0.13	0.00	- 0.03	- 0.01
16.	-14.74	- 5.22	- 0.33	+ 0.96	+ 0.76	+ 0.32	+ 0.05	- 0.04	- 0.05	- 0.02

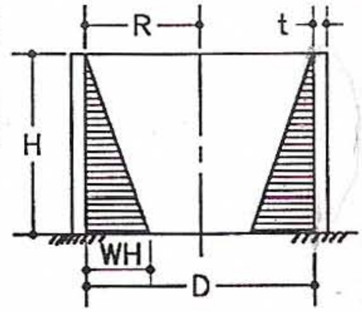
Table VI : Tension in circular rings
Moment Per m,M, applied at base
Hinged base, free top
T = coff . * MR/H^2
Positive sign indicates tension



$\frac{H^2}{Dt}$	Coefficients at point									
	0.0H	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H
0.4	+ 2.70	+2.50	+ 2.30	+ 2.12	+1.91	+ 1.69	+ 1.41	+ 1.13	+ 0.80	+0.44
0.8	+ 2.02	+ 2.06	+ 2.10	+ 2.14	+ 2.10	+ 2.02	+ 1.95	+ 1.75	+ 1.39	+0.80
1.2	+ 1.06	+ 1.42	+ 1.79	+ 2.03	+ 2.46	+ 2.65	+ 2.80	+ 2.60	+ 2.22	+1.37
1.6	+ 0.12	+ 0.79	+ 1.43	+ 2.04	+ 2.72	+ 3.25	+ 3.56	+ 3.59	+ 3.13	+2.01
2.0	- 0.68	+ 0.22	+ 1.10	+ 2.02	+ 2.90	+ 3.69	+ 4.30	+ 4.54	+ 4.08	+2.75
3.0	- 1.78	- 0.71	+ 0.43	+ 1.60	+ 2.95	+ 4.29	+ 5.66	+ 6.58	+ 6.55	+4.73
4.0	- 1.87	- 1.00	- 0.08	+ 1.04	+ 2.47	+ 4.31	+ 6.34	+ 8.19	+ 8.82	+6.81
5.0	- 1.54	- 1.03	- 0.42	+ 0.45	+ 1.86	+ 3.93	+ 6.60	+ 9.41	+11.0	+ 9.02
6.0	- 1.04	- 0.86	- 0.59	- 0.05	+ 1.21	+ 3.34	+ 6.54	+ 10.2	+13.0	+11.4
8.0	- 0.24	- 0.53	- 0.73	- 0.67	- 0.02	+ 2.05	+ 5.87	+ 11.3	+16.5	+6.06
10.	+ 0.21	- 0.23	- 0.64	- 0.94	- 0.73	+ 0.82	+ 4.79	+ 11.6	+19.4	+20.8
12.	+ 0.32	- 0.05	- 0.46	- 0.96	- 1.15	- 0.18	+ 3.52	+ 11.2	+21.8	+25.7
14.	+ 0.26	+ 0.04	- 0.28	- 0.76	- 1.29	- 0.87	+ 2.29	+ 10.5	+23.5	+30.3
16.	+ 0.22	+ 0.07	- 0.08	- 0.64	- 1.28	-1.30	+ 1.12	+ 9.67	+24.5	+34.6

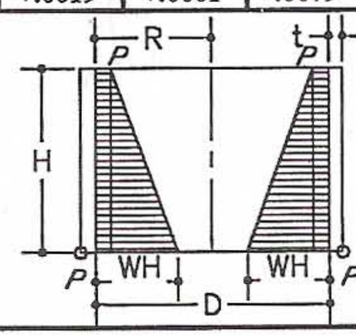
Table VII : **Moments in cylindrical wall**
Triangular Load
Fixed base, free top
Mom. = coeff . * wH^3
Positive sign indicates tension in the outside.





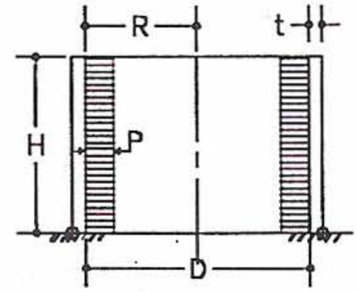
$\frac{H^2}{Dt}$	Coefficients at point									
	0.1H	0.21H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H	1.0H
0.4	+.0005	+.0014	+.0021	+.0007	-.0042	-.0150	-.0302	-.0529	-.0816	-.1205
0.8	+.0011	+.0037	+.0063	+.0080	+.0070	+.0023	-.0068	-.0224	-.0465	-.0795
1.2	+.0012	+.0042	+.0077	+.0103	+.0112	+.0090	+.0022	-.0108	-.0311	-.0602
1.6	+.0011	+.0041	+.0075	+.0107	+.0121	+.0111	+.0058	-.0051	-.0232	-0505
2.0	+.0010	+.0035	+.0068	+.0099	+.0120	+.0115	+.0075	-.0021	-.0185	-.0436
3.0	+.0006	+.0024	+.0047	+.0071	+.0090	+.0097	+.0077	+.0012	-.0119	-.0333
4.0	+.0003	+.0015	+.0028	+.0047	+.0066	+.0077	+.0069	+.0023	-.0080	-.0268
5.0	+.0002	+.0008	+.0016	+.0029	+.0046	+.0059	+.0059	+.0028	-.0058	-.0222
6.0	+.0001	+.0003	+.0008	+.0019	+.0032	+.0046	+.0051	+.0029	-.0041	-.0187
8.0	.0000	+.0001	+.0002	+.0008	+.0016	+.0028	+.0038	+.0029	-.0022	-.0146
10.	.0000	.0000	+.0001	+.0004	+.0007	+.0019	+.0029	+.0028	-.0012	-.0122
12.	.0000	.0001	+.0001	+.0002	+.0003	+.0013	+.0023	+.0026	-.0005	-.0104
14.	.0000	.0000	.0000	.0000	+.0001	+.0008	+.0019	+.0023	-.0001	-.0090
16.	.0000	.0000	-.0001	-.0002	-.0001	+.0004	+.0013	+.0019	+.0001	-.0079

Table VIII : **Moments in cylindrical wall**
Trapezoidal Load
Hinged base, free top
Mom. = coeff . *($wH^3 + pH^2$)
Positive sign indicates tension in the outside.



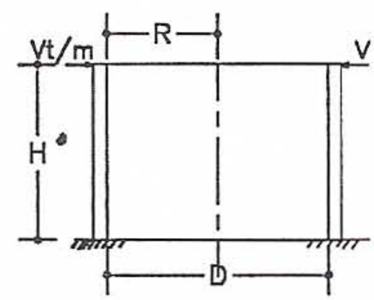
$\frac{H^2}{Dt}$	Coefficients at point									
	0.1H	0.21H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H	1.0H
0.4	+.0020	+.0072	+.0151	+.0230	+.0301	+.0348	+.0357	+.0312	+.0197	0
0.8	+.0019	+.0064	+.0133	+.0207	+.0271	+.0319	+.0329	+.0292	+.0187	0
1.2	+.0016	+.0058	+.0111	+.0177	+.0237	+.0280	+.0296	+.0263	+.0171	0
1.6	+.0012	+.0044	+.0091	+.0145	+.0195	+.0236	+.0255	+.0232	+.0155	0
2.0	+.0009	+.0033	+.0073	+.0114	+.0158	+.0199	+.0219	+.0205	+.0145	0
3.0	+.0004	+.0018	+.0040	+.0063	+.0092	+.0127	+.0152	+.0153	+.0111	0
4.0	+.0001	+.0007	+.0016	+.0033	+.0057	+.0083	+.0109	+.0118	+.0092	0
5.0	.0000	+.0001	+.0006	+.0016	+.0034	+.0057	+.0080	+.0094	+.0078	0
6.0	.0000	.0000	+.0002	+.0008	+.0019	+.0039	+.0062	+.0078	+.0068	0
8.0	.0000	.0000	-.0002	.0000	+.0007	+.0020	+.0038	+.0057	+.0054	0
10.	.0000	.0000	-.0002	-.0001	+.0002	+.0011	+.0025	+.0043	+.0045	0
12.	.0000	.0000	-.0001	-.0002	.0000	+.0005	+.0017	+.0032	+.0039	0
14.	.0000	.0000	-.0001	-.0001	-.0001	.0000	+.0012	+.0026	+.0033	0
16.	.0000	.0000	.0000	-.0001	-.0002	-.0004	+.0008	+.0022	+.0029	0

Table IX : **Moments in cylindrical wall**
Rectangular Load
Fixed base, free top
Mom. = coeff. * pH^2
Positive sign indicates tension in the outside



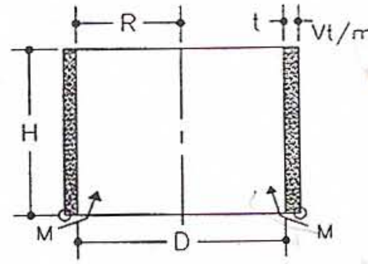
$\frac{H^2}{Dt}$	Coefficients at point									
	0.1H	0.21H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H	1.0H
0.4	-.0023	-.0093	-.0227	-.0439	-.0710	-.1018	-.1455	-.2000	-.2593	-.3310
0.8	.0000	-.0006	-.0025	-.0083	-.0185	-.0362	-.0594	-.0917	-.1325	-.1835
1.2	+.0008	+.0026	+.0037	+.0029	-.0009	-.0089	-.0227	-.0468	-.0815	-.1178
1.6	+.0011	+.0036	+.0062	+.0077	+.0068	+.0011	-.0093	-.0267	-.0529	-.0876
2.0	+.0010	+.0036	+.0066	+.0088	+.0089	+.0059	-.0019	-.0167	-.0389	-.0719
3.0	+.0007	+.0026	+.0051	+.0074	+.0091	+.0083	+.0042	-.0053	-.0223	-.0483
4.0	+.0004	+.0015	+.0033	+.0052	+.0068	+.0075	+.0053	-.0013	-.0145	-.0365
5.0	+.0002	+.0008	+.0019	+.0035	+.0051	+.0061	+.0052	+.0007	-.0101	-.0293
6.0	+.0001	+.0004	+.0011	+.0022	+.0036	+.0049	+.0048	+.0017	-.0073	-.0242
8.0	.0000	+.0001	+.0003	+ .0008	+.0018	+.0031	+.0038	+.0024	-.0040	-.0184
10.	.0000	-.0001	.0000	+.0002	+.0009	+.0021	+.0030	+.0026	-.0022	-.0147
12.	.0000	.0000	-.0001	.0000	+.0004	+.0014	+.0024	+.0022	-.0012	-.0123
14.	.0000	.0000	.0000	.0000	+.0002	+.0010	+.0018	+.0021	-.0007	-.0105
16.	.0000	.0000	.0000	-.0001	+.0001	+.0006	+.0012	+.0020	-.0005	-.0091

Table X : **Moments in cylindrical wall**
Shear per m,V, applied at top
Fixed base, free top
Mom. = coeff. * VH
Positive sign indicates tension in the outside



$\frac{H^2}{Dt}$	Coefficients at point									
	0.1H	0.21H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H	1.0H
0.4	+0.093	+.172	+0.240	+0.300	+0.354	+0.402	+0.448	+0.492	+0.535	+0.578
0.8	+0.085	+0.145	+0.185	+0.208	+0.220	+0.224	+0.223	+0.219	+0.214	+0.208
1.2	+0.082	+0.132	+0.157	+0.164	+0.159	+0.145	+0.127	+0.106	+0.084	+0.062
1.6	+0.079	+0.122	+0.139	+0.138	+0.125	+0.105	+0.081	+0.056	+0.030	+0.004
2.0	+0.077	+0.115	+0.126	+0.119	+0.103	+0.080	+0.056	+0.031	+0.006	-0.019
3.0	+0.072	+0.100	+0.100	+0.086	+0.066	+0.044	+0.025	+0.006	-0.010	-0.024
4.0	+0.068	+0.088	+0.081	+0.063	+0.043	+0.025	+0.010	-0.001	-0.010	-0.019
5.0	+0.064	+0.078	+0.067	+0.047	+0.028	+0.013	+0.003	-0.003	-0.007	-0.011
6.0	+0.062	+0.070	+0.056	+0.036	+0.018	+0.006	0.000	-0.003	-0.005	-0.006
8.0	+0.057	+0.058	+0.041	+0.021	+0.007	0.000	-0.002	-0.003	-0.002	-0.001
10.	+0.053	+0.049	+0.029	+0.012	+0.002	-0.002	-0.002	-0.002	-0.001	0.000
12.	+0.049	+0.042	+0.022	+0.007	0.000	-0.002	-0.002	-0.001	0.000	0.000
14.	+0.046	+0.036	+0.017	+0.004	-0.001	-0.002	-0.001	-0.001	0.000	0.000
16.	+0.044	+0.031	+0.012	+0.001	-0.002	-0.002	-0.001	0.000	0.000	0.000

Table XI **:** **Moments in cylindrical wall**
Moment Per m, M, applied at base
Hinged base, free top
Mom. = coeff. * M
Positive sign indicates tension in outside



$\frac{H^2}{Dt}$	Coefficients at point									
	0.1H	0.21H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H	1.0H
0.4	+.013	+.051	+.109	+.196	+.296	+.414	+.547	+.692	+.843	1
0.8	+.009	+.040	+.090	+.164	+.253	+.375	+.503	+.659	+.824	1
1.2	+.006	+.027	+.063	+.125	+.206	+.316	+.454	+.616	+.802	1
1.6	+.003	+.011	+.035	+.078	+.152	+.253	+.393	+.570	+.775	1
2.0	-.002	-.002	+.012	+.034	+.096	+.193	+.340	+.519	+.748	1
3.0	-.007	-.022	-.030	-.029	+.010	+.087	+.227	+.426	+.692	1
4.0	-.008	-.026	-.044	-.051	-.034	+.023	+.150	+.354	+.645	1
5.0	-.007	-.024	-.045	-.061	-.057	-.015	+.095	+.296	+.606	1
6.0	-.005	-.018	-.040	-.058	-.065	-.037	+.057	+.252	+.572	1
8.0	-.001	-.009	-.022	-.044	-.068	-.062	+.002	+.178	+.515	1
10.	0	-.002	-.009	-.028	-.053	-0.067	-.031	+.123	+.467	1
12.	0	-.000	-.003	-.016	-.040	-.064	-.049	+.081	+.424	1
14.	0	0	0	-.008	-.029	-.059	-.060	+.048	+.387	1
16.	0	0	+.002	-.003	-.021	-.051	-.066	+.025	+.354	1

Table XII **:** **Shear at base of cylindrical wall**

$$V = \text{coef.} * \begin{cases} wH^2 & \text{(triangular)} \\ pH & \text{(rectangular)} \\ M/H & \text{(mom. at base)} \end{cases}$$

Positive sign indicates shear acting inward

$\frac{H_2}{Dt}$	Triangular Load, fixed base	Rectangular Load, fixed base	Triangular or Rectangular Load, hinged base	Moment at edge
0.4	+ 0.436	+ 0.755	+ 0.245	- 1.58
0.8	+ 0.374	+ 0.552	+ 0.234	- 1.75
1.2	+ 0.339	+ 0.460	+ 0.220	- 2.00
1.6	+ 0.317	+ 0.407	+ 0.204	- 2.28
2.0	+ 0.299	+ 0.370	+ 0.189	- 2.57
3.0	+ 0.262	+ 0.310	+ 0.158	- 3.18
4.0	+ 0.236	+ 0.271	+ 0.137	- 3.68
5.0	+ 0.213	+ 0.243	+ 0.121	- 4.10
6.0	+ 0.197	+ 0.222	+ 0.110	- 4.49
8.0	+ 0.174	+ 0.193	+ 0.096	- 5.18
10	+ 0.158	+ 0.172	+ 0.087	- 5.81
12	+ 0.145	+ 0.158	+ 0.079	- 6.38
14	+ 0.135	+ 0.147	+ 0.073	- 6.88
16	+ 0.127	+ 0.137	+ 0.068	- 7.36

Table XIII .Supplementary coeff. for values of H^2 / Dt Greater than 16 *:
(Extension of Tables I toXII)

Coeff. at point

Table I

$\frac{H^2}{Dt}$	.75H	.80H	.85H	.90H	.95H
20	+0.716	+0.654	+0.520	+0.325	+0.115
24	+0.746	+0.702	+0.577	+0.372	+0.137
32	+0.782	+0.768	+0.663	+0.459	+0.182
40	+0.800	+0.805	+ 0.731	+0.530	+0.217
48	+0.791	+0.828	+ 0.785	+0.593	+0.254
56	+0.763	+0.838	+ 0.824	+0.636	+0.285

Table II

$\frac{H^2}{Dt}$	.75H	.80H	.85H	.90H	.95H
20	+0.812	+ 0.817	+ 0.756	+0.603	+ 0.344
24	+0.816	+ 0.839	+ 0.793	+0.647	+ 0.377
32	+0.814	+ 0.861	+ 0.847	+0.271	+ 0.436
40	+0.802	+ 0.866	+ 0.880	+0.778	+ 0.483
48	+0.791	+ 0.864	+ 0.900	+0.820	+ 0.527
56	+0.781	+ 0.859	+ 0.911	+0.852	+ 0.563

Table III

$\frac{H^2}{Dt}$	.75H	.80H	.85H	.90H	.95H
20	+0.949	+0.825	+0629	+0.379	+0128
24	+0.986	+0.879	+0.694	+0.430	+0.149
32	+1.026	+0.953	+0.788	+0.519	+0.189
40	+1.040	+0.996	+ 0.859	+0.591	+0.226
48	+1.043	+1.022	+ 0.911	+0.652	+0.262
56	+1.040	+1.035	+ 0.949	+0.705	+0.294

Table IV

$\frac{H^2}{Dt}$	.75H	.80H	.85H	.90H	.95H
20	+1.062	+1.017	+0.906	+0.703	+ 0.394
24	+1.066	+1.039	+0.943	+0.747	+ 0.427
32	+1.064	+1.061	+0.997	+0.821	+ 0.486
40	+1.052	+1.066	+1.030	+0.878	+ 0.533
48	+1.041	+1.064	+1.050	+0.920	+ 0.577
56	+1.021	+ 1.059	+ 1.061	+0.952	+ 0.613

Table V

$\frac{H^2}{Dt}$	.00H	.05H	.10H	.15H	.20H
20	-16.44	- 9.98	- 4.90	- 1.59	+0.22
24	-18.04	-10.34	- 4.54	- 1.00	+0.68
32	-20.84	-10.72	- 3.70	- 0.04	+1.26
40	-23.34	-10.86	- 2.86	+0.72	+1.56
48	- 25.52	-10.82	- 2.06	+1.26	+1.66
56	- 27.54	-10.68	- 1.36	+1.60	+1.62

Table VI

$\frac{H^2}{Dt}$	.75H	.80H	.85H	.90H	.95H
20	+15.30	+25.9	+36.9	+43.3	+ 35.3
24	+13.20	+25.9	+40.7	+51.8	+ 45.3
32	+ 8.10	+23.2	+45.9	+65.4	+ 63.6
40	+ 3.28	+19.2	+46.5	+77.9	+83.5
48	- 0.70	+14.1	+45.1	+87.2	+103.0
56	- 3.40	+9.2	+42.2	+94.0	+121.0

Table VII

$\frac{H^2}{Dt}$	.80H	.85H	.90H	.95H	1.00H
20	+.0015	+.0014	+.0005	-.0018	- .0063
24	+.0012	+.0012	+.0007	-.0013	- .0053
32	+.0007	+.0009	+ .0007	-.0008	- .0040
40	+.0002	+.0005	+.0006	-.0005	- .0032
48	.0000	+.0001	+ .0006	-.0003	- .0026
56	.0000	.0000	+ .0004	-.0001	- .0023

Table VIII

$\frac{H^2}{Dt}$	.75H	.80H	.85H	.90H	.95H
20	+.0008	+ .0014	+ .0020	+.0024	+ .0020
24	+.0005	+ .0010	+ .0015	+.0020	+ .0017
32	.0000	+ .0005	+ .0009	+.0014	+ .0013
40	.0000	+ .0003	+ .0006	+.0011	+ .0011
48	.0000	+ .0001	+ .0004	+.0008	+ .0010
56	.0000	.0000	+ .0003	+.0007	+ .0008

Table IX

$\frac{H^2}{Dt}$	.80H	.85H	.90H	.95H	1.00H
20	+.0015	+.0013	+.0002	-.0024	- .0073
24	+.0012	+.0012	+.0004	-.0018	- .0061
32	+.0008	+.0009	+.0006	-.0010	- .0046
40	+.0005	+.0007	+.0007	-.0005	- .0037
48	+.0004	+.0006	+.0006	-.0003	- .0031
56	+.0002	+.0004	+.0005	-.0001	- .0026

Table X

$\frac{H^2}{Dt}$	.05H	.10H	.15H	.20H	.25H
20	+0.032	+0.039	+0.033	+0.023	+ 0.014
24	+0.031	+0.035	+0.028	+0.018	+ 0.009
32	+0.028	+0.029	+0.020	+0.011	+ 0.004
40	+0.026	+0.025	+0.015	+0.006	+ 0.001
48	+0.024	+0.021	+0.011	+0.003	0.000
56	+0.023	+0.018	+0.002	+0.002	0.000

Table XI

$\frac{H^2}{Dt}$	.80H	.85H	.90H	.95H	1.00H
20	- 0.015	+0.095	+0.296	+0.606	+1.000
24	- 0.037	+0.057	+0.250	+0.572	+1.000
32	- 0.062	+0.002	+0.178	+0.515	+1.000
40	- 0.067	-0.031	+0.123	+0.467	+1.000
48	- 0.064	-0.049	+0.081	+0.424	+1.000
56	- 0.059	-0.060	+ 0.048	+0.387	+1.000

Table XII

$\frac{H^2}{Dt}$	Tri. Fixed	Rect. Fixed	T.orR. Hinged	Mom. at Edge
20	+0.114	+ 0.122	+ 0.062	- 8.20
24	+0.102	+ 0.111	+ 0.055	- 8.94
32	+0.089	+ 0.096	+ 0.048	- 10.36
40	+0.080	+ 0.086	+ 0.043	- 11.62
48	+0.072	+ 0.079	+ 0.039	- 12.76
56	+0.067	+ 0.074	+ 0.036	- 13.76

* For points not shown in the supplementary tables, ring tension & moment may be determined approximately by sketching curves similar to those in the text.

Table XIV : **Moments in circular slab with center support and fixed edge under uniform load : Moment = coeff. * pR^2**

Positive sign indicates compression in surface Loaded

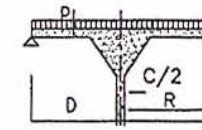
	c/D	0.05R	0.10R	0.15R	0.20R	0.25R	0.30R	0.40R	0.50R	0.60R	0.70R	0.80R	0.90R	1.0R
Radial Moments (M_r)	0.05	-.2100	-.0729	-.0275	-.0026	+.0133	+.0238	+.0342	+.0347	+.0277	+.0142	-.0049	-.0294	-.0589
	0.10		-.1433	-.0624	-.0239	-.0011	+.0136	+.0290	+.326	+.0276	+.0158	-.0021	-.0255	-.0541
	0.15			-.1089	-.0521	-.0200	+.0002	+.0220	+.0293	+.0269	+.0169	+.0006	-.0216	-.0490
	0.20				-.0862	-.0429	-.0161	+.0133	+.0249	+.0254	+.0176	+.0029	-.0178	-.0441
	0.25					-.0698	-.0351	+.0029	+.0194	+.0231	+.0177	+.0049	-.0143	-.0393
Tangential Moment (M_ϕ)	0.05	-.0417	-.0700	-.0541	.0381	-.0251	-.0145	+.0002	+.0085	+.0118	+.0109	+.0065	-.0003	-.0118
	0.10		-.0287	-.0421	-.0354	-.0258	-.0168	-.0027	+.0059	+.0099	+.0098	+.0061	-.0009	-.0108
	0.15			-.0218	-.0284	-.0243	-.0177	-.0051	+.0031	+.0080	+.0086	+.0057	-.0006	-.0098
	0.20				-.0172	-.0203	-.0171	-.0070	+.0013	+.0063	+.0075	+.0052	-.0003	-.0088
	0.25					-.0140	-.0150	-.0083	-.0005	+.0046	+.0064	+.0048	.0000	-.0078

Table XV : **Moments in circular slab with center support and hinged edge under uniform load : Moment = coeff. * pR^2**

Positive sign indicates compression in surface Loaded

	c/D	0.05R	0.10R	0.15R	0.20R	0.25R	0.30R	0.40R	0.50R	0.60R	0.70R	0.80R	0.90R	1.0R
Radial Moments (M_r)	0.05	-.3658	-.1388	-.0640	-.0221	+.0058	+.0255	+.0501	+.0614	+.0629	+.0566	+.0437	+.0247	
	0.10		-.2487	-.1180	-.0557	-.0176	+.0081	+.0391	+.0539	+.0578	+.0532	+.0416	+.0237	0
	0.15			-.1869	-.0977	-.0467	-.0135	+.0258	+.0451	+.0518	+.0494	+.0393	+.0226	0
	0.20				-.1465	-.0800	-.0381	+.0109	+.0352	+.0452	+.0451	+.0368	+.0215	0
	0.25					-.1172	-.0645	-.0055	+.0245	+.0381	+.0404	+.0340	+.0200	0
Tangential Moments (M_r)	0.05	-.0731	-.1277	-.1040	-.0786	-.0569	-.391	-.0121	+.0061	+.0175	+.0234	+.0251	+.0228	+.0168
	0.10		-.0498	-.0768	-.0684	-.0539	-.0394	-.0153	+.0020	+.0134	+.0197	+.0218	+.0199	+.0145
	0.15			-.0374	-.0516	-.0470	-.0375	-.0175	-.0014	+.0097	+.0163	+.0186	+.0172	+.0123
	0.20				-.0293	-.03667	-.0333	-.0184	-.0042	+.0065	+.0132	+.0158	+.0148	+.0103
	0.25					-.0234	-.0263	-.0184	-.0062	+.0038	+.0103	+.0132	+.0122	+.0085

Table XVI : Moments in circular slab with center support and hinged edge due to moment per m., M, applied at edge :
Moment = coeff. * M
Positive sign indicates compression in top surface.



	c/D	0.05R	0.10R	0.15R	0.20R	0.25R	0.30R	0.40R	0.50R	0.60R	0.70R	0.80R	0.90R	1.00R
Radial Moments (Mr)	.05	-2.65	-1.12	-.622	-.333	-.129	+.029	+.268	+.450	+.596	+.718	+.824	+.917	+1.0
	.10		-1.95	-1.03	-.584	-.305	-.103	+.187	+.394	+.558	+.692	+.808	+.909	+1.0
	.15			-1.59	-.930	-.545	-.280	+.078	+.323	+.51	+.663	+.790	+.900	+1.0
	.20				-1.37	-.842	-.499	-.057	+.236	+.451	+.624	+.768	+.891	+1.0
	.25					-1.20	-.765	-.216	+.130	+.392	+.577	+.740	+.880	+1.0
Tangential Moment (MΦ)	.05	-.530	-.980	-.847	-.688	-.544	-.418	-.211	-.042	+.095	+.212	+.314	+.405	+.486
	.10		-.388	-.641	-.608	-.158	-.419	-.233	-.072	+.066	+.185	+.290	+.384	+.469
	.15			-.319	-.472	-.463	-.404	-.251	-.100	+.035	+.157	+.263	+.363	+.451
	.20				-.272	-.372	-.368	-.261	-.123	+.007	+.129	+.240	+.340	+.433
	.25					-.239	-.305	-.259	-.145	-.0200	+.099	+.214	+.320	+.414

Table XVIII : Load on center support for circular slab
Load = coeff.x : (pR^2 hinged & fixed)
(M -moment at edge)

c/D	0.05	0.10	0.15	0.20	0.25
Hinged	1.320	1.387	1.463	1.542	1.625
Fixed	0.839	0.919	1.007	1.101	1.200
M at Edge	8.16	8.66	9.29	9.99	10.81

Table XIX : Stiffness of Circular Plate with Center Support :
$k = \text{coeff.} \times Et^3 / R$

c/D	0.05	0.10	0.15	0.20	0.25
Coeff.	0.290	0.309	0.332	0.358	0.387
Without Center Support : $k = 0.104\ Et^3 / R$					

Table XX : Stiffness of Cylindrical Wall, near edge hinged, far edge free : $k = \text{coeff.} \times Et^3 / H$

H^2/Dt	0.4	0.8	1.2	1.6	2	3	4	5	6	8	10	12	14	16	20	24	32	40	48	50
coeff.	.139	.27	.345	.399	.445	.548	.635	.713	.783	.903	1.01	1.11	1.2	1.28	1.43	1.57	1.81	2.03	2.22	2.4

VI.4. Loads and Forces Acting on Bunkers and Silos :

a) Bunkers :

The height of the bunker is so small that the plane of natural repose of the filling material does not intersect the opposite wall, i.e. $h \tan \alpha < L$, or practically $h < 1.5L$. In this case the side pressure can be calculated according to the earth pressure theory : horizontal press. intensity $p_2 = \gamma\, h \tan^2 (45 - \rho/2)$

vertical " " $p_1 = \gamma\, h$

The friction between the filling material and the walls can be neglected.

b) Silos :

Silos are generally very deep ($h \tan \alpha > L$ i.e. $h > 1.5L$) and therefore the wedge of the material causing the pressure intersects the opposite walls and hence not fully acting.

The forces acting on silos are :

Vertical press :

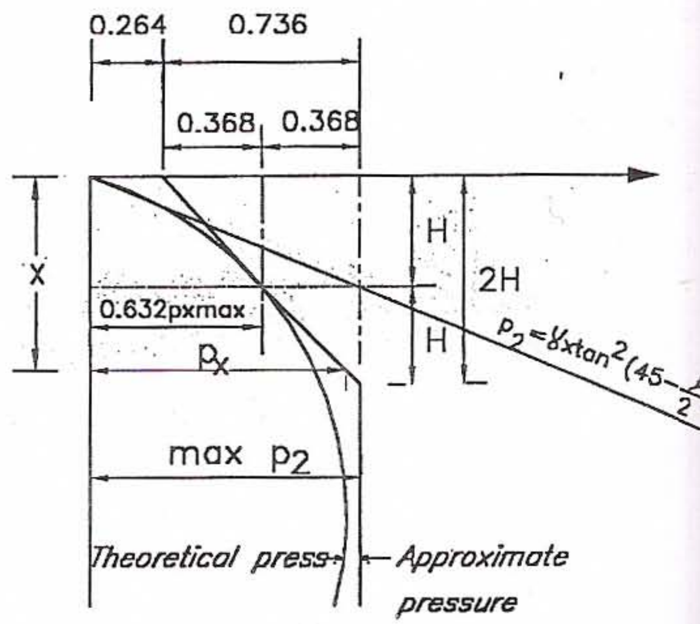
$$p_1 = \frac{\gamma}{k} \left(1 - \frac{1}{e^{kx}}\right),$$

max. $p_1 = \gamma / k$

horiz. press :

$$p_2 = \gamma \left(1 - \frac{1}{e^{kx}}\right) \cdot \frac{A}{o \tan \rho}$$

at $x = \infty$ max. $p_2 = \dfrac{\gamma}{\frac{o}{A} \tan \rho'}$



where :

o = Circumference of cell.

A = Cross-sectional area of cell

ρ = Angle of repose of filling material.

ρ' = Angle of fricition between filling material and silo walls.

$$k = \frac{o}{A} \cdot \tan \rho' \cdot \tan^2 \left(45 - \frac{\rho}{2}\right)$$

The pressure on silo can be approx. assumed as shown in fig. In this case max. p_2 occurs at $2H = 2 \max p_2 / \gamma \tan^2 (45 - \rho/2)$.

c) Properties of Some Filling Materials :

	corn	maiz	grains	cement	sand gravel	coal powder	coal stone	stones
γ kg/m3	820	650	700-850	-1200 1600	1800	700	850	1800
ρ^o	25-28	25	25-30	25	30-35	25	30	30

CHAPTER VII

REINFORCED CONCRETE

A) *Design of Sections*
(working stress design method)
B) *Details of Deep Beams and Short cantilevers.*

El-Bhairy R.C.Design Handbook Chapter (7) Reinforcd Concrete: Design and Details - Contents

VII.1. LOAD DISTRIBUTION ON TWO WAY SLABS

1. *Load Distribution According to U.A.R.*

r	1.00	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.00
α	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
β	0.35	0.29	0.25	0.21	0.18	0.16	0.14	0.12	0.11	0.09	0.08

2. *Load Distribution According to Marcus :*

r	1.00	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.00
α	.396	.473	.543	.606	.660	.706	.746	.778	.806	.830	.849
β	.396	.323	.262	.212	.172	.140	.113	.093	.077	.063	.053

3. *Load Distribution According to Grashoff :*

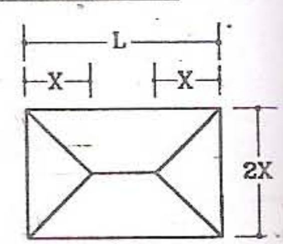
r	1.00	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.00
α	.500	.595	.672	.742	.797	.834	.869	.893	.914	.928	.941
β	.500	.405	.328	.258	.203	.166	.131	.107	.086	.072	.059

$r = \frac{m.b}{m.a}$; where m= **0.87 for continuity at one end of the slab**

=0.76 for continuity at both ends of the slab

VII.2. EQUIVALENT LOAD FOR DESIGN OF BEAMS :

supporting two way Slabs :



$\frac{L}{2x}$	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
α	.667	.725	.769	.803	.829	.852	.870	.885	.897	.908	.917
β	.500	.545	.583	.615	.643	.667	.688	.706	.722	.737	.750

VII.3. DIMENSIONING OF REINFORCED CONCRETE RECTANGULAR SECTIONS SUBJECT TO SIMPLE BENDING OR ECCENTRIC FORCES WITH BIG ECCENTRICITIES

Tables of k_1 & k_2 for : $f_s = 800 - 2400$ kg/cm^2

n = 15 (units in kgs & cms) $\beta = \frac{d'}{d} = 0.1$

f_c	f_s	800	1000	1200	1400	1600	1800	2000	2200	2400
25	k_1	.538	.568	.604	.636	.676	.702	.731	.760	.785
	k_2	715	909	1105	1301	1499	1697	1894	2092	2292
30	k_1	.459	.490	.518	.547	.575	.598	.621	.645	.668
	k_2	704	897	1091	1286	1483	1679	1877	2075	2274
35	k_1	.408	.433	.457	.479	.502	.521	.542	.562	.590
	k_2	694	885	1078	1273	1469	1665	1862	2060	2256
40	k_1	.369	.390	.411	.430	.449	.466	.484	.502	.520
	k_2	686	875	1068	1260	1455	1651	1846	2044	2240
45	k_1	.339	.357	.375	.392	.408	.423	.438	.454	.472
	k_2	678	866	1056	1248	1441	1636	1832	2028	2226
50	k_1	.314	.330	.345	.361	.375	.389	.401	.415	.427
	k_2	671	857	1046	1237	1430	1628	1818	2014	2210
55	k_1	.296	.308	.321	.334	.347	.359	.371	.383	.394
	k_2	665	849	1037	1227	1419	1611	1806	2000	2194
60	k_1	.277	.289	.301	.313	.324	.335	.347	.360	.367
	k_2	659	842	1028	1217	1408	1600	1793	1987	2184
65	k_1	.262	.273	.284	.294	.305	.317	.324	.340	.344
	k_2	654	835	1021	1208	1398	1589	1782	1975	2170
70	k_1		.259	.279	.279	.288	.296	.306	.315	.325
	k_2		829	1013	1200	1389	1578	1771	1963	2158
75	k_1		.247	.256	.265	.274	.282	.289	.298	.306
	k_2		823	1006	1192	1379	1570	1761	1952	2144
80	k_1		.237	.245	.253	.261	.268	.276	.283	.290
	k_2		818	1000	1185	1371	1561	1750	1941	2132
85	k_1		.227	.235	.242	.250	.255	.263	.274	.277
	k_2		813	994	1178	1363	1552	1740	1931	2122
90	k_1		.219	.226	.233	.240	.246	.250	.258	.265
	k_2		809	988	1171	1357	1543	1731	1920	2110
95	k_1		.211	.217	.224	.230	.236	.243	.248	.252
	k_2		804	983	1165	1349	1535	1722	1912	2102
100	k_1		.204	.210	.216	.222	.229	.233	.239	.245
	k_2		800	978	1159	1342	1528	1715	1904	2004

DIMENSIONING OF R.C. RECTANGULAR SECTIONS SUBJECT TO SIMPLE BENDING OR ECCENTRIC FORCES WITH BIG ECCENTRICITIES

TABLES OF k_1 & k_2 FOR : $f_s = 1200\ kg/cm^2$

1.For Simple Bending :

$$d = k_1\sqrt{\frac{M}{b}},\ A_s = \frac{M}{k_2 . d};$$

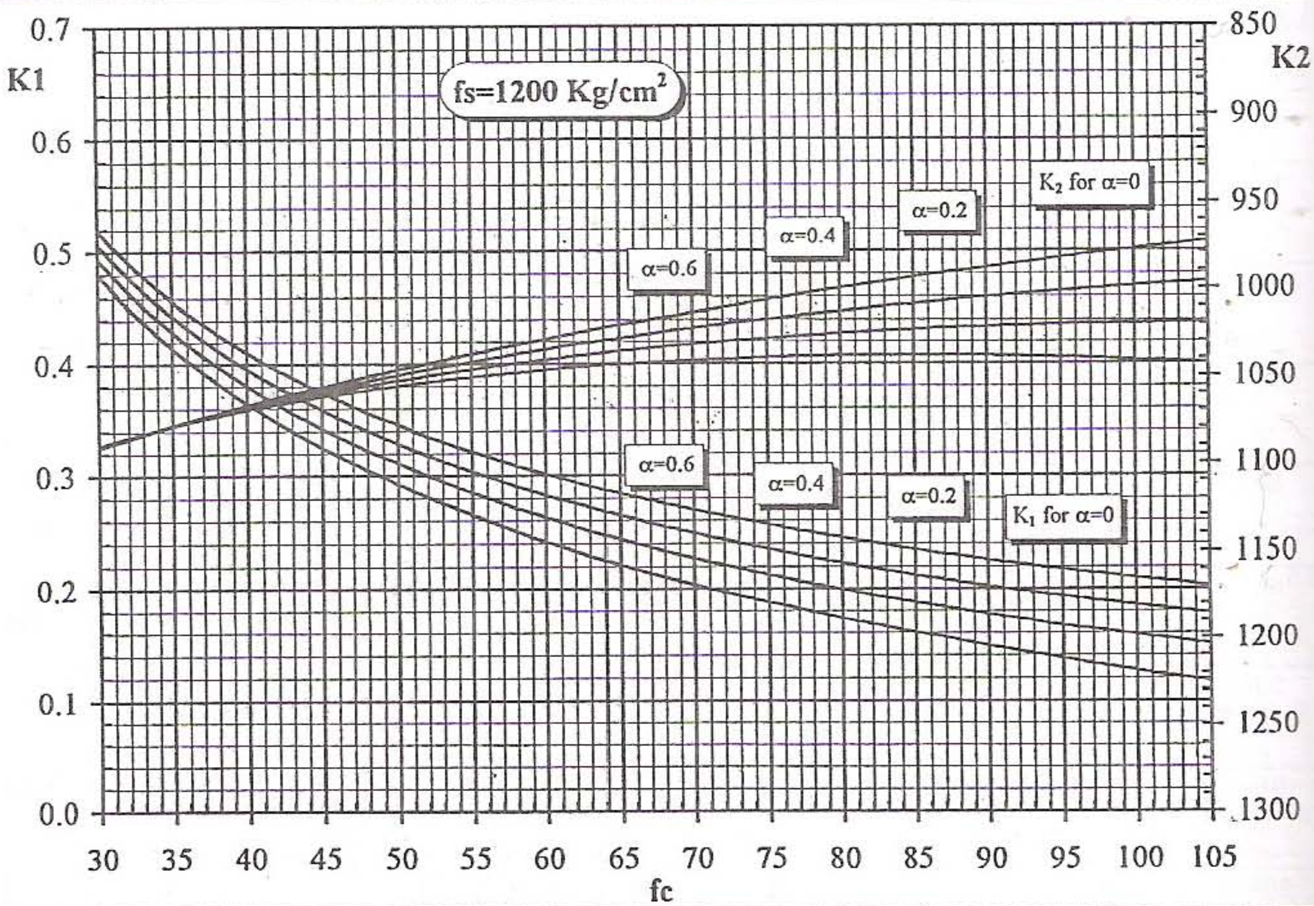
2.For Eccentric Force :

$$d = k_1\sqrt{\frac{M_s}{b}},\ A_s = \frac{M_s}{k_2 . d} \pm \frac{N}{f_s}$$

(Units in kgs. and cm.)

$e_s = e - t/2 + cover$ (tension) $e_s = e + t/2 - cover$ (Compression)

f_c	$\alpha = 0$		$\alpha = 0.2$		$\alpha = 0.4$		$\alpha = 0.6$	
	K_1	K_2	K_1	K_2	K_1	K_2	K_1	K_2
45	.375	1056	.359	1058	.342	1060	.325	1062
50	.345	1046	.329	1049	.311	1052	.292	1055
55	.321	1037	.303	1041	.284	1046	.265	1050
60	.301	1028	.282	1034	.263	1040	.241	1046
65	.284	1021	.264	1028	.244	1035	.221	1042
70	.269	1013	.249	1022	.229	1032	.204	1041
75	.256	1006	.235	1017	.212	1028	.188	1039
80	.245	1000	.223	1013	.199	1026	.173	1038
85	.235	994	.212	1009	.187	1023	.160	1038
90	.226	988	.202	1005	.177	1022	.148	1038
95	.217	983	.193	1002	.167	1021		
100	.210	978	.185	999	.159	1020		



DIMENSIONING OF R.C. RECTANGULAR SECTIONS SUBJECT TO SIMPLE BENDING OR ECCENTRIC FORCES WITH BIG ECCENTRICITIES

TABLES OF k_1 & k_2 FOR : $f_s = 1400\ kg/cm^2$

1.For Simple Bending :

$$d = k_1 \sqrt{\frac{M}{b}}, A_s = \frac{M}{k_2 . d};$$

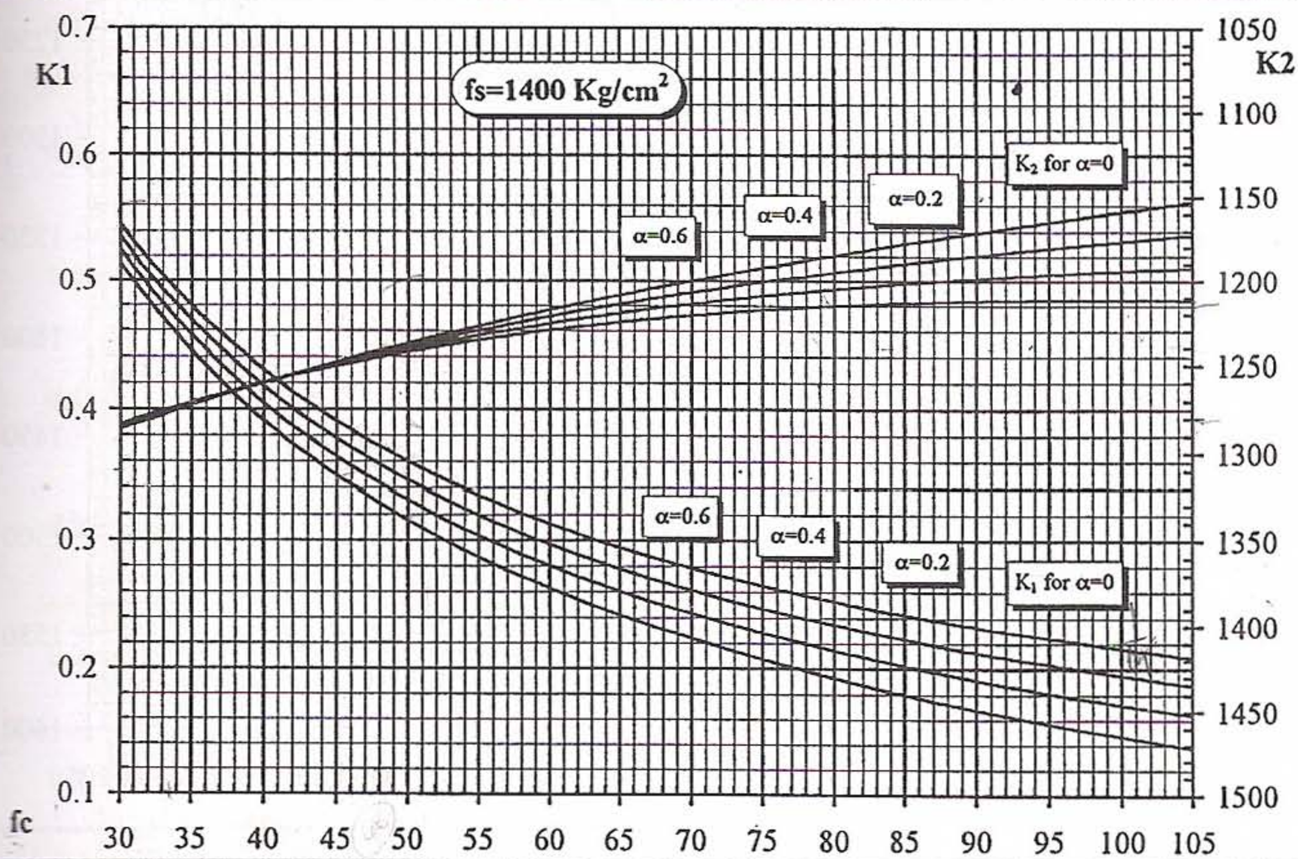
2.For Eccentric Force :

$$d = k_1 \sqrt{\frac{M_s}{b}}, A_s = \frac{M_s}{k_2 . d} \pm \frac{N}{f_s}$$

(Units in kgs. and cm.)

$e_s = e - t/2 + \text{cover}$ (tension) $e_s = e + t/2 - \text{cover}$ (Compression)

f_c	$\alpha = 0$		$\alpha = 0.2$		$\alpha = 0.4$		$\alpha = 0.6$	
	K_1	K_2	K_1	K_2	K_1	K_2	K_1	K_2
45	.392	1248	.378	1249	.363	1250	.350	1251
50	.361	1237	.346	1239	.331	1241	.317	1243
55	.334	1227	.319	1230	.304	1233	.287	1235
60	.313	1217	.297	1221	.280	1226	.263	1230
65	.294	1208	.279	1214	.261	1219	.242	1225
70	.279	1200	.262	1207	.243	1214	.223	1221
75	.265	1192	.247	1200	.228	1209	.208	1217
80	.253	1185	.234	1195	.214	1205	.193	1215
85	.242	1178	.223	1190	.202	1201	.180	1213
90	.233	1171	.213	1185	.192	1198	.168	1212
95	.224	1165	.203	1180	.181	1196	.157	1211
100	.216	1159	.195	1176	.171	1194	.147	1211



DIMENSIONING OF R.C. RECTANGULAR SECTIONS SUBJECT TO SIMPLE BENDING OR ECCENTRIC FORCES WITH BIG ECCENTRICITIES

TABLES OF k_1 & k_2 FOR : $f_s = 1600\ kg/cm^2$

1.For Simple Bending :

$$d = k_1\sqrt{\frac{M}{b}},\ A_s = \frac{M}{k_2 \cdot d};$$

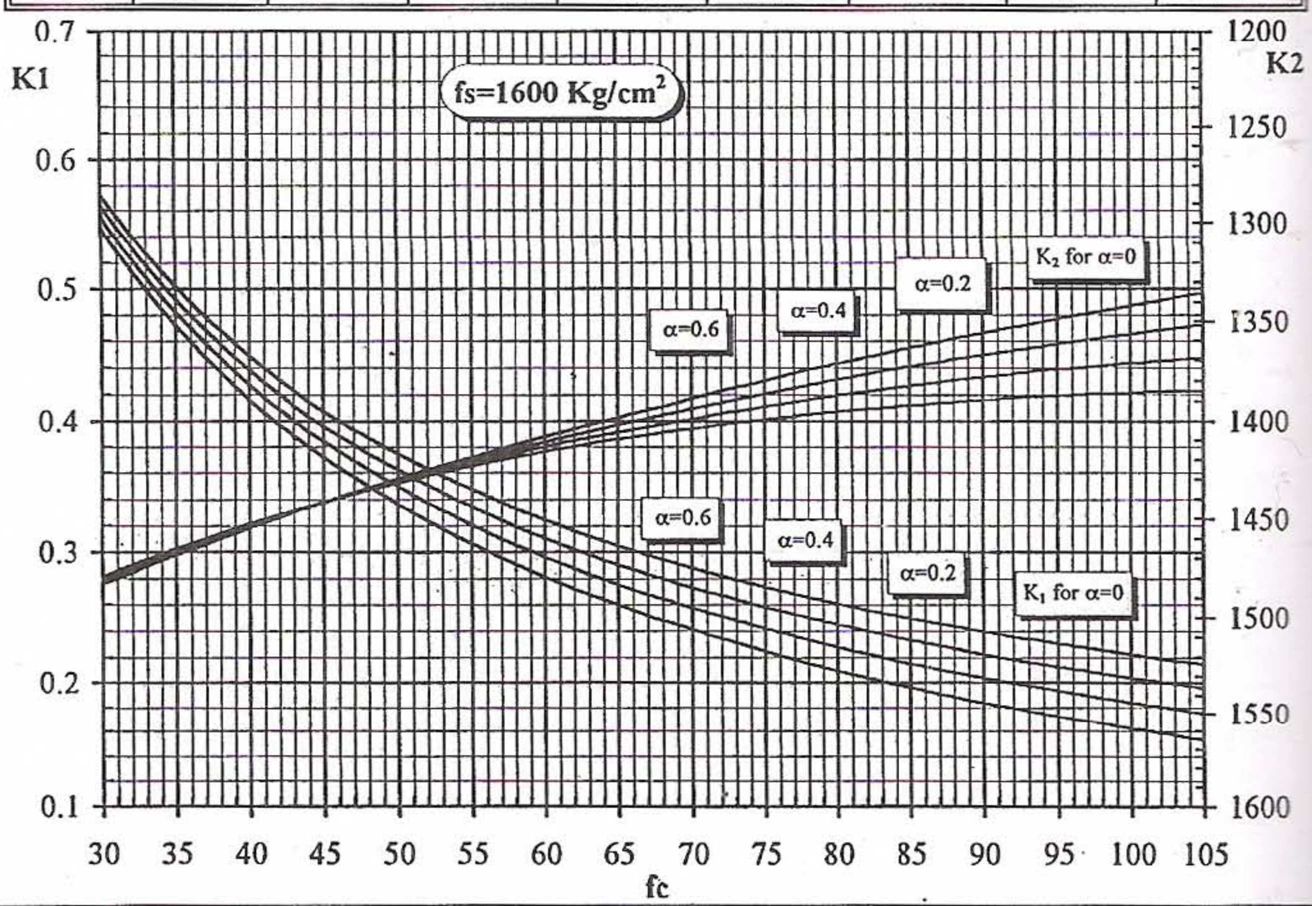
2.For Eccentric Force :

$$d = k_1\sqrt{\frac{M_s}{b}},\ A_s = \frac{M_s}{k_2 \cdot d} \pm \frac{N}{f_s}$$

(Units in kgs. and cm.)

$e_s = e - t/2 + \text{cover (tension)}$ $\quad e_s = e + t/2 - \text{cover (Compression)}$

f_c	$\alpha = o$		$\alpha = 0.2$		$\alpha = 0.4$		$\alpha = 0.6$	
	K_1	K_2	K_1	K_2	K_1	K_2	K_1	K_2
45	.408	1441	.397	1441	.386	1441	.373	1441
50	.375	1430	.361	1431	.349	1432	.336	1432
55	.347	1419	.334	1240	.320	1422	.307	1423
60	.324	1408	.311	1411	.296	1413	.281	1416
65	.305	1398	.284	1402	.276	1406	.260	1409
70	.288	1389	.273	1394	.268	1399	.239	1405
75	.274	1379	.256	1386	.242	1392	.224	1399
80	.261	1371	.244	1379	.227	1387	.209	1395
85	.250	363	.233	1373	.215	1382	.196	1392
90	.240	1357	.222	1368	.204	1379	.183	1390
95	.230	1349	.212	1362	.193	1374	.172	1387
100	.222	1342	.204	1357	.183	1371	.102	1386



DIMENSIONING OF R.C. RECTANGULAR SECTIONS SUBJECT TO SIMPLE BENDING OR ECCENTRIC FORCES WITH BIG ECCENTRICITIES

TABLES OF k_1 & k_2 FOR : $f_s = 1800\ kg/cm^2$

1.For Simple Bending :

$$d = k_1\sqrt{\frac{M}{b}}, A_s = \frac{M}{k_2 . d};$$

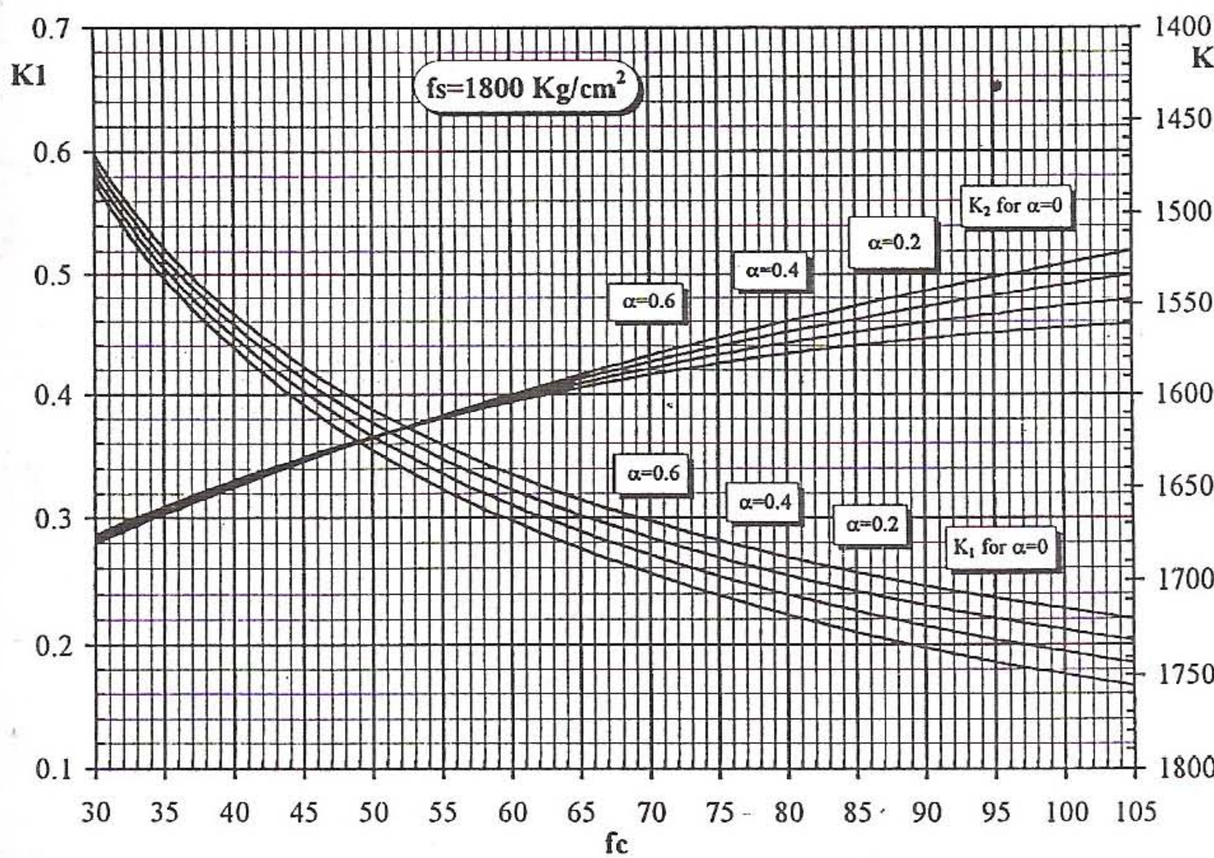
2.For Eccentric Force :

$$d = k_1\sqrt{\frac{M_s}{b}}, A_s = \frac{M_s}{k_2 . d} \pm \frac{N}{f_s}$$

(Units in kgs. and cm.)

$e_s = e - t/2 + cover$ (tension) $e_s = e + t/2 - cover$ (Compression)

f_c	α= 0		α = 0.2		α= 0.4		α = 0.6	
	K_1	K_2	K_1	K_2	K_1	K_2	K_1	K_2
45	.423	1636	.411	1636	.403	1635	.392	1634
50	.389	1628	.378	1623	.366	1623	.355	1623
55	.359	1611	.348	1611	.336	1612	.324	1612
60	.335	1600	.323	1602	.311	1603	.298	1604
65	.317	1589	.303	1591	.289	1594	.275	1596
70	.296	1578	.283	1582	.269	1585	.255	1589
75	.282	1570	.268	1575	.254	1579	.239	1584
80	.268	1561	.254	1567	.239	1572	.223	1578
85	.255	1552	.242	1559	.226	1566	.209	1574
90	.246	1543	.231	1551	.215	1556	.197	1569
95	.236	1535	.220	1546	.204	1556	.186	1566
100	.229	1528	.212	1540	.195	1551	.176	1563



DIMENSIONING OF R.C. RECTANGULAR SECTIONS SUBJECT TO SIMPLE BENDING OR ECCENTRIC FORCES WITH BIG ECCENTRICITIES

TABLES OF k_1 & k_2 FOR : $f_s = 2000\ kg/cm^2$

1.For Simple Bending :

$$d = k_1\sqrt{\frac{M}{b}},\ A_s = \frac{M}{k_2 . d};$$

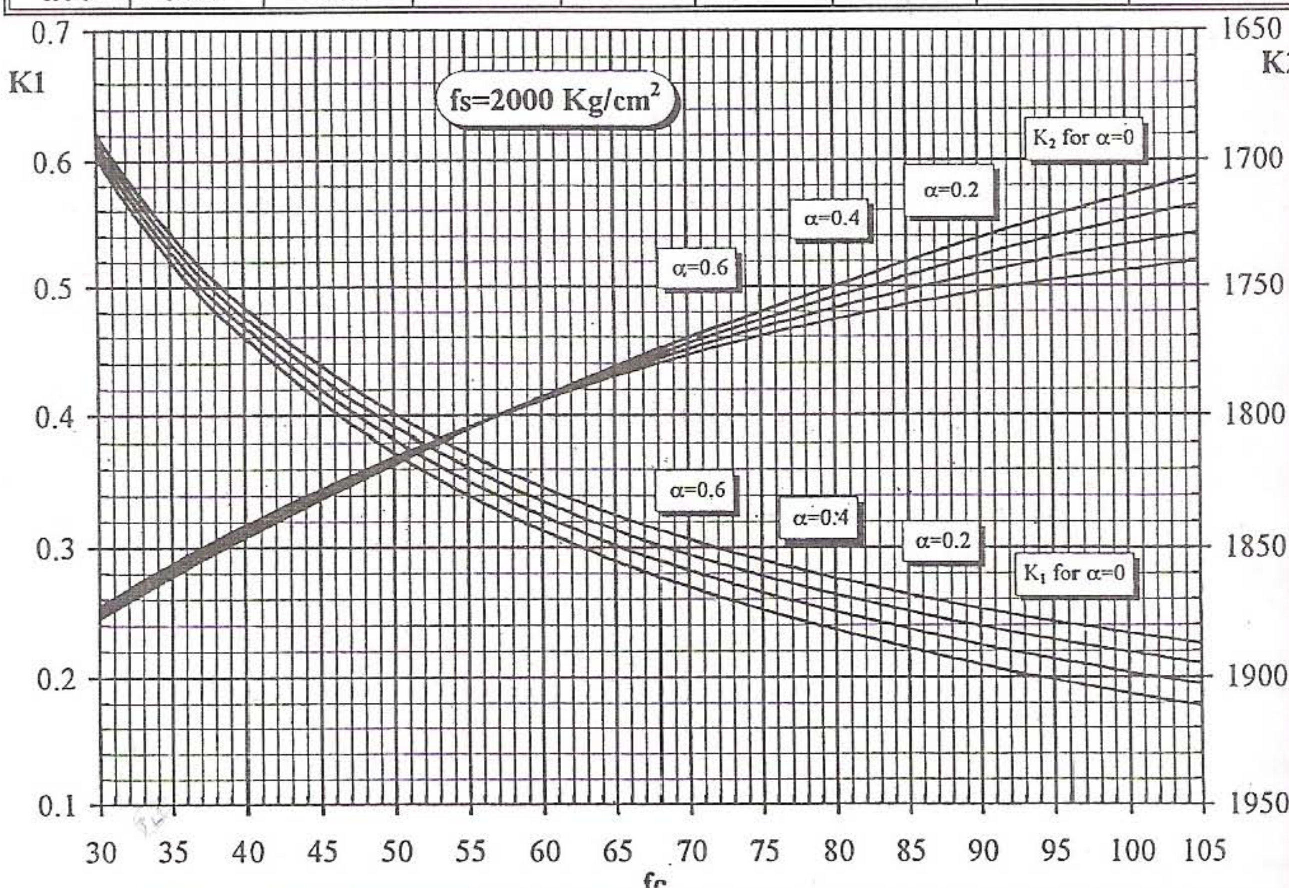
2.For Eccentric Force :

$$d = k_1\sqrt{\frac{M_s}{b}},\ A_s = \frac{M_s}{k_2 . d} \pm \frac{N}{f_s}$$

(Units in kgs. and cm.)

$e_s = e - t/2 + cover$ (tension) $e_s = e + t/2 - cover$ (Compression)

f_c	$\alpha = 0$		$\alpha = 0.2$		$\alpha = 0.4$		$\alpha = 0.6$	
	K_1	K_2	K_1	K_2	K_1	K_2	K_1	K_2
45	.438	1832	.429	1831	.421	1829	.411	1828
50	.401	1818	.389	1816	.382	1817	.372	1815
55	.371	1806	.361	1806	.350	1805	.339	1805
60	.347	1793	.335	1793	.325	1794	.312	1794
65	.324	1782	.313	1783	.302	1784	.289	1785
70	.306	1771	.294	1773	.282	1775	.269	1777
75	.289	1761	.277	1763	.265	1766	.251	1769
80	.276	1750	.263	1754	.250	1759	.236	1763
85	.263	1740	250	1748	.236	1751	.222	1753
90	.250	1731	.239	1738	.226	1745	.209	1752
95	.243	1722	.229	1730	.214	1739	.198	1747
100	.233	1715	.219	1724	.205	1735	.188	1744



DIMENSIONING OF R.C. RECTANGULAR SECTIONS SUBJECT TO SIMPLE BENDING OR ECCENTRIC FORCES WITH BIG ECCENTRICITIES

TABLES OF k_1 & k_2 FOR : $f_s = 2200\ kg/cm^2$

(Units in kgs. and cm.)

1.For Simple Bending :

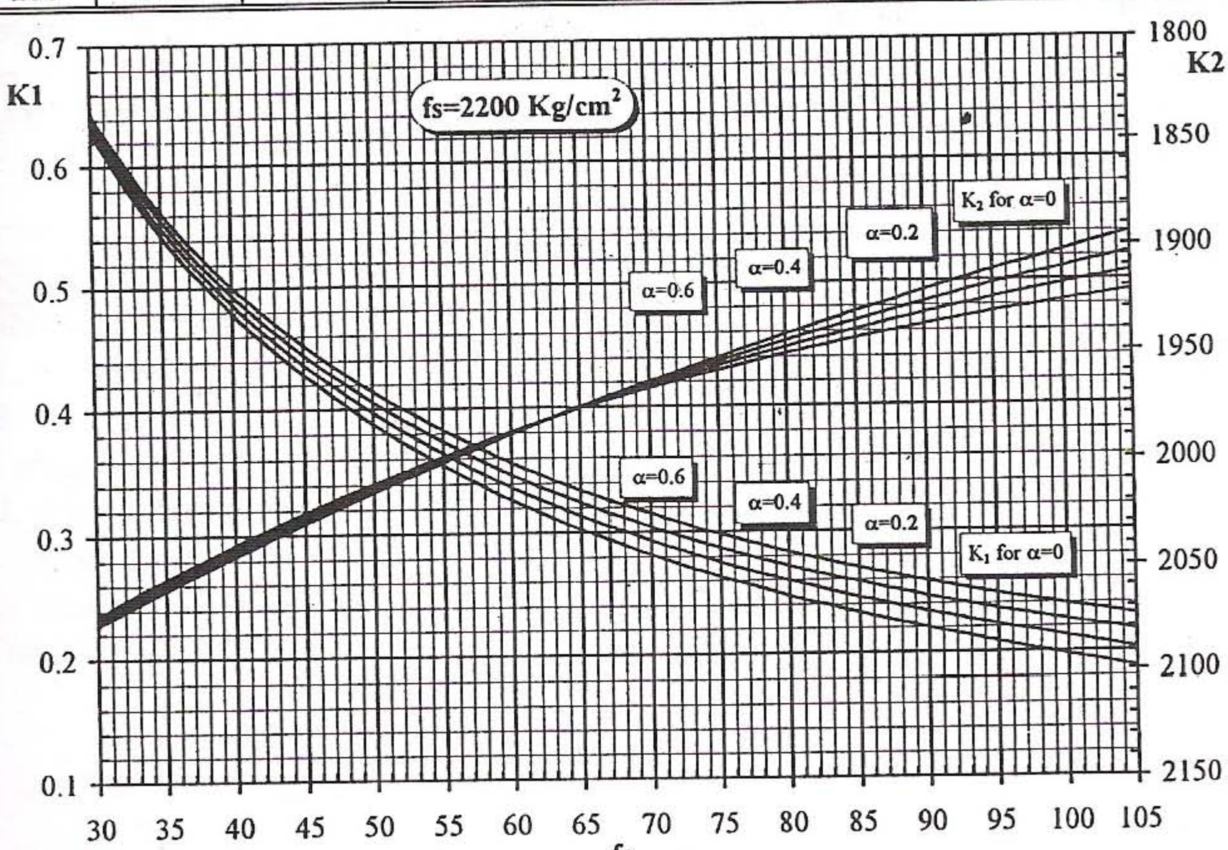
$$d = k_1\sqrt{\frac{M}{b}}, A_s = \frac{M}{k_2 . d};$$

2.For Eccentric Force :

$$d = k_1\sqrt{\frac{M_s}{b}}, A_s = \frac{M_s}{k_2 . d} \pm \frac{N}{f_s}$$

$e_s = e - t/2 + cover$ (tension) $e_s = e + t/2 - cover$ (Compression)

f_c	$\alpha = 0$		$\alpha = 0.2$		$\alpha = 0.4$		$\alpha = 0.6$	
	K_1	K_2	K_1	K_2	K_1	K_2	K_1	K_2
45	.454	2028	.445	2027	.436	2025	.427	2023
50	.415	2014	.406	2012	.397	2011	.388	2009
55	.383	2000	.374	1998	.365	1997	.356	1996
60	.360	1987	.348	1987	.336	1987	.324	1986
65	.340	1975	.328	1975	.316	1975	.304	1975
70	.315	1963	.302	1964	.289	1966	.276	1976
75	.298	1952	.285	1954	.272	1956	.259	1958
80	.283	1941	.279	1944	.265	1947	.251	1950
85	.274	1931	.260	1935	.245	1939	.230	1943
90	.258	1920	.242	1925	.227	1931	.212	1937
95	.248	1912	.232	1919	.216	1925	.200	1932
100	.239	1904	.223	1912	207	1920	.191	1928



VII. DIMENSIOING OF T- and L- SECTIONS.

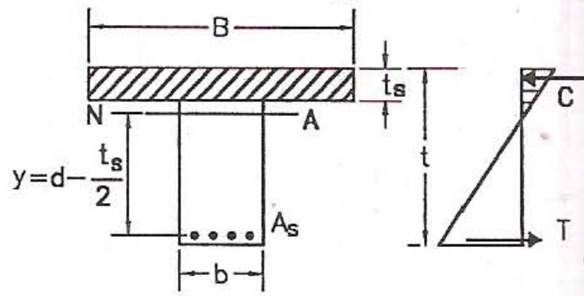
T - Sections may be devided with respect to their dimensioning into two groups :

a) T - Sections with $B/b_o > 5$

b) T - Sections with $B/b_o \leq 5$

a) T - Sections with $B/b_o > 5$

In this case the compression zone is very big and the compression stresses will be very low. The neutral axis will be very near to the flange . The compression in web can be neglected . The comp. force can be considered as acting in the middle of the flange.



$$A_s = \frac{M_s}{(d - t_s/2).\,f_s} \pm \frac{N}{f_s}$$

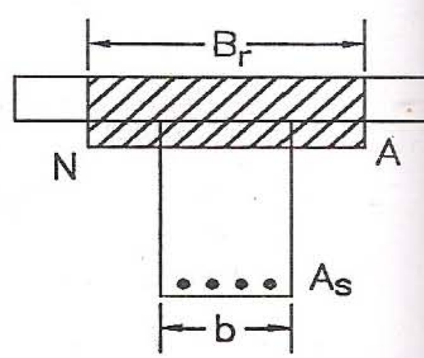
(for simple bending : N = o .)

And the max. comp. stress in the flange is :

$$f_c = \frac{f_s}{d - t_s/2}\left(\frac{f_s \,.\, d}{B \,.\, t_s} + \frac{t_s}{30}\right) - \frac{N}{B.\,t_s\,(1 - t_s/2d)}$$

b) T - Sections with $B/b_o \leq 5$

The compression stresses are usually high. They must not exceed 0.75 of the allowable f_c for rectangular section . The neutral axis lies outside the flange. The compression in web can be neglected and the comp. zone can be assumed as a rectangle with a reduced breadth (B_r) . The dimensioning of a T - Sec. is in this case exactly the same as rectangular section as follows :



$z = k_z\sqrt{\frac{M_s}{B}}$ (k_z variable with f_c & f_s, see table a)

$B_r = r.B$ (r = reduction factor depending on $\frac{t_s}{z}$ & $\frac{B}{bo}$ see table b or the curve).

$$d = k_1\sqrt{\frac{M_s}{B_r}}$$

$$A_s = \frac{M_s}{k_2 .\, d} \pm \frac{N}{f_s}$$

k_1 & k_2 are the same as for rectangular sec.).

Table (a) : *Values of k (position of N.A. in T-Sections):*

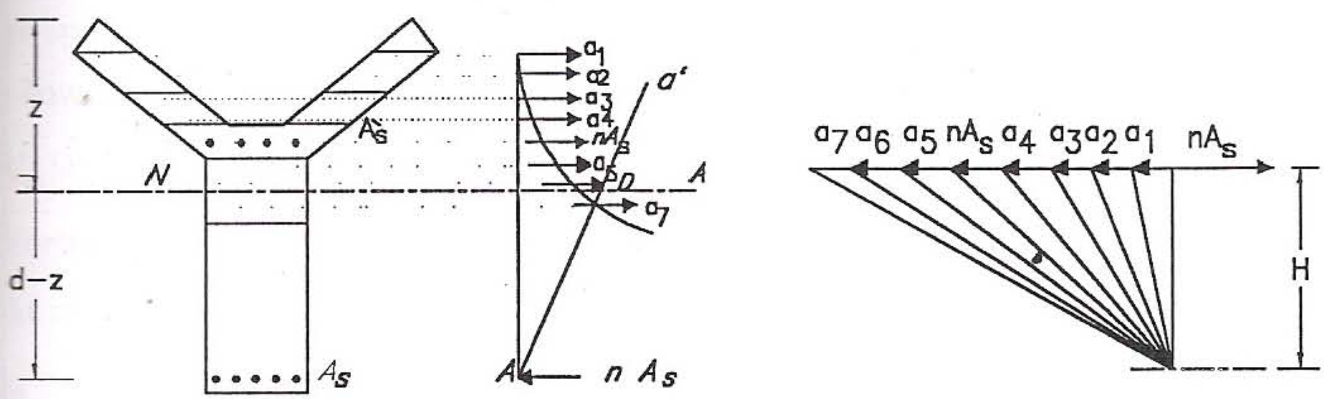
f_s =	1200	1400	1600	1800	2000	2200	2400
f_c = 30	.141	.133	.126	.119	.114	.110	.105
40	.137	.129	.122	.117	.112	.107	.103
50	.133	.126	.119	.114	.110	.105	.101
average	.14	.130	.122	.115	.112	.107	.103

Table (b) : *Reduction Factor r (Br = r.B) :*

	t_s/z								
B/b_o	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2
2.0	1.0	1.0	.98	.96	.92	.88	.82	.76	.68
2.5	1.0	.99	.98	.95	.90	.85	.78	.70	.62
3.0	1.0	.99	.97	.94	.89	.83	.76`	.67	.57
3.5	1.0	.99	.97	.94	.89	.82	.74	.65	.54
4.0	1.0	.99	0.97	.93	.88	.81	.73	.63	.52
5.0	1.0	.99	0.97	.93	.87	.80	.71	.61	.49

VII.6 *Graphical Determination of Stresses*

Graphical method for the determination of stresses is generally used for complicated forms of sections. We proceed as follows :



a) Determination of the N.A. of the section :

1. devide the area of the section into small strips of area a1 , a2 , ...
2. consider the area of each strip as a force acting in the centre of gravity , the steel is to be introduced with n-times its area.
3. draw the force polygon , polar diagram and link polygon as shown. The horizontal through the point of intersection Aa` & Bb` gives the position of the N.A.

b) The stresses in concrete & steel can be determined from the relations :

$$f_c = \frac{M.z}{I_{nv}} \quad \& \quad f_s = \frac{M\,(d - z)\,n}{I_{nv}}$$

where:

I_{nv} = the moment of interia of the virtual area about the N.A. = 2H x Area ABD

VII.4. DESIGN AND DETAILS OF DEEP BEAMS

Definition :

Deep beams are those which have depth to effective span ratio greater than :

d/L >	0.8	for simple beams.
	0.4	for continuous beams.

A) Simple supported deep beams subject to uniform loads :

Tension $T = \max. M_o / Y_{ct}$ and $A_s = T / f_s$, in which :

Y_{ct} (Lever arm) = 0.86 L (but not more than 0.87 d)

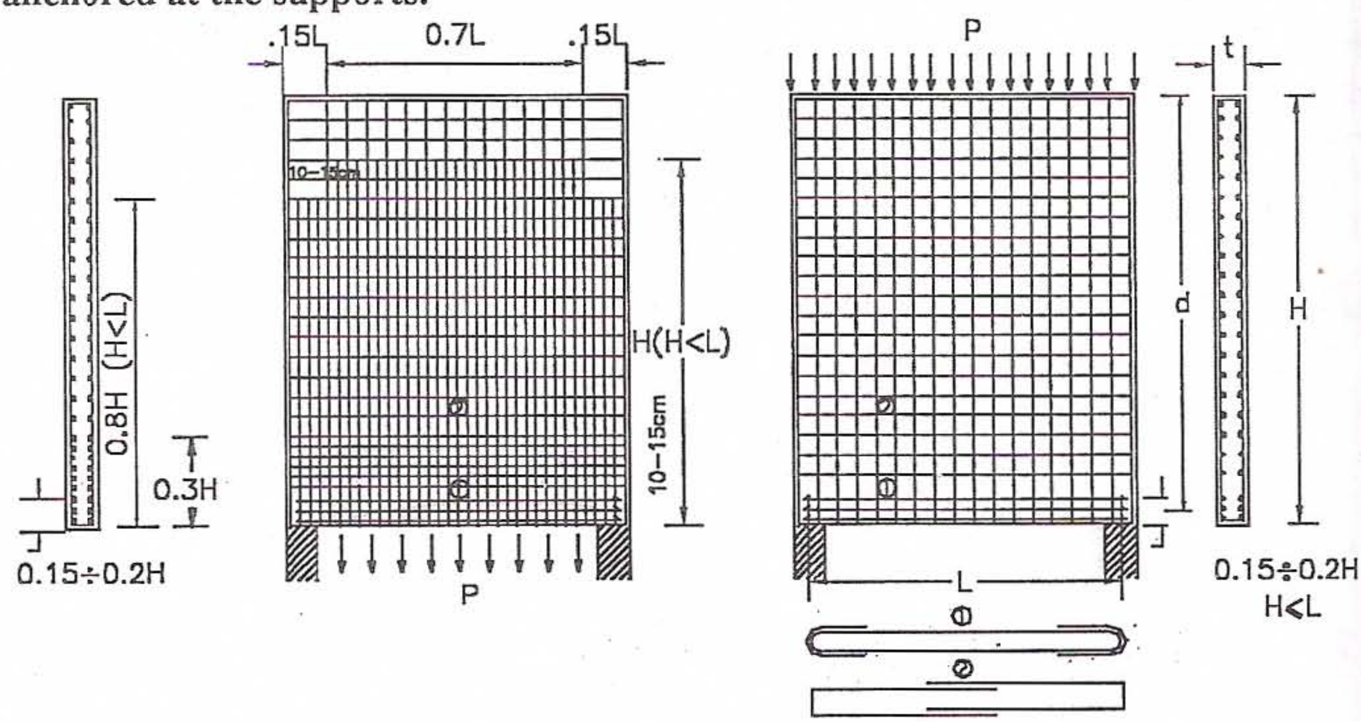
$Y_{ct} = 0.87$ d for $H \leq L$	and $Y_{ct} = 0.86$ L for $H > L$, i.e.
$T = \frac{\max. Mo}{0.87\ d}$ " " "	and $T = \frac{\max. Mo}{0.86\ L}$ " " "

For uniform distributed load w/m\` , we have : $M_o = wL^2/8$

then :-

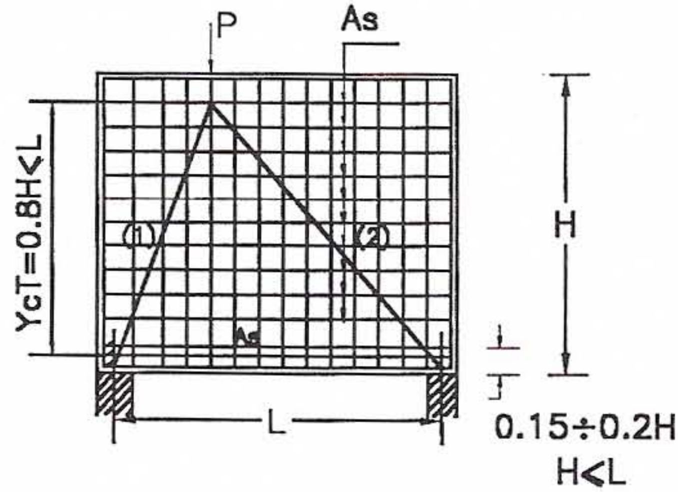
$T = 0.15\ wL^2/d$ for $H \leq L$ and $T = 0.15\ wL$ for $H > L$

The tension steel is to be extended over the whole length of the span and to be well anchored at the supports.



For loads hung at the bottom, vertical reinforcements carrying the full reaction are to be introduced for the full height over a length 0.7 L.

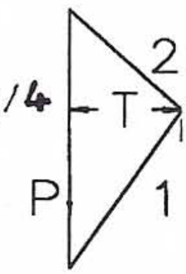
B) Simply supported deep beams subject to concentrated loads :



They may be calculatad in the same way as uniform loads introducting the corresponding values of Mo .

The tension can also be directly determined from the triangle of forces shown in figure.

$A_s = T / f_s$, $A_{s'} = A_s / 4$



The angle $\alpha \geq 45^o$.

c) Continuous deep beams :

The bending moments M may be calculated according to theory of elasticity as applied to slender beams .

The tension T is given by : $T = M / Y_{ct}$ in which :

$Y_{ct} = 0.43$ L at midspan and $Y_{ct} = 0.37$ L at support (but not more than 0.87 d).

For uniformly distributed load w /m` , we have :

At mid-span ($y_{ct} = 0.43$ L <0.87d)

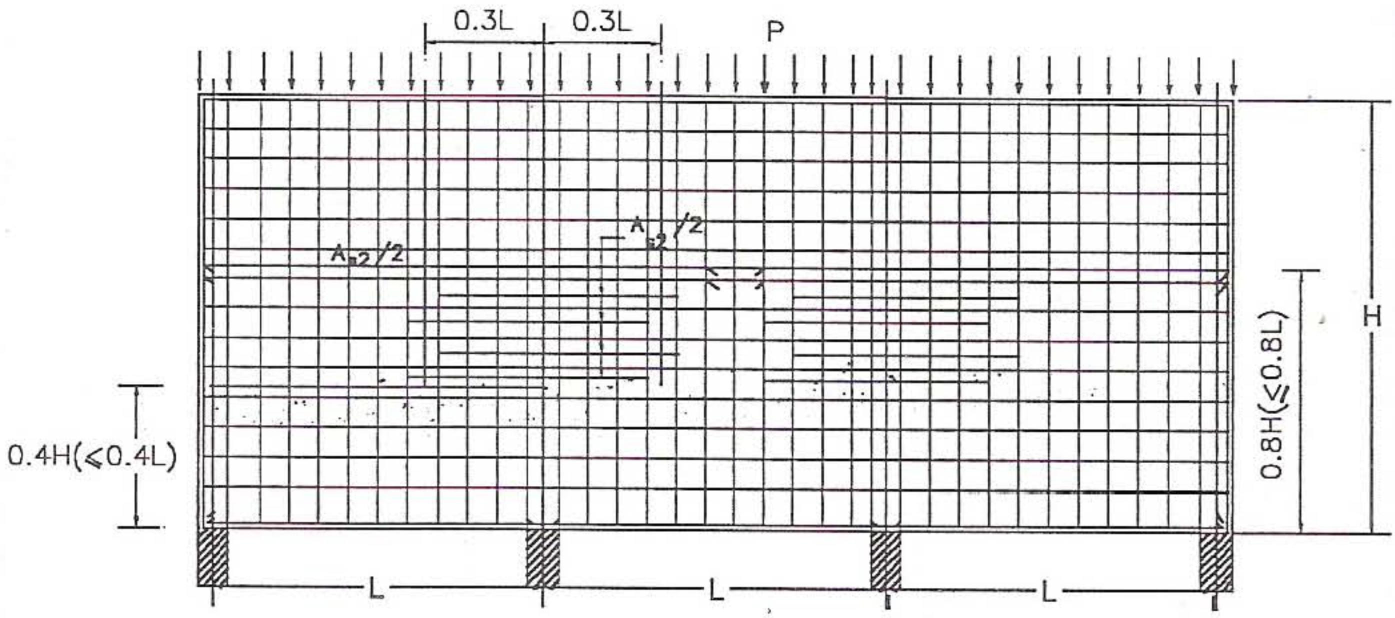
	$L \leq 2H$	$L > 2H$
Outer span	$T = \dfrac{wL^2 / 12}{0.43\ L} = 0.20\ wL$	$T = \dfrac{wL^2 / 12}{0.87\ d} = 0.10 \dfrac{wL^2}{d}$
Inner spans	$T = \dfrac{wL^2 / 16}{0.43\ L} = 0.15\ wL$	$T = \dfrac{wL^2 / 16}{0.87\ d} = 0.07 \dfrac{wL^2}{d}$

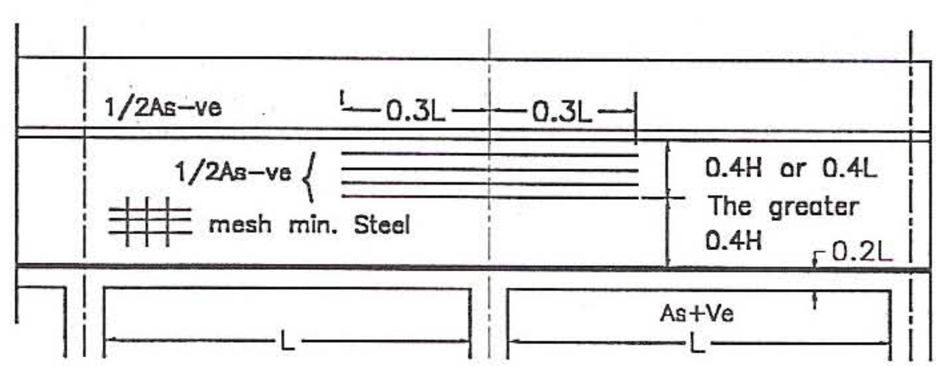
Over support ($Y_{ct} = 0.37$ L)

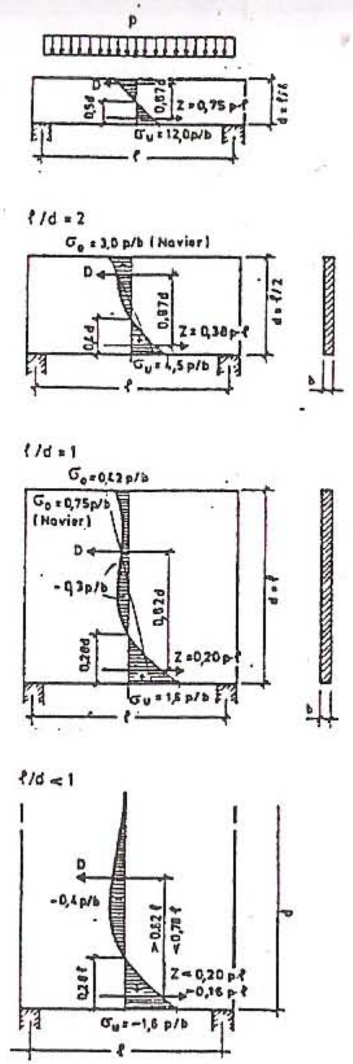
	$L \leq 2H$	$L > 2H$
At inner support of outer span	$T = \dfrac{wL^2 / 10}{0.37\ L} = 0.27\ wL$	$T = \dfrac{wL^2 / 10}{0.87\ d} = 0.12 \dfrac{wL^2}{d}$
Other inner supports	$T = \dfrac{wL^2 / 12}{0.37\ L} = 0.23wL$	$T = \dfrac{wL^2 / 12}{0.87\ d} = 0.10 \dfrac{wL^2}{d}$

It is recommended to extend the full amount of the tension steel at the middle of the spans to the supports.

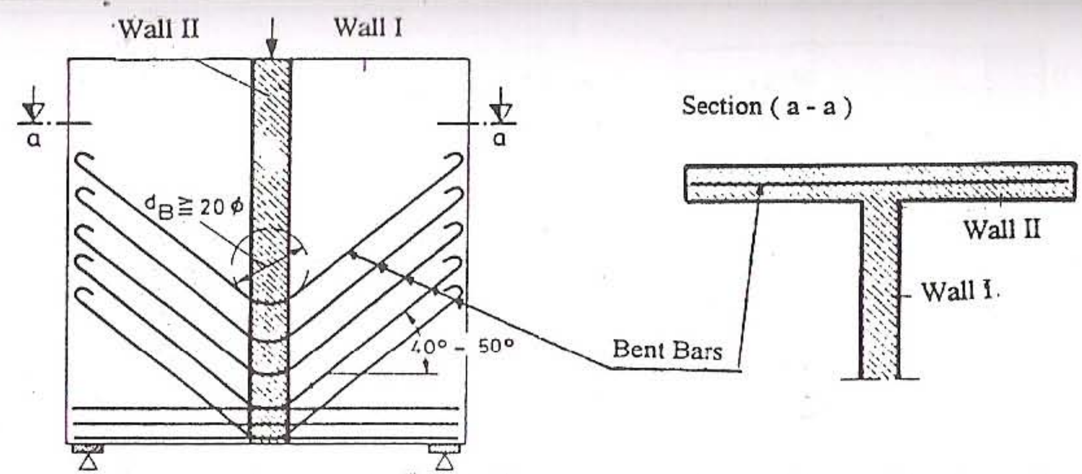
Half the tension steel required at the conter lines of the supports must. be arranged over the full length of the adjoining spans and the other half to be extended to a distance equal to 0.3 L on each side of the center line of the support as shown in the following figure :





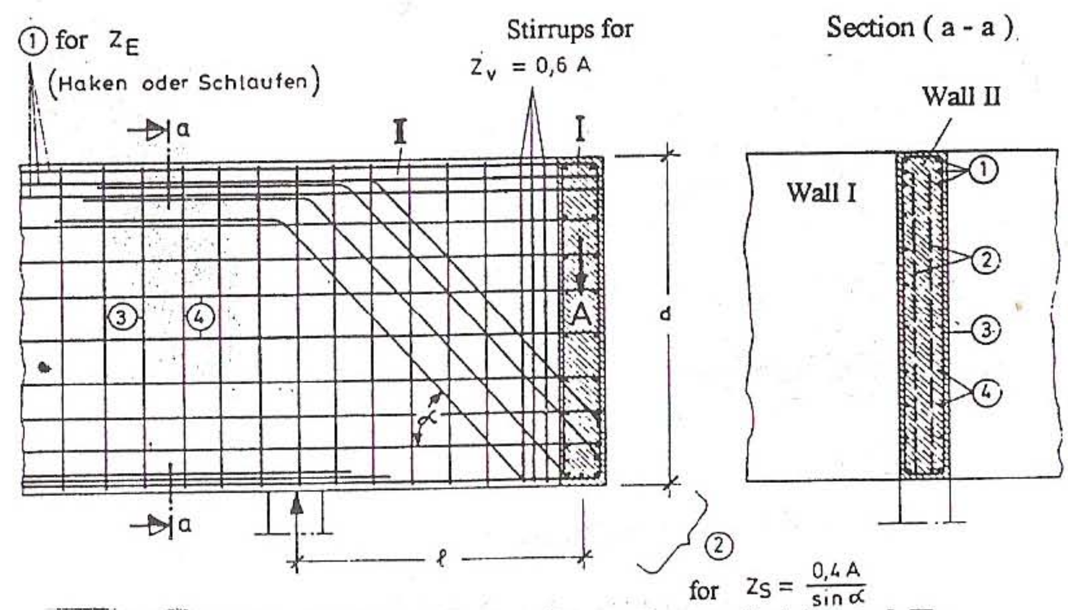


Normal Stress Distribution For Deep Beams With Variable Span/Depth Ratio

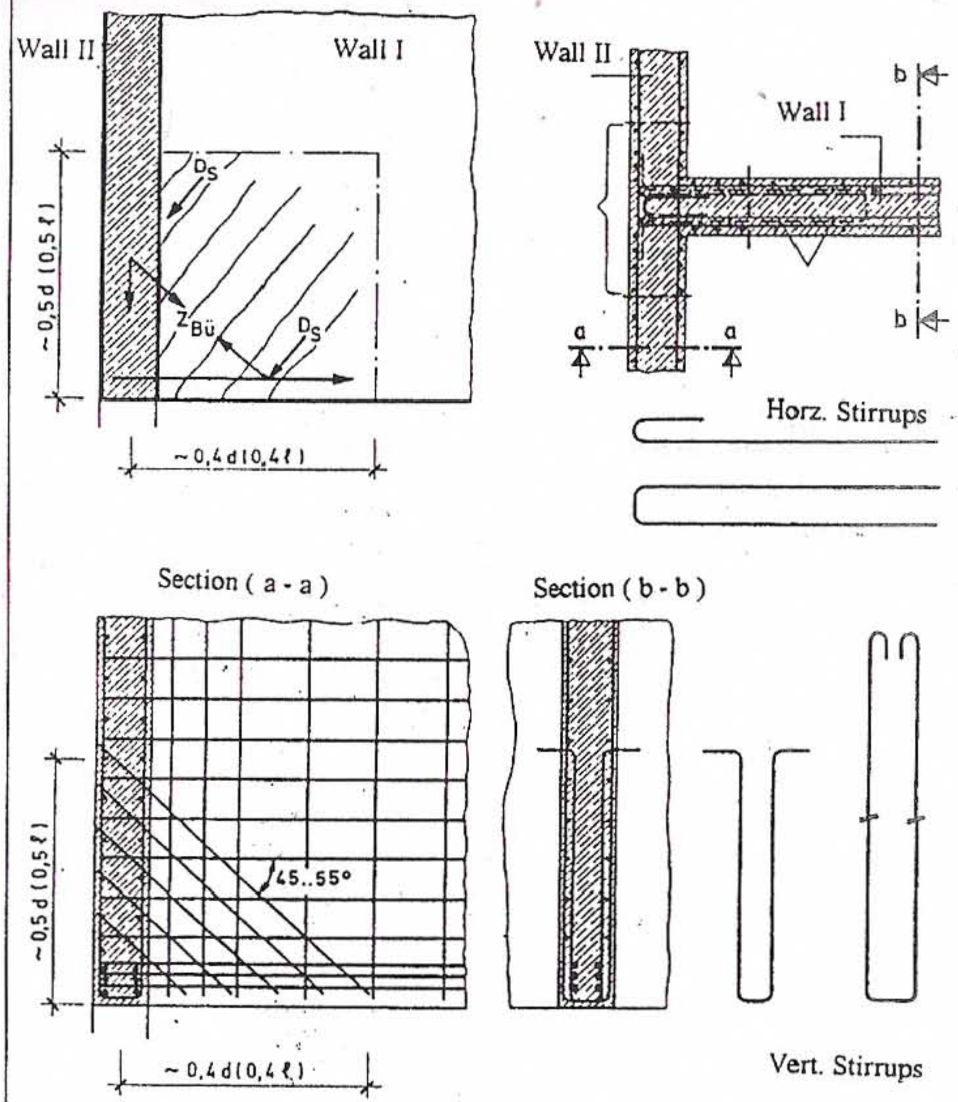


Details Of Rft. Of Simply Supported Deep Beam Subjected To Concentrated Load At Mid Span Section.

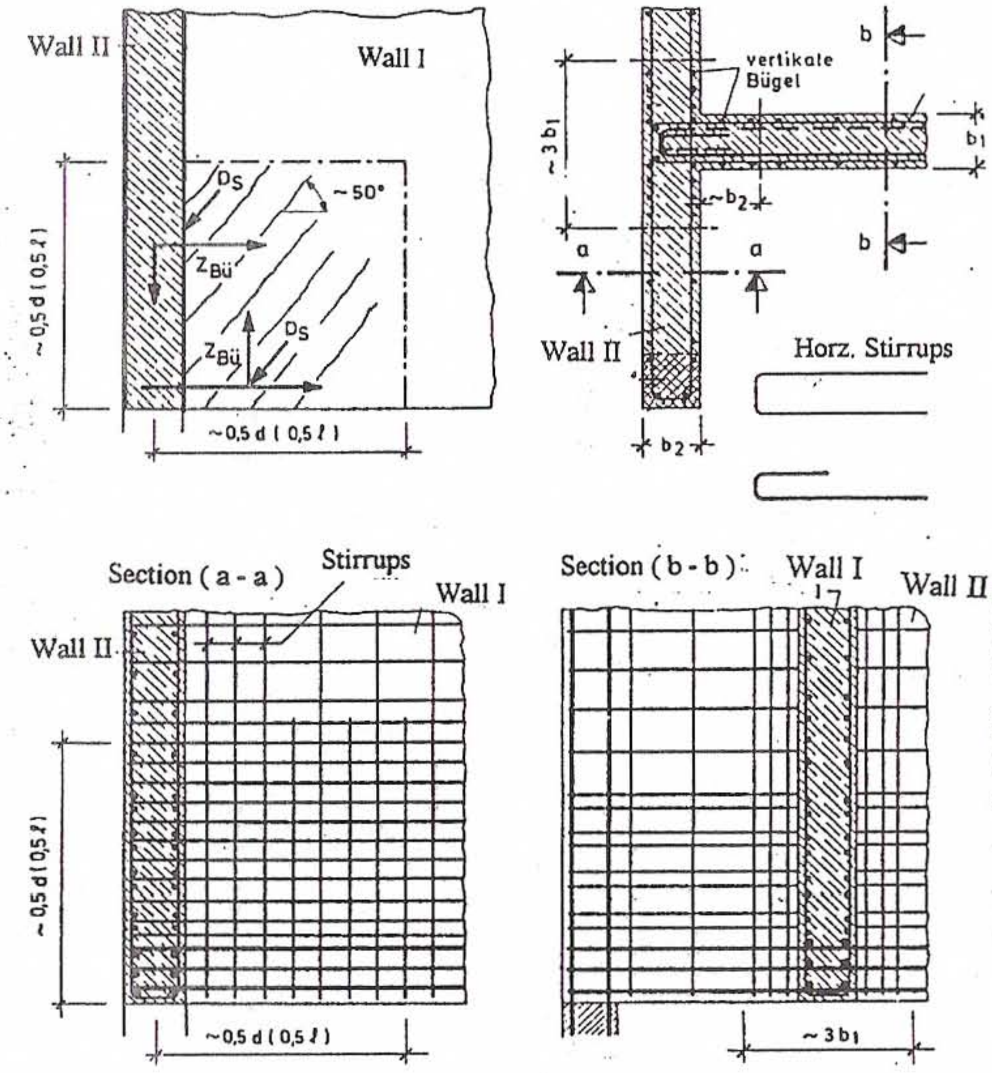
Note: Mesh Reinforcement Is Not Shown



Details Of Rft. Of Cantilever Deep Beam Subjected To Concentrated Load At Cantilever End.

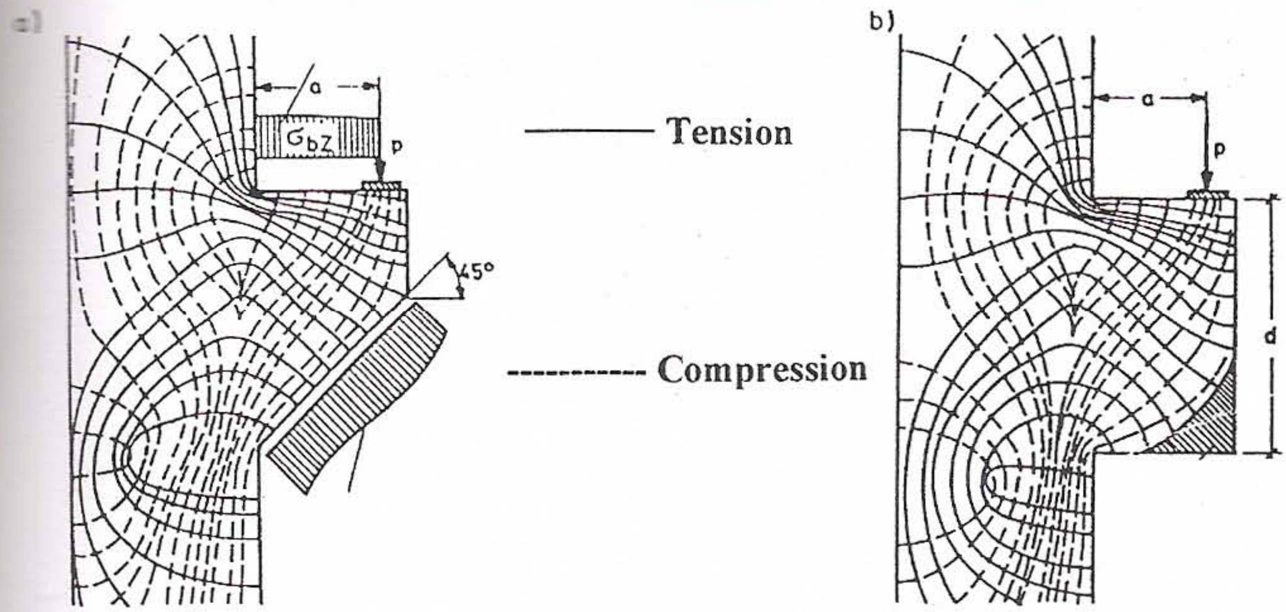


A-) Inclined Stirrups

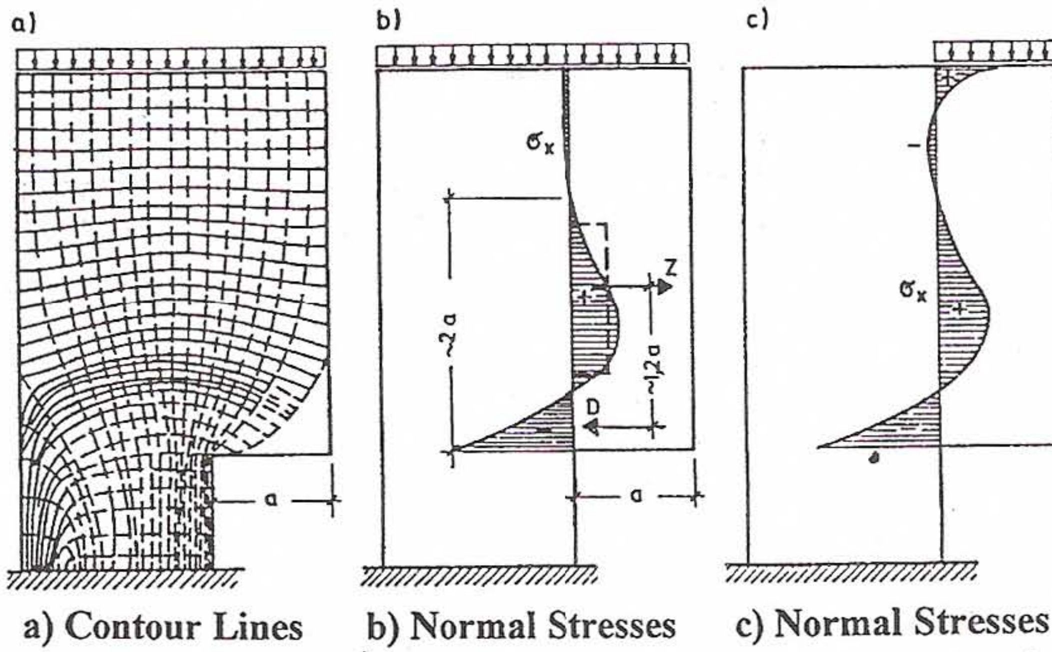


B-) Vertical And Horizontal Stirrups

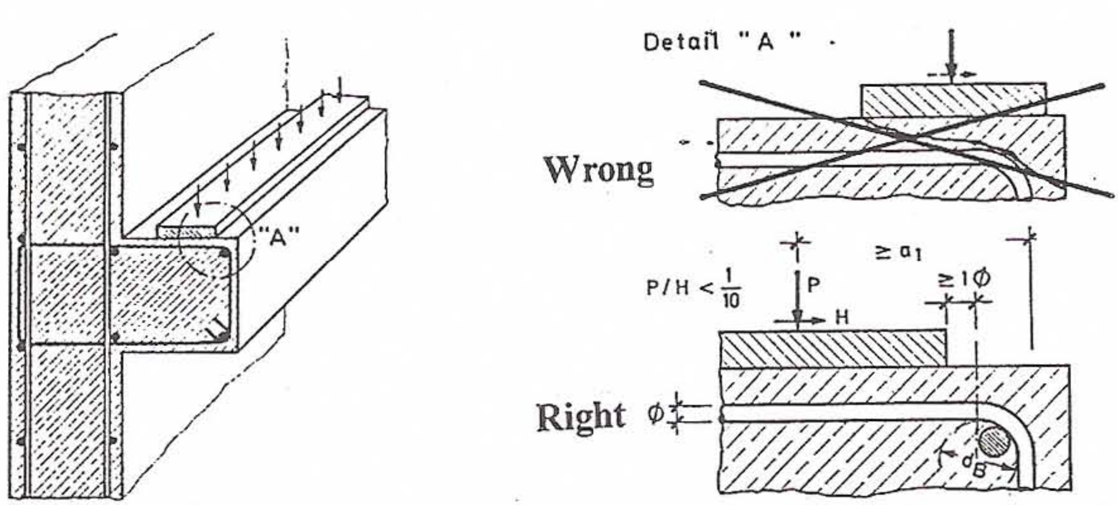
Details Of Rft. Of Deep Beam (Wall II) Supporting Another Deep Beam (Wall I).

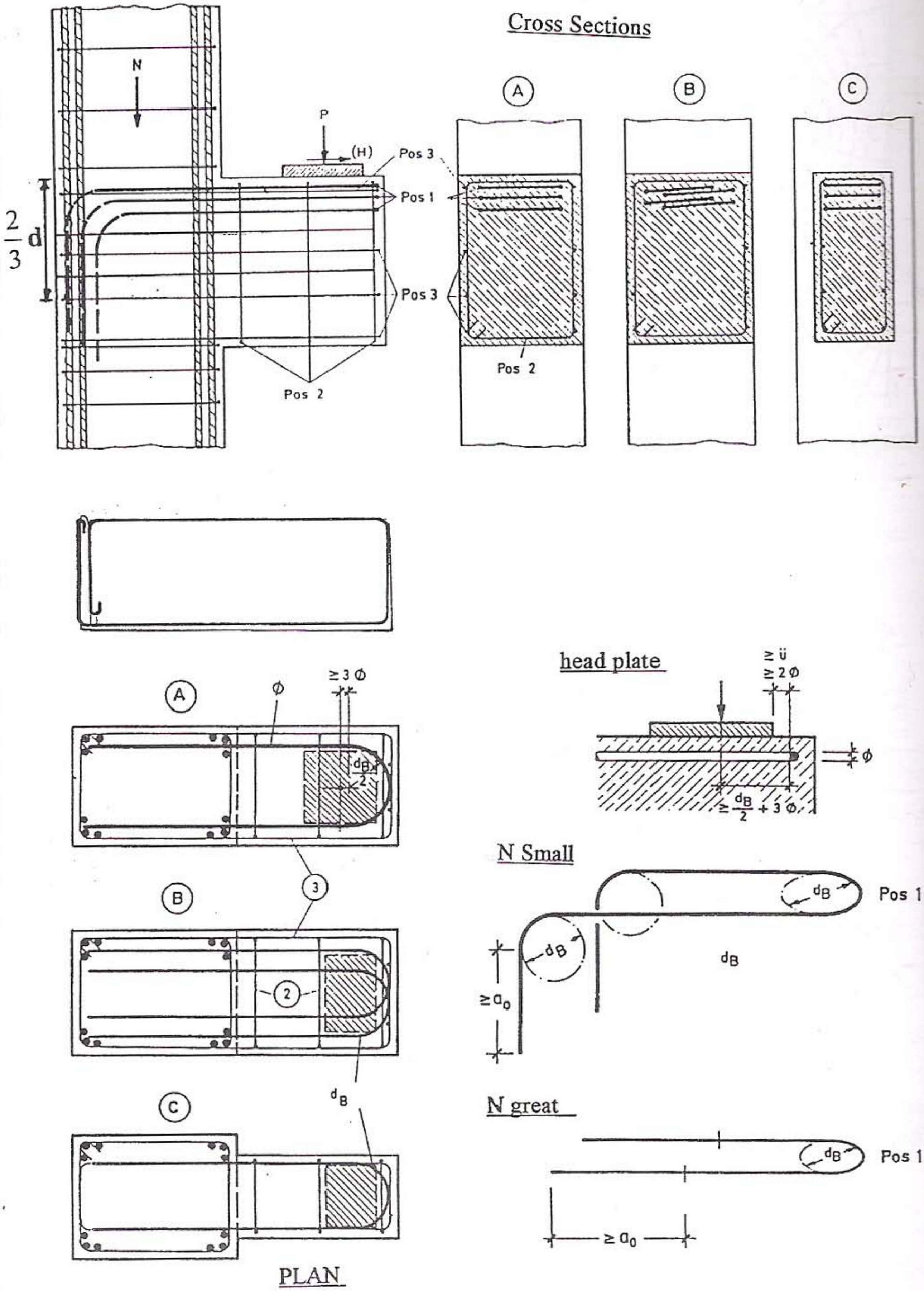


Tension and compression contour lines in short cantilevers



Deep short cantilever subjected to uniform load





Details Of Reinforcement Of Short Cantilever

1. Introduction

Reinforced concrete slabs can be designed according to one of the following methods.

a) Elastic design : *which includes;*

1)Rules of the code of practice: which are limited to specified cases of loading (only for uniform loads) and special shapes of slabs (only rectangular).

2)Elastic theory of plates : such as tables of Czerny (P.75-122) for rectangular slabs subjected to uniform or triangular loads (water pressure). Also, for circular slabs (P.128-145) and for triangular and trapezoidal shapes (P . 175 - 177).

b) Plastic design of slabs: *such as:*

1- Upper bound solution (yield line analysis).
2- Lower bound solution (always safe solution).
3- Strip method (Hillerborg).

2- CONCEPT OF THE YIELD LINE ANALYSIS

When a reinforced concrete slab is subjected to increasing load, initially we observe that the slab behaves elastically. As the load is gradually increased, cracking of concrete on the tension side begin at section of Max moment. The stiffness of the cracked section is reduced and a redistribution of the moment in the slab takes place. As the load is increased further, that reinforcement, will yield in the central area of the slab. Although that section of max. moment will continue to deform, its resistance moment will not increase and consequently an even greater redistribution of moment takes place.

The application of increased load will cause the reinforcement in even more sections to yield and further propagation of yield lines until the boundary of the slab.

Design of Slabs

Elastic Design

Code Rules

Only for uniform loads and for rectangular slabs for flat slabs with regular column arrangement.

Theory of elasticity

General solutions given in form of tables such as:

1- Czerny: (P.75 – 122) for triangular and uniform loads.
2- Bares: (P.175 – 177) for triangular & trapezoidal shapes.
3- Circular slabs: (P.128 – 143).
4- Pucher: Influence areas.

Plastic Design

Upper Bound
Solutions:
(may be unsafe)
Yield line analysis
(Johansen)

Lower Bound
Solutions:
(always safe)
strip method
(Hillerborq)

At this stage the slab is carrving its ultimate load . Any slight increase of the load will now cause a state of unstable equilibrium and the slab will continue to deform until crushing of concrete occurs. The condition when the yield lines just reach the slab boundary may be regarded as the collapse criteria of the slab.

The system of the yield lines or fracture lines is called yield line pattern.

3. Rules governing the choice of the yield line pattrens.

The rules governing possible yield line patterns may be summarized as follows in order to help in selecting a geometrically possible pattern:

a) Yield lines are straight, they can't change direction unless joined by another yield line.

b) Yield lines begin to from at points of maximum elastic bending moment in the slab and end at the slab boundary.

c) A yield line or yield line produced, must pass throug the point of intersection of the axes of rotation of the two slab parts connected by the yield line.

d) Axes of rotation generally lie along line of supports (beams), and pass over point support (pivot points) (columns) with any angle.

4- Virtual work method.

Having postulated a yield line pattern at failure, the first step is to give any convenient point in the slab a virtual deflection δ, in terms of which the corresponding deflections of all other parts of the slab may be calculated. The total work done by the loads to produce this deflection is given by

$$\text{External work done} = \Sigma\,(\dot{w}\,\delta)$$

which is usually simplified to

$$\text{External work done} = \Sigma \iint w\,\delta\,dx\,dy$$

where w = distributed load on the slab at collapse.

The energy absorbed in rotation at a yield line is the total moment along the yield line multiplied by the rotation at the yield line. If θ is the rotation, which can be calculated in terms of the virtual deflection δ, then the total internal work done on all the lines is given by

$$\text{Internal work done} = \Sigma (m_b \; L_b \; \theta_b)$$

where m_b = ultimate moment per unit of yield line length,

L_b = Yield line length.

The solution for the slab is obtained by equating the external work done to the internal energy absorbed,

$$\Sigma (w\,\delta) = \Sigma (m_b \; L_b \; \theta_b)$$

This method can be summarized in the following steps:

1st step; assume a yield line pattern (mechanism) at failure.

2nd step; choose a convenient point in the slab and give this point a virtual displacement. The deflection of all parts of the slab can now be expressed in terms of δ.

The loss of work due to this displacement is given by ;

external work = $$\Sigma \iint w\,\delta \, dx \;\; dy = \Sigma \; w\,\delta \qquad (1)$$

3rd *step; The work done in a yield line is the total moment along the yield* line multiplied by the rotation of the yield line hence:

Internal work = $$\Sigma \, m_b \; L_b \; \theta_b \qquad (2)$$

where m_b = the ultimate moment/ unit length along yield line

L_b = lenght of yield line.

θ_b = rotation of yield line.

4th step; the solution of the slab is obtained by equating the loss in external work to the gain in internal work.

$$\text{i,e. } \Sigma \; w\,\delta = \Sigma \, m_b \; L_b \; \theta_b \qquad (3)$$

5th step; for minimum energy (minimum ultimate load or maximum moment).

$$\frac{\delta m}{\delta \beta} = 0 \;\; ; \;\; \frac{\delta m}{\delta \gamma} = 0 \;\; ; \;\; \frac{\delta m}{\delta \psi} = 0 \qquad (4)$$

6th step ; substitute values of the variables (β , γ , Ψ....) into equation (3) to get the value of m_b

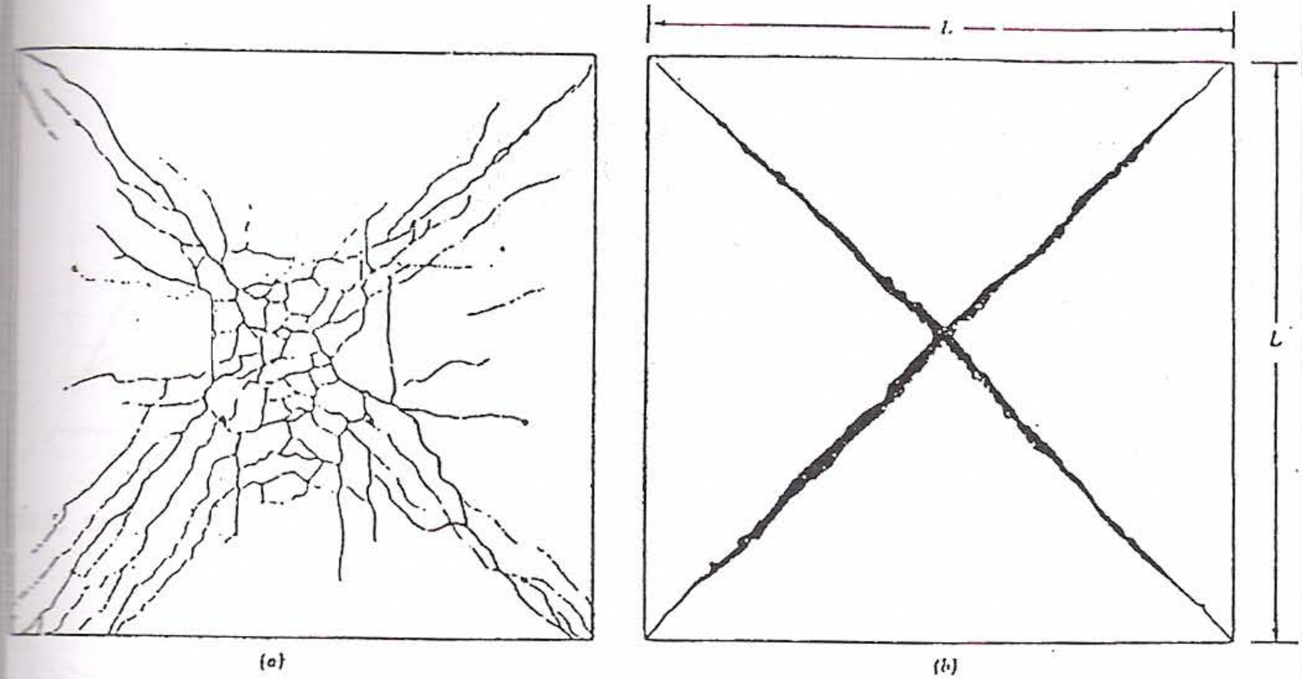


Fig.(1) Yield line in a square slab on stiff beams. (a) Actual cracking at failure. (b) Assumed [Yield lines] or [Fracture lines.]

III. Notations

Symbol		Meaning
———		Simply supported edge
———		fixed supported edge
- - - - -		free edge
▨		column
∼∼∼		Sagging yield line (m) (+ve moment)
∼∼∼∼		Hogging yield line (m') (-ve moment)
—·—·—		Axis of rotation
	w	Uniform distributed load w (t/m^2)
═══	q	Line load (t/m')
+	p	Concentrated load (t)

IV. Typical yield line patterns.

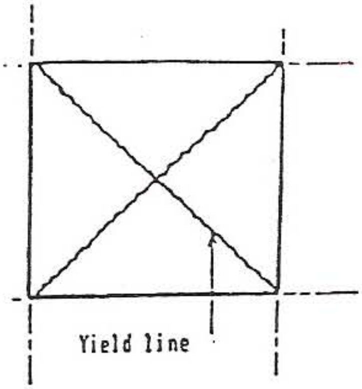


Fig.(2) Square Slab

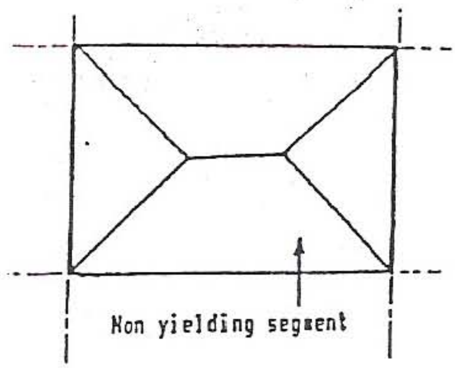


Fig.(3) Rectangular Slab

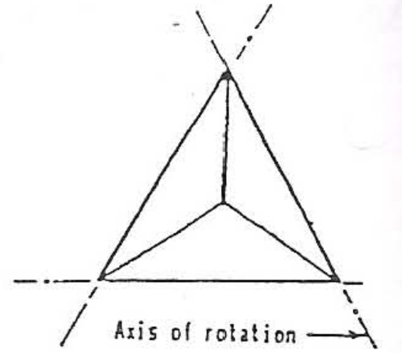
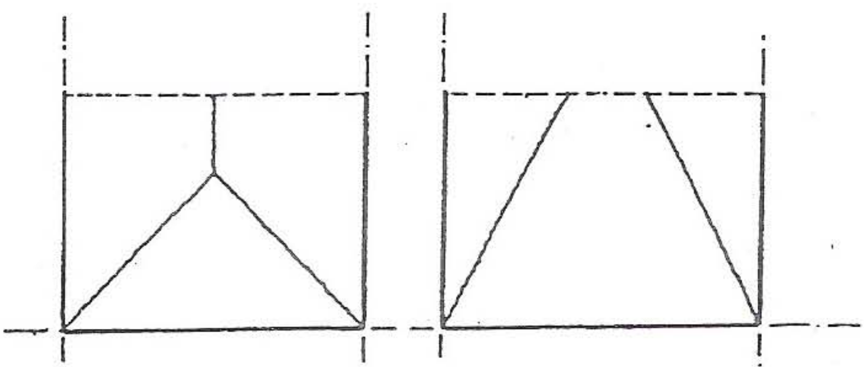


Fig.(4) Triangular Slab



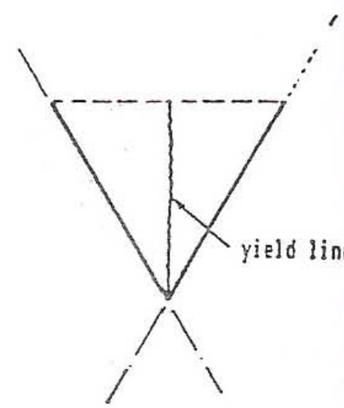


Fig.(5) Rectangular slab simply supported on three sides

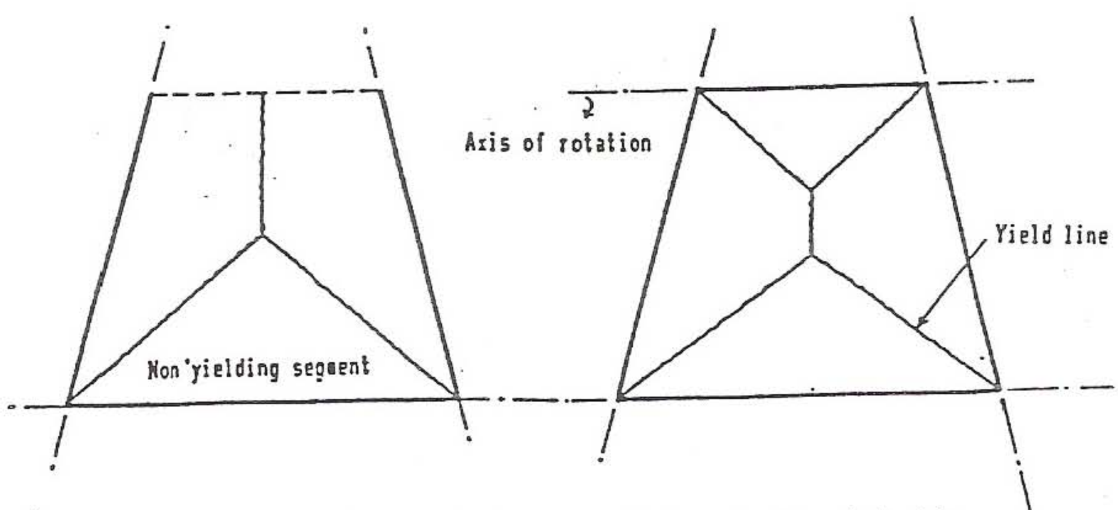


Fig.(6) Trapizoidal slabs simply supported on 4 sides & 3 sides.

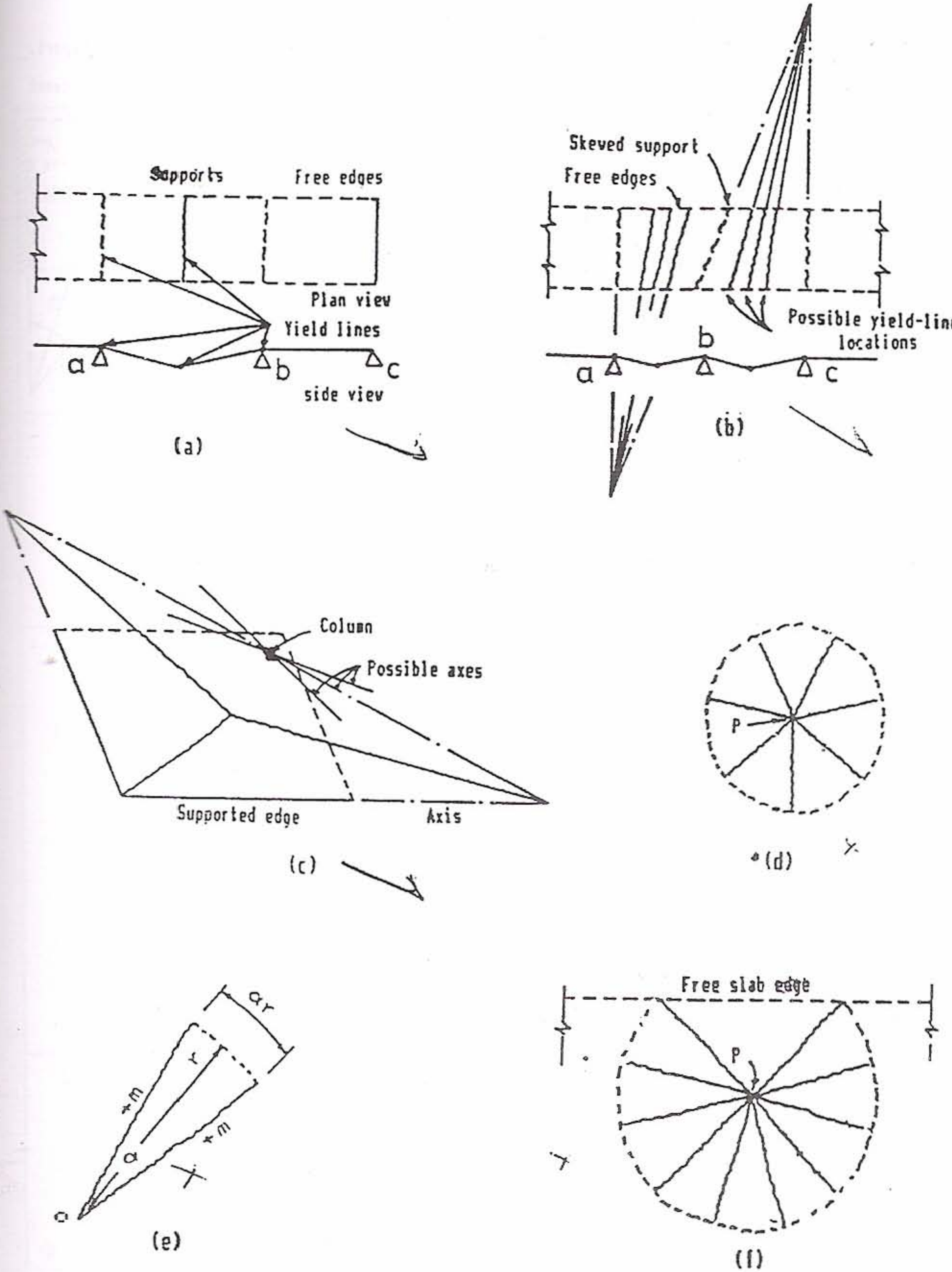


Fig.(7) Yield-line patterns. (a) Continuous one-way slab,right supports at a and b. (b) Continuous slab with a skewed support at b. (c) Slab supported on two adjacent edges and a column. (d) Fan pattern. (e) A fan segment. (f) Fan pattern near free edge.

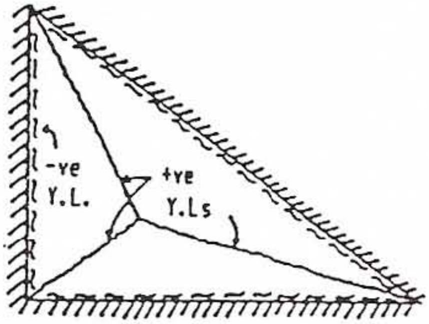


Fig.(8) Triangular slab fixed on all sides.

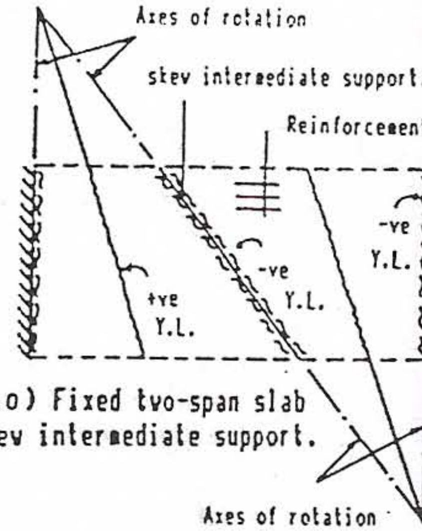


Fig.(10) Fixed two-span slab with skew intermediate support.

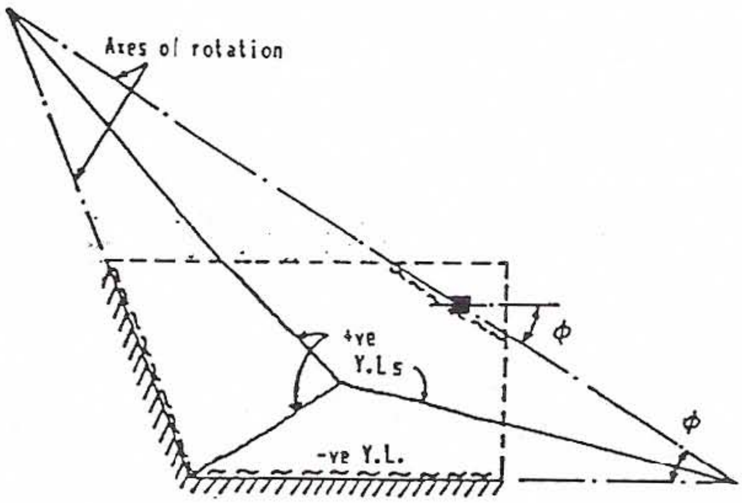
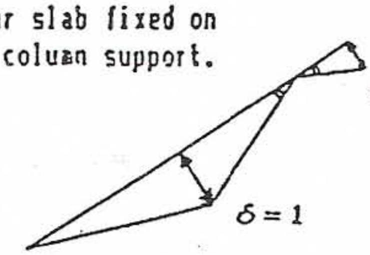


Fig.(9) Irregular slab fixed on two sides with one column support.



Rotation about the yield line.

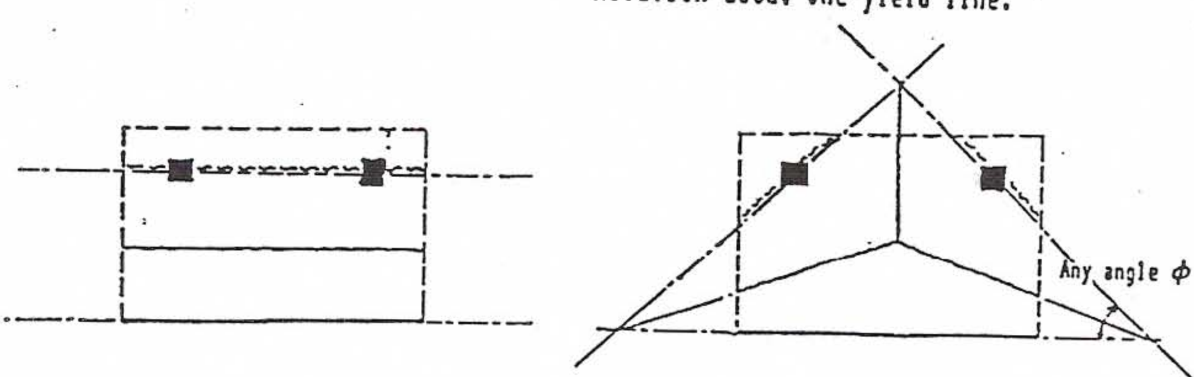
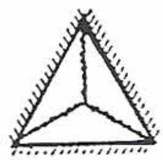
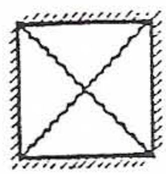
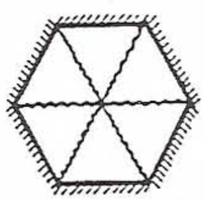
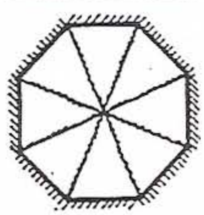


Fig.(11) Rectangular slab supported on two columns at one side. Both patterns shown are geometrically possible.

N SIDED SLABS WITH REGULAR SHAPE

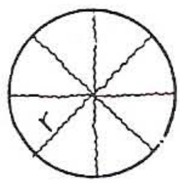
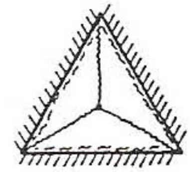
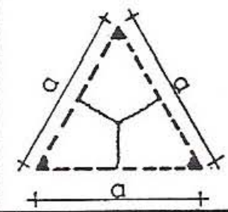
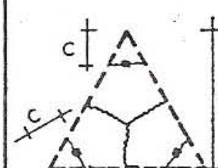
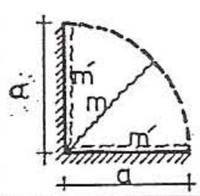
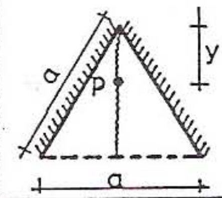
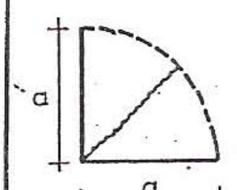
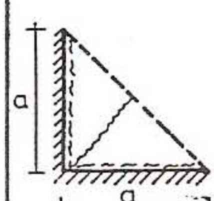
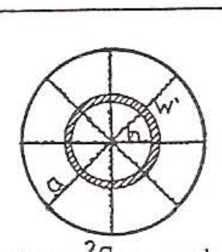
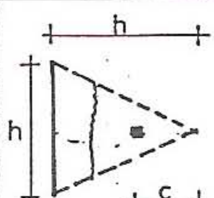
N	Yield line shape	Uniform load	Concentrated load
3	 Triangle	$m = \frac{W}{3.12\ (1+\phi)}$	$m = \frac{P}{10.3(1+\phi)}$
4	 Square	$m = \frac{W}{24\ (1+\phi)}$	$m = \frac{P}{8\ (1+\phi)}$
6	 Hexagonal	$m = \frac{W}{20.8\ (1+\phi)}$	$m = \frac{P}{6.93\ (1+\phi)}$
8	 octagonal	$m = \frac{W}{19.8\ (1+\phi)}$	$m = \frac{P}{6.63\ (1+\phi)}$
∞	 Circle	$m = \frac{W}{18.8\ (1+\phi)}$ $m = \frac{W}{6\pi\ (1+\phi)}$	$m = \frac{P}{6.24\ (1+\phi)}$ $m = \frac{P}{2\pi\ (1+\phi)}$

Where ;

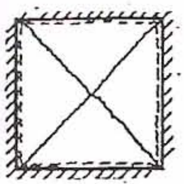
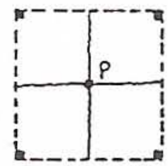
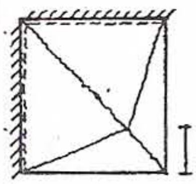
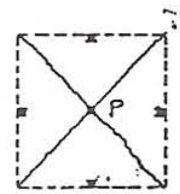
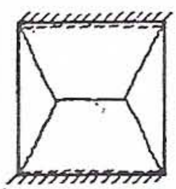
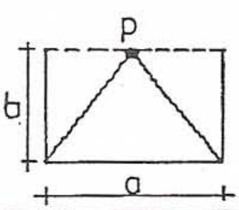
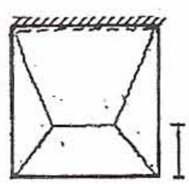
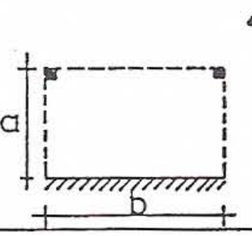
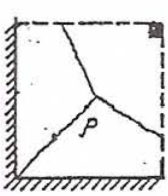
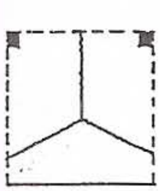
W = w x area of the slab (uniform Load)
P : Concentrated load at the slab center
$\phi = m'/m$

MOMENT OF SOME REGULAR SLABS UDER UNIFORM

AND CONCENTRATED LOADS

	SHAPE	EQUATION MOMENT		SHAPE	EQUATION MOMENT
1		$m = \frac{w\ r^2}{6(1+\phi)}$ $m = \frac{W}{18.8(1+\phi)}$ $m = \frac{P}{2\Pi\ (1+\phi)}$	1		$m = \frac{W}{31.2(1+\phi)}$ $m = \frac{P}{10.38(1+\phi)}$
2		$m = \frac{W}{14.1}$ $m = \frac{P}{5.66}$	2		$m = \frac{W}{4.45}$ $m = \frac{P}{2.6}$
3		$m = \frac{P}{5.2}$	3		$m = \frac{w}{\sqrt{3}}(\frac{7}{18} - \frac{c}{h})$ $m = \frac{w}{2\ 3}\ (1-3(\frac{c}{2h})^{2/3}$
4		$m = m'$ $m = \frac{W\ r^2}{14.4}$	4		$m = \frac{P\ y}{a}\ (\frac{1}{4\phi+3})$ at $\phi = 0$ $m = \frac{P\ Y}{3\ a}$
5		$m = \frac{W\ r^2}{7.2}$	5		$m = \frac{W}{6(1+2\phi)}$
6		Line load $m = \frac{w'b(a-b)}{a}$	6		$m = \frac{1}{8}\ w\ h^2 z$ where $z = \frac{(2/3+c/h)}{(1/2+c/h)}$

MOMENT OF SOME REGULAR SLABS UDER UNIFORM AND CONCENTRATED LOADS

	SHAPE	EQUATION MOMENT		SHAPE	EQUATION MOMENT
1		$m = \frac{W}{24\ (1+\phi)}$ at $\phi = 1$ $m=m'= W/48$	1		$m = \frac{P}{4}$ $m = \frac{W}{8}$
2		$m=m'= \frac{W}{35}$	2		$m = \frac{P}{8}$
3		$m=m'= \frac{W}{35}$	3		$m = \frac{P}{\frac{a}{b} + \frac{4b}{a}}$
4		$m=m'= \frac{W}{29.3}$	4		$m = \frac{Pd(a^2+b^2)}{a^2 b}$
5		$m= \frac{W}{10.67(1+\phi)}$ $m= \frac{P}{4}$	5		
6		$m = \frac{W}{8.55}$ $m=m'= \frac{W}{4.2}$	6		

Flat slabs.

a) Flat slab without column head & without drop pannel (without edge beam)

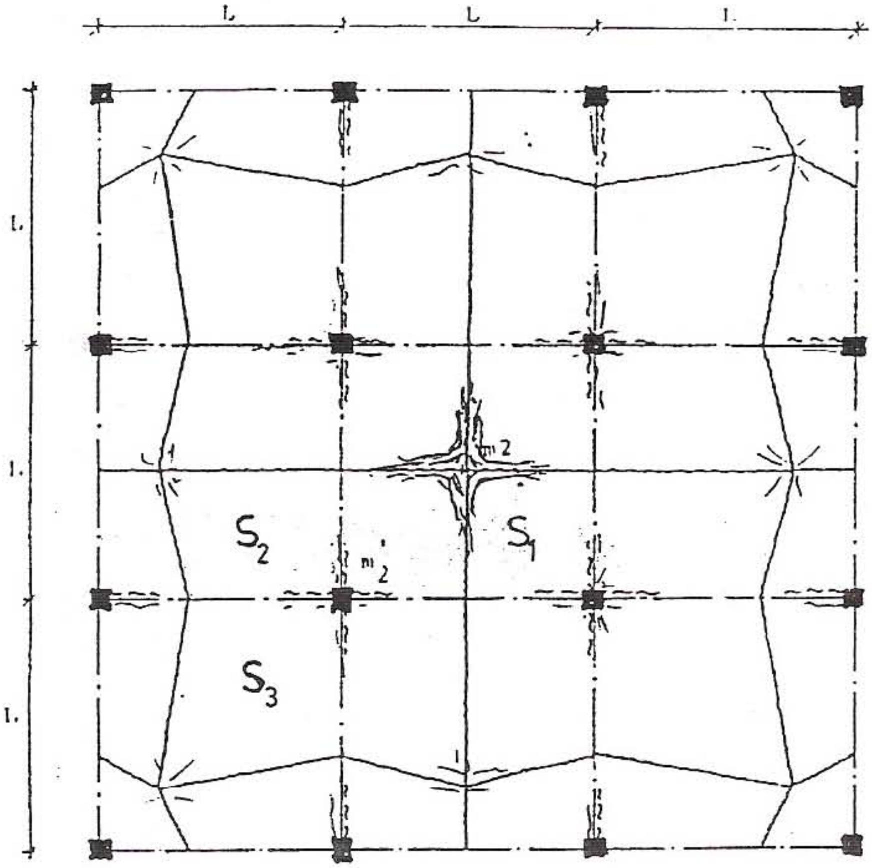


Fig.(48)

for m' = m

m1 : +ve moment in edge span 0.42 L from outer edge $= \frac{wL^2}{11.6}$

m2 : +ve moment in other spans $= \frac{wL^2}{16}$

m_1' : -ve moment in 1st int. support $= \frac{wL^2}{11.6}$

m_2' : -ve moment in other support $= \frac{wL^2}{16}$

It is recommended that the +ve (m) should cover the span & the -ve m should cover $\frac{1}{2}$ span.

PART(II)

EXPLANATIONS AND COMMENTS

TO

CODE TEXT

2001

CHAPTER (3) : GENERAL DESIGN CONSIDERATIONS
CHAPTER (4) : LIMIT STATE DESIGN METHOD
CHAPTER (5) : WORKING STRESS DESIGN METHOD
CHAPTER (6) : ANALYSIS OF STRUCTURAL ELEMENTS
CHAPTER (10) : PRESTRESSED CONCRETE

El-Behairy R.C. Design Handbook

PART (II)

Applications to The New Egyptian Code 2001 for Design and Construction of R.C.structures

CHAPTER (3)
GENERAL DESIGN CONSIDERATIONS

3-1 *Design methods*:

This chapter deals with the basic concepts for the design methods of reinforced concrete sections under the effect of the different kinds of loads. These methods ensure that all members of the structure shall be proportioned to an adequate strength in accordance with the provisions of this code, which are based on statistical bases,to satisfy the requirements for use and workability during the structure life time without any deformations or defected cracks and with an adequate factor of safety against failure.

This can be satisfied by the use of one the following two methods for the design of reinforced concrete structures:

1- Limit state design method.

2- Elastic (working stress) design method.

3-1-1 *LIMIT STATE DESIGN METHOD:*

It is not allowed to use this method for design of R.C. structures, with characteristic concrete strength after 28 days less than 20 N/mm^2 . The conditions given in article (3-1) are satisfied by taking adequate load factors of safety for the working loads to obtain the ultimate loads at which the structure reaches one of the limit states mentioned in article (3-2-1-1) .

By calculating these limits, all factors which negatively affect the strength of the structure in resisting the loads resulting from material strength reduction factors and the approved tolerances in construction and calculations are to be taken into consideration.

The ultimate limit states are:

3-1-1-1- Ultimate strength limit state:

It is the limit which guarantees - on statistical basis - that the structure, or any part of it will not fail as a result of reaching its ultimat limit strength. This limit state controls the behaviour of all parts of the structure at failure (article 4-2).

3-1-1-2 *Stability limit state :*

It is the limit which guarantees - on statistical basis-that the structure will not fail due to buckling (art. 6-4), overturning, uplift, or sliding.

3-1-1-3 *Serviceability limit states:*

They are the limit states which when ignored, the use and the strength of the structure are negatively affected. They are divided into the following states:

a)**Deformation and deflection limit states :** They are the states which ensure to prevent excessive deformations or deflections more than the allowed values (article 4-3-1) .

b)**Limit state of cracking :** It is the state which ensures the control of crack width, which affects negatively the performance of the structure or limits its use, or reduces its life time, or affects the appearance of any part of it (article 4-3-2).

3-1-2 *ElASTIC (WORKING STRESS) DESIGN METHOD*:

The conditions of article (3-1) are satisfied by the determination of allowable working stresses for the different structural elements. The structure is designed so that the resulting stresses from working loads should not exceed the allowable stresses. Design should also ensure the stability of the structure to prevent the occurance of buckling (art. 3-1-1-2), excessive deformations and deflections, and to control the crack width (art. 3-1-1-3).

3-2 *Safety conditions:*

Safety of the structure is satisfied when the strength of the different sections are greater than internal stresses resulting from the different cases of loading and the direct and indirect strains.

The structure as a whole and each part of it should be safe and usable during the structure lifetime.

3-2-1 *Safety determination by the limit state design method*

3-2-1-1 *Load determination*

***a)* Service loads :**

They are the expected loads under the working conditions. The probability of the increase of these loads should not be more than 5%. These loads are taken from the Egyptian code for loads. They include dead and live loads. They also include wind, soil & liquid pressure, dynamic loads, settlement of supports, creep, shrinkage, and change of temperature effect. Also, seismic loads should be taken into consideration in seismic zones according to their intensity.

b) Ultimate load values :

The ultimate load values are calculated by multiplying the working loads defined in art (3-2-1-1-a) by the following load factors;

1-For members subjected to live loads, where wind and seismic loads may be neglected:

$$U = 1.4\,D + 1.6\,L \qquad (3\text{-}1)$$

where D = Dead loads L = Live loads

U = Ultimate loads (factored loads)

2-For cases when live loads dose not exceed 0.75 the dead loads then :

$$U = 1.5\,(D+L) \qquad (3\text{-}2)$$

3-For members subjected to live loads, plus lateral pressure due to liquid or soil:

$$U = 1.4\,D + 1.6\,[E+L] \qquad (3\text{-}3)$$

where E = Lateral loads.

U should not be less than the value given in eqn. (3-1)

In case of lateral pressure of liquids contained in closed members such as tanks, the value 1.6 E , in equation (3-3) and (3-7) is to be replaced by 1.4 E.

4)If there are loads due to wind pressure (W) or due to earthquakes (S), The ultimate loads is to be taken the bigger of the following equtions:

U = 0.8 (1.4 D + 1.6 L + 1.6 W)	(3-4)
U = 0.8 (1.4 D + 1.6 L + 1.6 S)	(3-5)

Provided that U should not be less than the value given in eq n (3-1), and it is not allowed to add seismic loads plus wind loads.

5)In cases of loading when decreasing dead load results in increase of straining actions in some sections, the ultimate load factor for dead load is taken (0.9).

6-In cases when the dead loads increase the stability of the structure or reduce the internal stresses ,the ultimate loads are replaced in articles (1,3,4,5) by the following ultimate load factors

U = 0.9 D	3-6
U = 0.9 D + 1.6 E	3-7
U = 0.9 D + 1.3W	3-8
U = 0.9 D + 1.3 S	3-9

7-The effect of temperature changes, settlement of supports, creep, shrinkage (T) is taken as follows :

U= 0.8 (1.4 D + 1.6 L + 1.4 T)	(3-10)
It should not be less than : U= 1.4 (D + T)	(3-11)

The effect of these straining action is calculated according to art. (3-3)

The dynamic loads (K) may be taken as an equivalent additional static load :

U= 1.4 D + 1.6 L + 1.6 K	(3-12)

Taking into consideration what mentioned in eq n (3-6)

C) Load factors for the design using the elastic method and for serviceability limit states :

1) By the design of structures using the elastic method (art.3-1-2) and by the calculations of the serviceability limit states (art. 3-1-1-3 deflection, and crack width using the elastic method) design loads are considered to be equal to the working loads (art. 3-2-1-1-a).
2) In cases of loading when decreasing dead load result in increase of straining actions in some sections, the ult. load factor is taken (0.9) for dead loads.
3) If the dead loads increase the stability of the structure, the working loads are taken as follows:

1-	0.9 D	(3-13)
2-	0.9 D + W or 0.9 D + S	(3-14)

Note : The allowable stresses are increased, if the wind or seismic loads are included, as in article (5-3-1-9).

3-2-1-2 Strength reduction factors (γ)

These factors represent the safety factors required to consider the different factors affecting negatively the ultimate strength of the sections.

They are represented in the small differences allowed statistically in the errors of the dimensions of the sections, deviation of concrete and steel qualities from those assumed in the design, and the small errors in the calculations due to the design assumptions.

The strength reduction factors differ according to the type of the applied load (flexure, shear ......etc), also according to the importance of the member in the structure.

The values of these factors are as follows:

1- Ultimate strength limit state:

The material reduction factor for the concrete strength (γ_C) and for the steel rft. (γ_S) are taken as follows :

Table : (3-2) values of material reduction factors γ_c & γ_s

a	- Axial and eccentric tension -Flexure-Shearing forces-Bond - Torsional moments-Bearing	$\gamma_C = 1.50$ $\gamma_S = 1.15$	3-15-a 3-15-b
b	- Eccentric compression force e/t $\geq$ 0.05	$\gamma_C = 1.5\{(\frac{7}{6}) - (\frac{e/t}{3})\} \geq 1.5$ $\gamma_S = 1.15\{(\frac{7}{6}) - (\frac{e/t}{3})\} \geq 1.15$	3-16-a 3-16-b

Table: γ_C & γ_S for case of eccentric compression (e/t $\geq$0.05)

e/t	0.05	0.1	0.2	0.3	0.4	0.5
$\gamma_C = 1.5\{(\frac{7}{6}) - (\frac{e/t}{3})\} \geq 1.5$	1.725	1.70	1.65	1.60	1.55	1.50
$\gamma_S = 1.15\{(\frac{7}{6}) - (\frac{e/t}{3})\} \geq 1.15$	1.323	1.30	1.265	1.226	1.188	1.15

2- *Serviceability limit states*

The material reduction factors for concrete and steel for all serviceability limit states are taken as follows :

- Deflection, Deformation, Cracking $\gamma_c = \gamma_s = 1.0$ (3-17)

3-2-2 *Safety determination by the elastic method :*

The safety of the structure when using the elastic method of design is determined according to chapter (5).

3-3 Internal actions:

a- Shrinkage loads : taken from code for loads.

b- Temperature loads : taken from code for loads.

c-Long - term strain of concrete (Creep): taken according to article (2-3-3-5)

C-1 Definition :

Creep is defined as the excessive deformation that take place under effect of some or all the service loads, which increases with time, the plastic deformations (ε_{cr}) is added to the instantaneous elastic deformation (ε_o) resulting from direct loading of service loads.

C-2: Factors affecting creep.

- Creep increases as loading stresses increase.
- Creep decreases if the age of concrete increase.when loaded.
- Creep increases as the cement content increase.
- Creep increases as the water-cement ratio increase.
- Creep increases as the member thickness decreases.

C-3: Estimation of creep strains (ε_{cr}) in ordinary conditions

instantaneous strain (ε_o) equals to: $\varepsilon_o = f_o / E_c$

The strain due to creep is taken: $\varepsilon_{cr} = \phi \cdot \varepsilon_o$

Then, the total strain ε_t is taken: $\varepsilon_t = \varepsilon_o + \varepsilon_{cr} = (f_o / E_c) \cdot (1+\phi)$ (2-4-b)

E_c is calculated from eqn (2-1)

The value of ϕ is taken according to table (2-9)

Table (2-9) values of ϕ

Weather condition	Dry Weather R.H = 55 %		Wet Weather R.H = 75 %	
$B_{(mm)}$	$\geqslant$ 600	$\leqslant$ 200	$\geqslant$ 600	$\leqslant$ 200
Shrinkage Stage	Dry shrinkage ($\varepsilon \times 10^{-3}$)			
3 – 7 days	0.31	0.43	0.21	0.26
7 – 60 days	0.30	0.32	0.21	0.26
> 60 days	0.28	0.19	0.20	0.16
Age at loading	ϕ			
3 – 7 days	2.90	3.80	2.10	2.70
7 – 60 days	2.50	3.00	1.90	2.20
> 60 days	2.00	1.70	1.70	1.40

R.H → Relative Humidity

$B = 2\ A_c/P_c$

Where: A_c = R.C Section Area mm^2

P_c = R.C Section Perimeter mm

CHAPTER (3) (SUMMARY)

Methods of design

- Limit state design method
  - Ultimate strength Limit state
    - Flexure
    - Eccent. force
    - Shear
    - Torsion
    - Bearing
    - Bond
    - Development length
  - Stability limit state
    - Buckling
    - Overturning
    - Uplift
    - Sliding
  - Serviceability limit state
    - Deflection
    - Deformation
    - Crack width control
- Elastic (working stress) design method
  - Flexure
  - Eccentric force
  - Shear
  - Torsion
  - Bearing
  - Bond
  - Development length

VALUES OF MATERIAL. REDUC. FACTORS FOR CONCRETE γ_C & STEEL γ_S

1) . Axial and eccentric tension . Bending moment. Shearing force . Torsional moment. Bearing . Bond	$\gamma_c = \mathbf{1.5}$ $\gamma_s = \mathbf{1.15}$
2) Eccentric compression where $e/t \geq 0.05$ (see table below)	$\gamma_C = 1.5\{(\frac{7}{6}) - (\frac{e/t}{3})\} \geq 1.50$ $\gamma s = 1.15\{(\frac{7}{6}) - (\frac{e/t}{3})\} \geq 1.15$
3) Working limit states : Deflection-Deformation-cracking	$\gamma_c = 1.0$ $\gamma_s = 1.0$

e/t	.05	0.1	0.2	0.3	0.4	0.5
γ_c	1.73	1.70	1.65	1.60	1.55	1.50
γ_s	1.323	1.30	1.265	1.226	1.18	1.15

Chapter (4)
LIMIT STATES DESIGN METHOD

Contents — *Page*

CHAPTER (4)
LIMIT STATES DESIGN METHOD

4-1- GENERAL CONSIDERATION:

This chapter deals with the application of the basis for the design of R.C. structures according to the limit states design method mentioned in chapter (3)

These limit states ensure enough safety against failure where the sections reach their ultimate limit strength and satisfy all serviceability requirements.

Limit states design method

4-2 Ultimate strength limit states
- 4-2-1 Flexure or eccentric forces
- 4-2-2 Shear
- 4-2-3 Torsion
- 4-2-4 Bearing
- 4-2-5 Bond, development length & splices

4-3 Serviceability limit states
- 4-3-1 Deformation and deflection limit states
- 4-3-2 Cracking limit states

4-2 ULTIMATE STRENGTH LIMIT STATES:

4-2-1 U.S.D : FLEXURE OR ECCENTRIC FORCES:

4-2-1-1 Basic assumptions and general considerations:

The ultimate strength limit state for sections subjected to flexure or combined flexure and eccentric force should satisfy the equilibrium conditions, compatibility of strain conditions, as well as the following considerations:

1- Strain distribution along the section are linear, therefore, the strains in the steel reinforcement and concrete are proportional to their distances from the neutral axis. This assumption is valid in all elements except for deep beams that have non-linear stress distribution.

2- Fig.(4-1) shows the idealized stress-strain relationship for steel reinforcement. Yield stress limits are as follows:

3-a fy is taken not more than 400 N/mm^2 for H.G. steel and wire mesh of ϕ >10mm.

3-b. If tests prove that fy for mild steel >240 N/mm^2, this value can be used but not more than 280 N/mm^2.

3-c. For cold wire mesh: yield stress $f_y \not> 300$ N/mm^2.

4- Limit state for cracking should be satisfied in the determination of f_y for the used steel according to clause (4-3-2).

5-Concrete strength in tension is neglected, while steel rft resists all tension stresses.

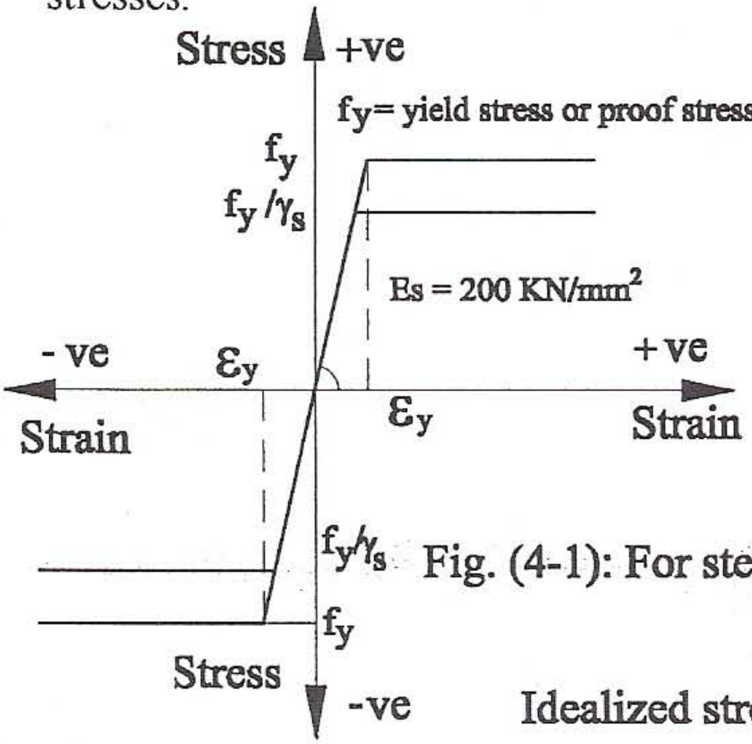


Fig. (4-1): For steel

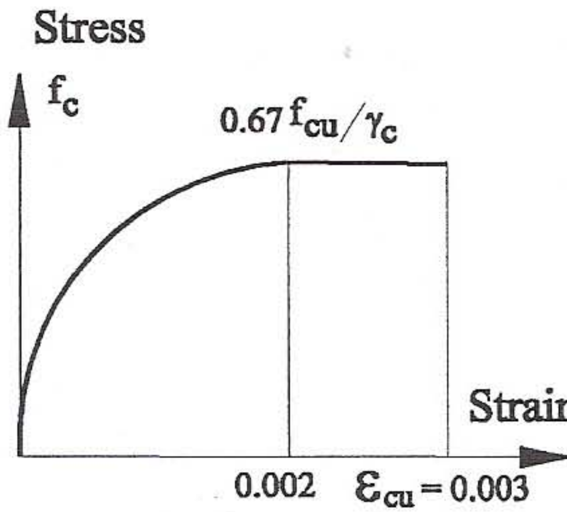


Fig. (4-2): For concrete

Idealized stress-strain curve

6- Fig.(4-2) shows the idealized stress-strain relationship for concrete in compression.

7- Maximum compression strains are taken as follows:

$\varepsilon_{cu} = 0.003$ for concrete sections subjected to flexure or combined flexure and axial force creating tension stress in part of the section.

$\varepsilon_{cu} = 0.002$ for concrete sections subjected to compressive axial force applied at the section centroid.

8- According to assumptions No. (6&7), the ultimate compressive stress distribution along the section is taken as shown in Fig (4-3).

9- Requirements for assumptions (6&7) could be satisfied in case of rectangular ,T -, L- and trapezoidal sections as shown in Fig. (4-4).

It is assumed that there is a uniform compression stress on the equivalent area shown in Fig (4-4), (Equivalent rectangular stress block) This concrete stress is equal to $0.67\ f_{cu}/\gamma c$.

10- Concerning circular sections and other sections not mentioned in assumption (9), the ultimate stress distribution is taken according to Fig. (4-3). An alternative distribution could be done through calculating the equivalent depth of rectangle for the cases that have the area of the equivalent rectangle is equal to that of the ultimate stress area and have the same centroid.

11- For sections subjected to biaxial bending moments only or with axial force, the ultimate stress distribution is as shown in Fig. (4-3) and as mentioned in assumption No. (10).

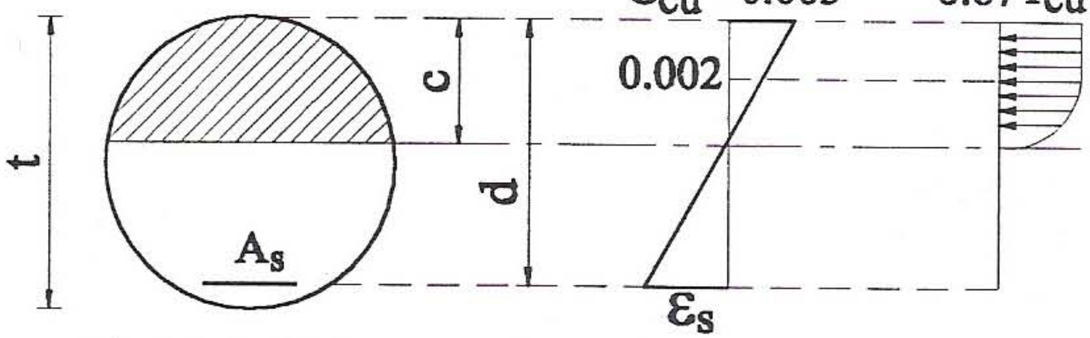


Fig. (4-3): Ultimate strains and stresses distributions

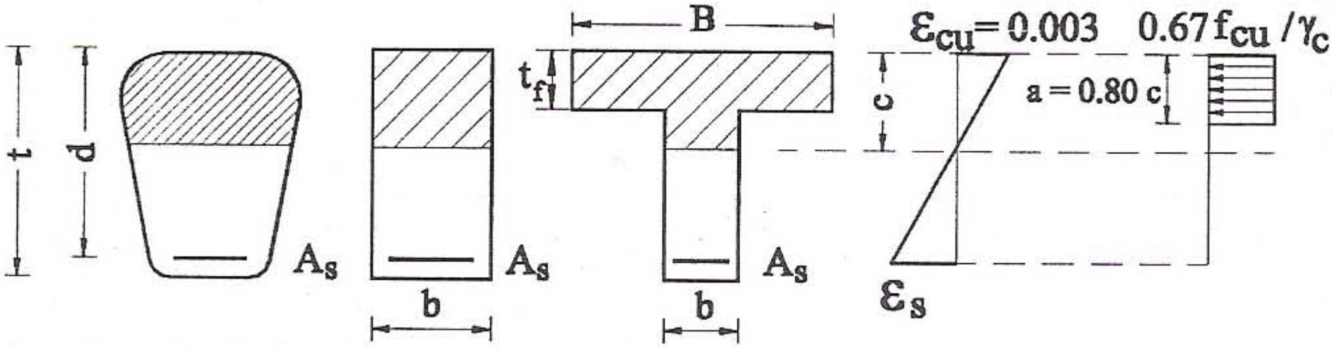


Fig. (4-4): Equivalent rectangle of compression stress distribution

Table (2-4): Mechanical properties of steel types

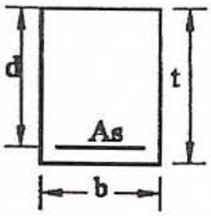
Steel type	Bar surface	Grade	Yeild stress or 0.2% proof stress N/mm² (min.)	Ultimate tensile strength N/mm² (min.)	Elongation percent (L=10 φ)*	Cold folding	
						φ mm	Diameter of curvature
Normal mild steel	smooth	350/240	240	350	20	≤25 >25	2 φ 3 φ
		450/280	280	450	18	≤25 >25	2 φ 3 φ
High grade steel	deformed	520/300	360	520	12	≤20 >20, ≤36	4 φ 5 φ
		600/400	400	600	10	20< φ ≤25 25≤ φ <36	4 φ 5 φ 6 φ
Cold ** formed welded wire mesh	Smooth or deformed or ribbed	520/450	450	520	8	--	--

* L = measurement length (mm), φ = bar diameter (mm).

** It is not allowed to use meshes with diameters less than 5 mm.

4-2-1-2 SECTIONS SUBJECTED TO FLEXURE :

a) With tension rft. only:



$M_u = (A_s\, f_y / \gamma_s)(d - a/2)$	4-1
$a = A_s\,(f_y/\gamma_s) / [\,0.67\, b\,(f_{cu}/\gamma_c)\,]$ $a/d \nless 0.1$ & $y_{ct} \ngtr 0.95\, d$	4-2

b Balanced section

$\varepsilon_y = f_y / (E_s\, \gamma_s)$	4-3
$\varepsilon_{cu} = 0.003$	

Is the limit between Brittle and ductile failure.
When concrete strain ε_{cu}=0.003, steel Yield strain ε_y reach their max limit at the same time.

c) Max. ultimate moment and max % of steel

Max allowed B.M. for sec with tension rft. only $M^*_{u\,max} = R_{max}\, bd^2\, f_{cu}/\gamma_c$	4-4
Max steel ratio for sec. $\mu^*_{max} = A_{s\,max}/bd$ with tension rft. only $= [0.67\,(f_{cu}/\gamma_c)/(f_y/\gamma_s)]\,(a_{max}/d)$	4-5

Values of Rmax & μ^*_{max} are given in table (4-1). For the case of moment redistribution by ± 10 %. Table (4-2) (see design aides).

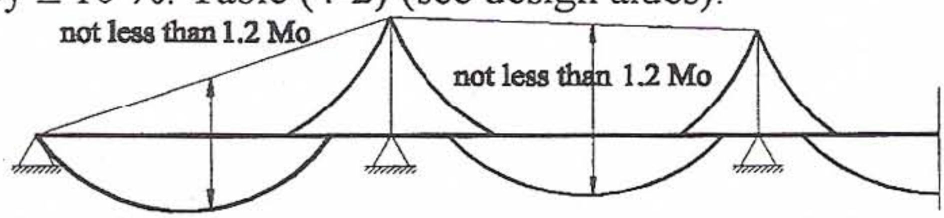
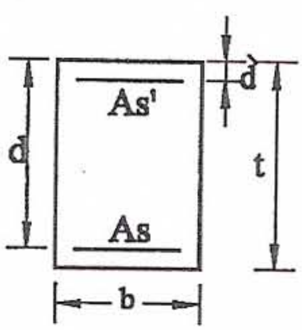


Fig. (4-5): Bending moment redistribution in beams

d) Design of rectangular sections with compression rft.



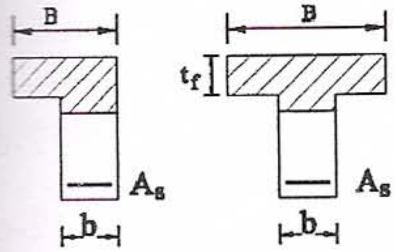
$M_u = R_{max}\,(f_{cu}/\gamma_c)\, b\, d^2 + (f_y/\gamma_s) A'_s\,(d - d')$ (4-6) $= M^*_{u\ max} + (f_y/\gamma_s)\, A'_s\,(d - d')$	
$A_s\,(f_y/\gamma_s) = 0.67\, a_{max}\, b\, f_{cu}/\gamma_c + A'_s\, f_y/\gamma_s$ (4-7) $= \mu^*_{max}\, b\, d\, f_y/\gamma_s + A'_s\, f_y/\gamma_s$ $A_s = \mu^*_{max}\, b\, d + A'_s$	

The following conditions must be satisfied:

a) Serviceability limit states for deflection and deformations
b) It is prefered that $A'_s/A_s \leq 0.4$
c) Min. comp. steel $\nless$ 10 % tension steel ($\mu' \nless 0.1\,\mu$).
d) Distance between stirrups $\ngtr 15\,\phi$.

e) Check that $\varepsilon_{cu}.E_s \geqslant f_y/\gamma_s$		f_y	24	36	40
This condition can be neglected if	(d`/d)	d ≤	0.2	0.15	0.1

e) T-and L section (Height of equivalent stress block (a) > t_f)



$M_u = 0.67\,(f_{cu}/\gamma_c) B\, t_f\,[d - (t_f/2)]$	(4-8-a)
$= A_s\,(f_y/\gamma_s)[d - (t_f/2)]$	(4-8-b)

f) Min. % of steel	1- As ,the least of	(1.1/fy)bd or 1.3 As req.	(4-9)
For R – Sec.	but not less than	0.25 % bd (st. 240/350	
T – Sec.		0.15 % bd (st. 360/520)	

For T & L sec (b) is the web width

g) Sections subjected to biaxial bending Mx & My:

Use simplified method article (6-4-6) or competability of strains.

For T-sections with flange in tension side: part of main rft. (≤As/3) could be distributed in the least of flange effective width or $L_0/10$.
Transversal rft. should be used to transfere shear force from these bars outside the web.

Eccentric compression force

- Small $P_u < 0.04\, f_{cu}\, A_c$
  Neglect P_u
  Design for flexure only
- Small Mu < P.emin
  neglect M_u
  design as column
- Compression failure
  $P_u > P_b$
  use interaction diagram
- Tension failure
  $P_u < P_b$
  use design charts for bending

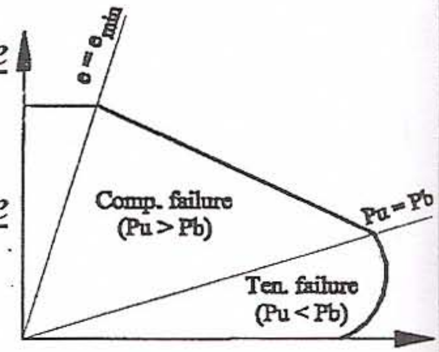
4-2-1-3 Sections subjected to Eccentric comp. force(combined flexure & axial comp)(M_u & P_u).

1) Small P_u: Design under flexure only and neglect axial force (use design chart for simple bending) if section is subjected to $P_u \leq 0.04\, f_{cu}\, A_c$ (4-10)
2) Small M_u: Sections subjected mainly to axial comp. force and small value of B.M < P.e $_{min}$; $e_{min} = M_u/P_u = 0.05$ t or 20 mm (4-11)
design section as column as follows:

Approximale design as column:

Tied Column		$P_u = 0.35\ f_{cu}\ A_c + 0.67\ A_{sc}\ f_y$	4-12-a
Spiral column	The least of	$P_u = 1.14\ (0.35\ f_{cu}\ A_c + 0.67\ f_y\ A_{sc})$ $P_u = 0.35 f_{cu}\ A_k + 0.67\ A_{sc}\ f_y + 1.38\ V_{sp}\ f_{yp}$	4-12-b
	V_{sp}	=Volume of spiral = $(\ \pi\ A_{sp}\ D_k\ /\ p\ )$ $\mu_{sp} \geq 0.36\,(f_{cu}/f_{yp})\,[\,A_c/A_k - 1\,]$ $\mu_{sp} = v_{sp}\ /\ A_k$	4-12-d 4-12-e 4-12-f

A_k = Concrete area surrounded by spiral stirrup.
A_{sp} = Cross sectional area of stirrup.
p = Pitch of spiral stirrups (ranging from 30 → 80 mm)
D_k = Diameter of concrete core inside C.L. of spiral stirrup.
f_{yp} = Yield stress of spiral stirrups.



3) Sections subjected to eccentric compression force

Compression failure i.e $P_u > P_b$

use interaction diagrams (see design aids (part II).

4) Sections subjected to eccentric compression force

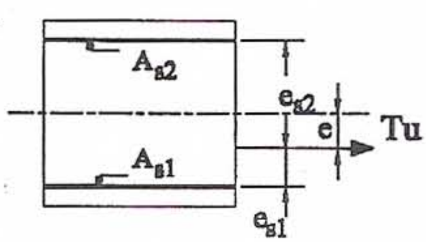
Tension failure i.e $P_u < P_b$

(use design chart for simple bending).

4-2-1-4 Sections subjected to eccentric tension (combined flexure and axial tension):

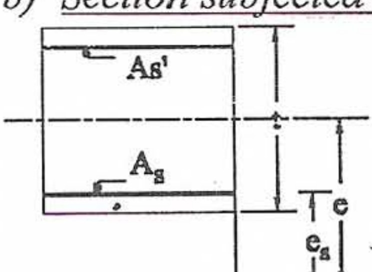
i) Sections subjected to axial tension or eccentric tension acting between steel rft.

$$\left(\frac{e}{t} < 0.5 \ \text{ or } \ e < \frac{d - d}{2}\right)$$



$$A_{s1} = \frac{T_u\, e_2}{(e_1 + e_2)}\,\frac{\gamma_s}{f_y} \quad , \quad A_{s2} = \frac{T_u\, e_1}{(e_1 + e_2)}\,\frac{\gamma_s}{f_y}$$

b) ***Section subjected to eccentric tension acting outside steel rft.*** $\left(\frac{e}{t} > 0.5\right)$



use design charts for simple bending :

M_u and T_u $\quad e = M_u / T_u \quad e_s = e - t/2 + cover$

$$M_{us} = T_u\, e_s \qquad A_s = \mu\, b\, d + \frac{T_u}{f_y / \gamma_s}$$

4-2-2 ULTIMATE STRENGTH LIMIT STATE

4-2-2-1 BEAMS

4-2-2-1-1 Nominal ultimate shearing force

a) For uniform load with comp. reaction at distance d/2

$Q_{max} = \frac{w\,L}{2} - w\,(\frac{c}{2} + \frac{d}{2})$, c is the width of the support

b) For concentrated load with $(a \leq 2d) : Q_{max} = P_u . \frac{a}{2d}$

c) For concentrated load with $(a < \frac{d}{2}) : Q_{max} = P_u$

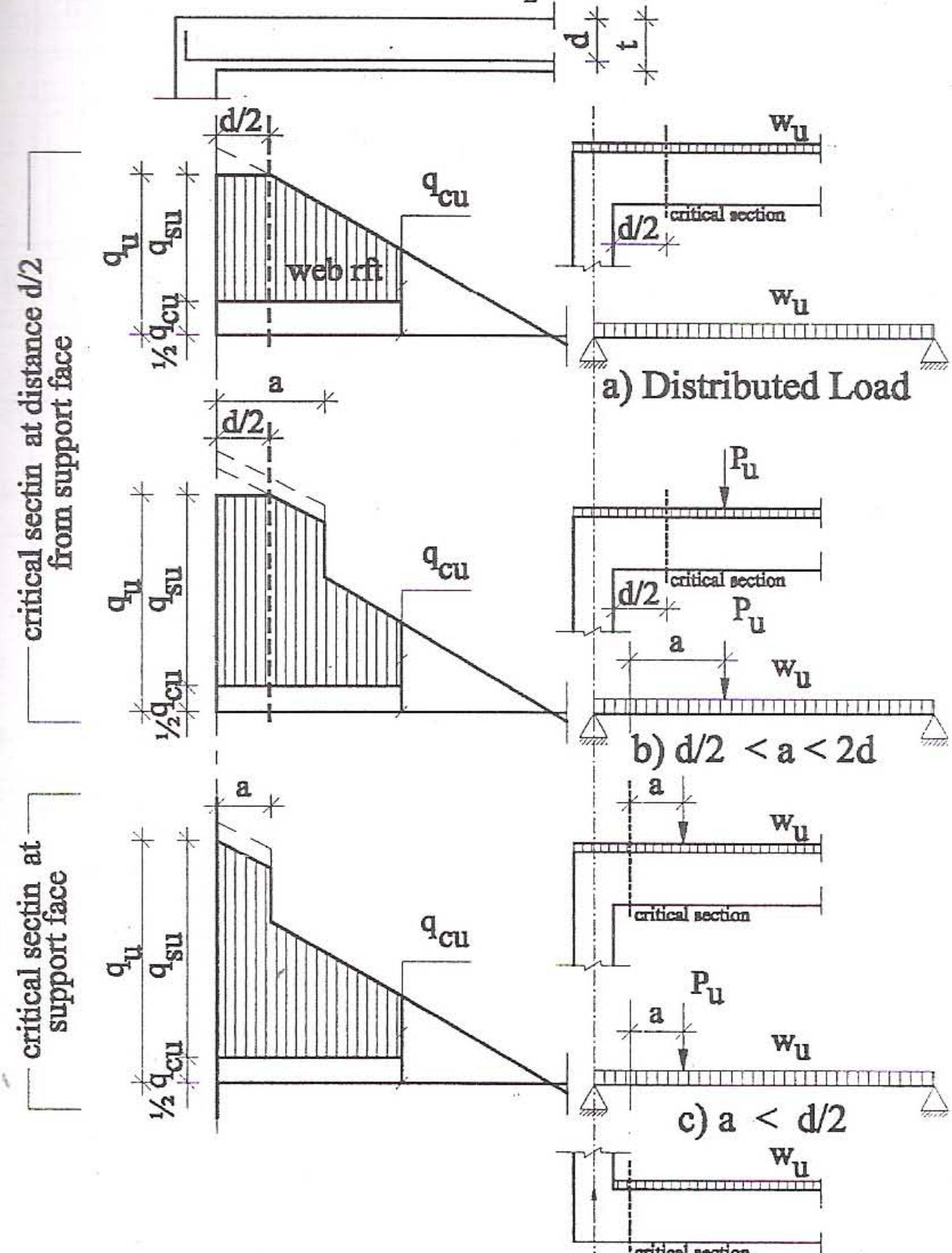


Fig. (4-7): Shear stress distribution and critical sections for beams

4-2-2-1-2 Nominal ultimate shear strength (N/mm^2)

Shear stress (Beams & slabs)	$q_u = Q_u / (bd)$	4-13
If variable depth ($\tan\beta \le 0.33$) If section height decrease while bending moment increase:	$Q_{ur} = Q_u - (M_u \tan\beta) / d$ $Q_{ur} = Q_u + (M_u \tan\beta) / d$	4-14
Max. shear stress with rft.	$q_{u\,max} = 0.7\sqrt{f_{cu}/\gamma_c} \le 3\ N/mm^2$	4-15
Combined shear and torsion	$q_{u\,max} = \delta_{si}\,(0.7\sqrt{f_{cu}/\gamma_c})$	
where: For soild sections	$\delta_{si} = 1/\sqrt{1+(q_{tu}/q_u)^2}$	4-16-a
For box sections	$\delta_{si} = 1/[1+(q_{tu}/q_u)]$	4-16-b
Torsion can be neglected if	$q_{tu} < 0.06\sqrt{f_{cu}/\gamma_c}$	4-17

4-2-2-1-3 Shear strength resisted by concrete (N/mm^2)

a) Max. concrete shear strength (without rft.):	$q_{cu} = 0.24\sqrt{f_{cu}/\gamma_c}$	4-18
b) In case of axial force P_u or T_u	$q_{cu} = \delta\,(0.24\sqrt{f_{cu}/\gamma_c})$	
For axial comp	$\delta_c = 1 + 0.07\,(P_u/A_c) \not> 1.5$	4-19
For axial tension	$\delta_t = 1 - 0.\,2\,(T_u/A_c)$	4-20
c) In case of combined shear & torsion q_{cu} can be neglected or reduced to be :	$q_{cu} = \delta_{si}\,(0.24)\sqrt{f_{cu}/\gamma_c}$	
d) In case of slabs, footing & beams with $t < 300$ mm.	$q_{cu} = 1.3 \times 0.24\sqrt{f_{cu}/\gamma_c}$	

4-2-2-1-4 Shear resisted by rft. for beams :

If $q_u > q_{cu}$ shear rft. should be used to resist q_{su}	$q_{su} = q_u - 0.5\,q_{cu}$	4-21

4-2-2-1-5 Shear reinforcement for beams :

VI. stirrups	$\mu = A_{st} / (b.s) = q_{su} / (f_y / \gamma_s)$	4-22
Inclined stirrups or bent bars	$q_{sub} = A_{sb} (f_y / \gamma_s)(\sin\alpha + \cos\alpha) / b S$	4-23
where	$q_{sub} = q_{su} - q_{sus}$	4-24
If = 45°	$q_{sub} = A_{sb} (f_y / \gamma_s) \sqrt{2} / b S$	4-25
For one raw of bent bars	$q_{sub} = A_{sb} (f_y / \gamma_s) \sin\alpha / bd$	4-26
On condition that	$q_{sub} \le 0.24 \sqrt{f_{cu} / \gamma_c}$	4-27

4-2-2-1-6 General recommendations for arranging web rft.

1)	$\mu_{min} = \frac{A_s}{b S} = \frac{0.4}{(f_y)}$	≮	st 240/350: 0.15 %	st 350/520: 0.10 %	≮ 5 ϕ 6 / m`	4-28
2)	If b ≥ 400 mm or b > t, use 4 branches stirrups with max spacing 250 mm.					
3)	If b >t: $\mu_{min} = A_{st\,min}/(b.s) = (0.4/(f_y)(q_u/q_{cu})$	where $(q_u/q_{cu}) < 1$				4-29
4)	The following structural members should be designed on basis that shear stress is resisted by conerete only according to: $(0.5\, q_{cu} \ge q_u)$ Taking into account that the concrete resitstance to shear (q_{cu}) is increased by 30%, i.e: $q_{cu} \le 0.5 \times 1.3 \times 0.24 \sqrt{f_{cu} / \gamma_c} = 0.16 \sqrt{f_{cu} / \gamma_c}$ These members include: 1- Footings and slabs 2- Beams with t < The bigger value of (250 mm or 2.5 t_f or 0.5 B_W) This case is valid for embedded beams and hollow slabs					4-30
5)	Max. spacing between v1 stir. D/2 ≯ 200mm					
6)	Max. spacing between bent bars = d or 1.5 d	If $q_{su} < 1.5\, q_{cu}$				
7)	All lines with `45° from beam center must cut a bent bar.					
8)	For shear rft. max. $f_y = 400$ N/mm² for all steel types					
9)	Avoid construction joints at zones with high shear stresses,otherwise check friction shear (4-2-2-4).					

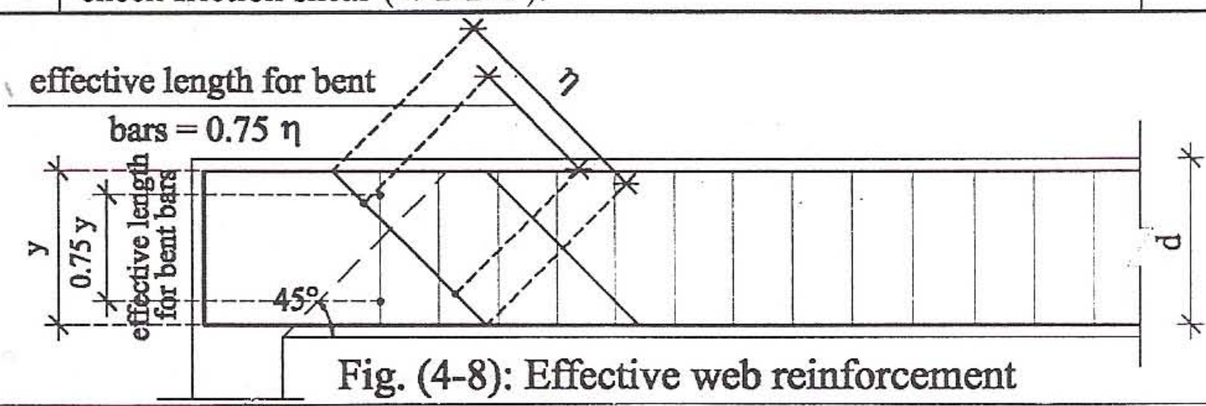


Fig. (4-8): Effective web reinforcement

4-2-2-2 Slabs and Footings :

1- Shear stresses in slabs & footings are calculated in the two directions as beams (one way shear) according to articles (4-2-2-1-1), (4-2-2-1-3), and (4-2-2-1-6-d).

2- Punching shear stresses are calculated according to article (4-2-2-3)

4-2-2-3 Punching shear

The critical section for punching shear in slabs and footing is at a distance d/2 around the circumference of the concentrated load .

b) Punching shear $q_{up} = Q_{up} / b_o\, d$ where $b_o = 2(a+b+2d)$		4-31
iii)In flat slabs, punching shear resulting from moments transfered from slabs to columns, should be taken into consideration (6-2-6-7)		
d) Nominal concrete punshing strength equal the least of:	$q_{cup} = 0.8[\frac{\alpha.d}{b_o} + 0.2]\sqrt{f_{cu}/\gamma_c}$ N/mm^2	4-32-a
	$q_{cup} = 0.316[0.5 + (a/b)]\sqrt{f_{cu}/\gamma_c}$ N/mm^2	4-32-b
	$\propto = 4$ interior col. $= 3$ external col. $= 2$ corner col.	
e) Max. $q_{cup} \le 0.316\sqrt{f_{cu}/\gamma_c}$ N/mm^2		4-33
f) Slab and footings thickness are determined as concrete only resist the punching shear as: $q_{cup} \geqslant q_{up}$		4-34

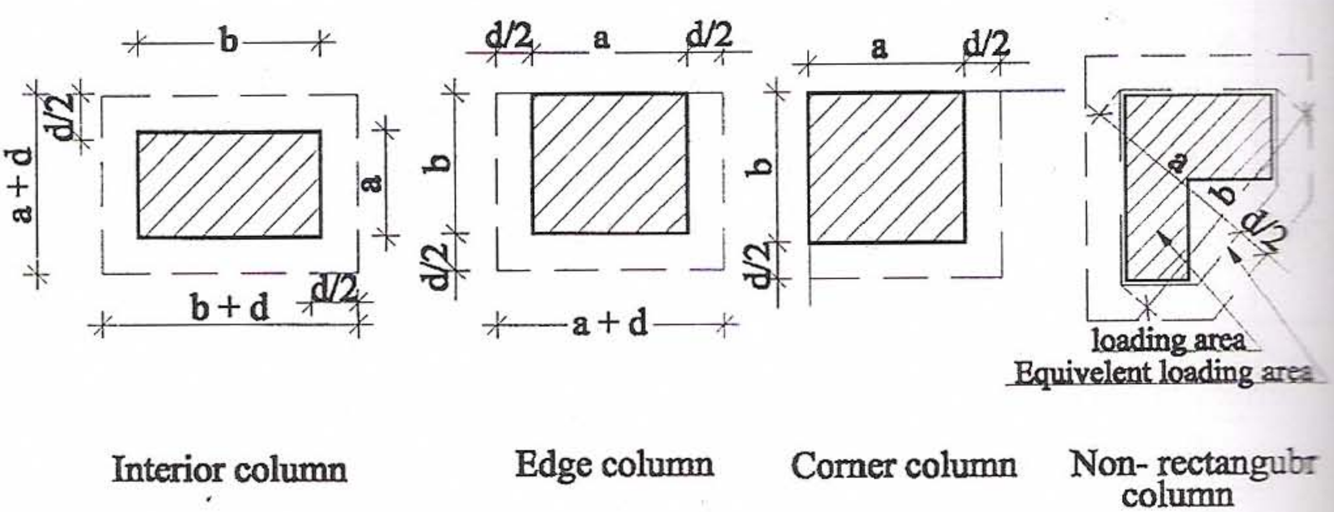


Fig. (4-9): Critical sections for punching shear

4-2-2-4 FRICTION SHEAR

i) This article is applied when shear is transfered by friction as in construction and casting joints.

ii) The shearing forces are resisted completely by rft. and concrete resistance is neglected. Rft. for friction shear is calculated as follows:

1) If the rft. perpendicular to shear plane
$$A_{sf} = Q_u / (\mu f_y / \gamma_s) \pm Nu/(fy/\gamma s) \quad (4\text{-}35)$$

2) If Rft. inclined by α_f to shear plane
$$A_{sf} = Q_u / [(f_y / \gamma_s)(\mu \sin\alpha_f + \cos\alpha_f)] \pm Nu / [(Fy/\gamma s)\sin\alpha_f] \quad (4\text{-}36)$$

c) μ = coefficient of friction

μ = 1.20 Monolethic concrete joints

= 0.80 rough surface with roughness depth $\simeq$ 5 mm.

= 0.50 roughness depth < 5 mm or steel element fixed on concrete members

d) Max friction shear stress : $Q_u / A_c \leq 0.15 f_{cu} / \gamma_c \leq 4.0$ N/mm²

where A_c is area of conc.resisting friction shear

e) For friction shear rft. max f_y = 400 N/mm² for all steel types.

f) In case of tension + equations (4-35) & (4-36) shear, the area of steel should be increased to resist the tension as in.

4-2-2-5 BRACKETS AND CORBELS

a) Short cantilevers with : a < d

b)

Main Reinf. A_s : the bigger of	$A_n + A_f$ or $A_n + (2/3) A_{sf}$	but not less than $\mu = A_s / bd > 0.03 f_{cu} / f_y$	4-37-a 4-37-b

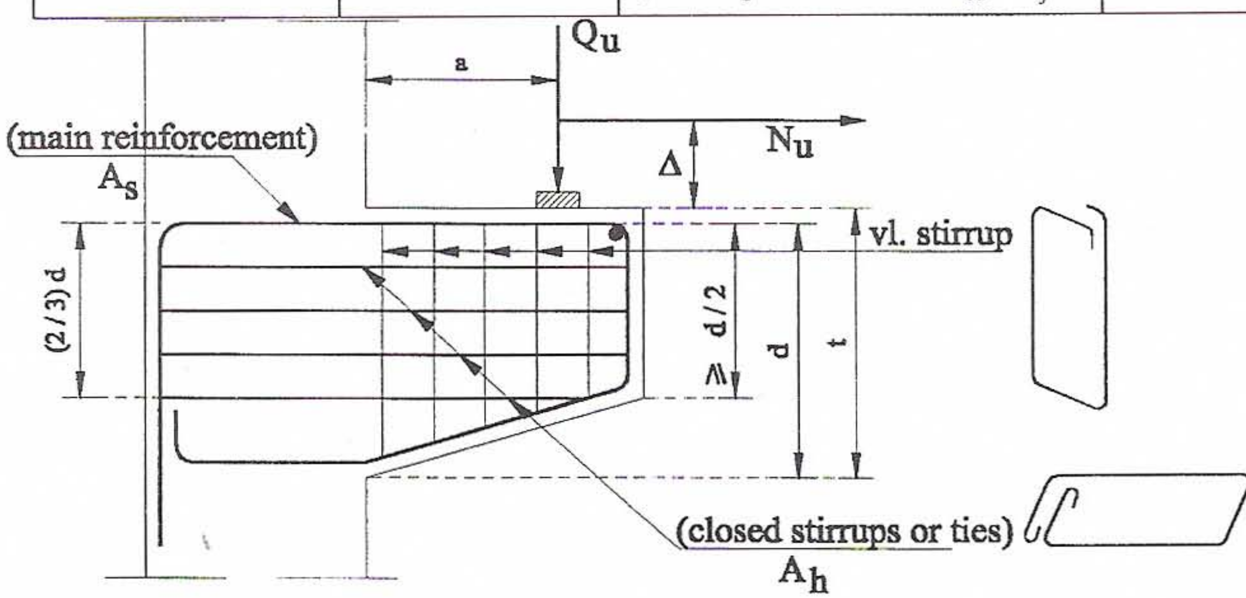


Fig. (4-10): Short cantilevers

A_f= Steel area required to resist flexure.

A_n= Steel area required to resist tension

$$M_u = Q_u.a + N_u\,(t + \Delta - d) \quad (4\text{-}38)$$

$$A_n = N_u / (f_y / \gamma_s) \quad (4\text{-}39)$$

A_{sf} = Steel area required to resist friction shear (eqn (4-35) or (4-36))

Q_u = max shearing force ≤ value in art. (4-2-2-4-b)

N_u = tension force (considered L.L.)

c) H1. Rft. (closed stirrups)

$$A_h = 0.5\,(A_s - A_n) \quad (4\text{-}40)$$

To be arranged at 2/3 d from top.

iv) For cases in which the short cantilever is exposed to torsion moment due to eccentricity of loads or due to horizontal forces, V1. stirrups should be added to satisfy the section resistance to torsion moments. In all cases the web rft. should not be less than the conditions mentioned in article (4-2-3)

v) Bearing strength must be checked according to art. (4-2-4) and loading area must not be extended beyond the straight part of main steel rft. (fig. 4-10)

4-2-2-6 DEEP BEAM IN SHEAR:

See art (6-3-2) for deep beam conditions

i) This article is applicable only for deep beams loaded at top surface by comp stresses

ii) The critical section for shear is at the following distance from support face:

for uniform load	$0.15\,L_n$	But not more than d/2 from face of support
for conc. load at distance (a) from support face	0.5 a	

c) Shear stress

$$q_u = Q_u / (b\,g) \quad (4\text{-}41)$$

where g = the least of clear span or effect. Dept

d) Max shear stress:	$q_{u\,amx} = \delta_d\,(0.70\sqrt{f_{cu}/\gamma_c})$ N/mm²w	4-15
	here $\delta_d = (\frac{1}{3})\left[2 + (0.4\,L_n / d)\right]$	4-42
e) Concrete shear strength	$q_{cu} = \delta_{dc}\,(0.24\sqrt{f_{cu}/\gamma_c})$ N/mm²	4-18
where:	$\delta_{dc} = 3.5 - 2.5\,(M_u / Q_u\,d)$	4-43
but should not be more than $q_{cu} \le 0.46\sqrt{f_{cu}/\gamma_c}$ N/mm²		4-44
f) If $q_u > q_{cu}$ use shear rft where $q_{su} = q_u - 0.5\,q_{cu}$		4-45

g) Calculations of shear Reinforcement

Shear stress	$q_{su} = \delta_v\, q_{suv} + \delta_h\, q_{suh}$	4-46-a
H1. shear stress	$q_{suh} = (A_h / S_h)(f_y / b\,\gamma_s)$	4-46-b
V1. shear stress	$q_{suv} = (A_v / S_v)(f_y / b\,\gamma_s)$	4-46-c
where :	$\delta_h = (11 - L_n / d) / 12$	4-46-d
and	$\delta_v = (1 + L_n / d) / 12$	4-46-f

where : A_h = H1. shear rft. & A_v = V1. shear rft.
S_h = Distance between H1. stirrups.
S_v = Distance between VL. stirrups.
L_n = clear span .

The shear rft. is continue with the same value along the all span.

			st 240/350	st 360/520
h) Min rft. for deep beam :	V1 Rft	$A_v/bS_v \geq$	0.2 %	0.15 %
	H1 Rft	$A_h/bS_h \geq$	0.25 %	0.20 %

Where S_v < the least of 2b or $^1/_3$ d & < 250 mm.
S_h < the least of 2b or $^1/_3$ d & < 250 mm.

i) Main rft must extend allover beam length with enough anchorage length to support.

j) Web rft. must be designed for cases of loading producing tension on loaded side and by lateral loads.

TABLE : VALUES OF ULTIMATE STRESSES OF CONCRETE (SHEAR – PUNCHING - FRICTION - TORSION - BOND - BEARING- MODULUS OF ELASTICITY)

Characteristic conc. Strength N/mm^2		f_{cu}	20.0	25.0	30.0	35.0	40.0
Shear only							
1-	Max. shear stress with rft. $q_{max} = 0.7\sqrt{f_{cu}/\gamma_c}$ (4-15)	q_{max}	2.55	2.86	3.13	3.38	3.61
2-	Conc. shear strength without rft. $q_{cu} = 0.24\sqrt{f_{cu}/\gamma_c}$ (4-18)	q_{cu}	0.88	0.98	1.07	1.16	1.24
3-	Conc. strength for slabs, footings & beams with d < 300 mm $q_{cu} = 1.3 \times 0.24\sqrt{f_{cu}/\gamma_c}$	q_{cu}	1.14	1.27	1.39	1.51	1.61
4-	Max concrete Punching strength : $q_{cup} = 0.316\sqrt{f_{cu}/\gamma_c}$	q_{cup}	1.15	1.29	1.41	1.53	1.63
5-	Frition shear strength $q_{ufric.} = Q_u / A_c = 0.15\, f_{cu}/\gamma_c$	q_u fric	2.0	2.5	3.0	3.50	4.00
6-	Deep beams: max concrete shear strength $q_{cu} = 0.46\sqrt{f_{cu}/\gamma_c}$	q_{cu}	1.68	1.88	2.06	2.22	2.37
7-	Torsion stresses can be neglected if : $q_{tu} < 0.06\sqrt{f_{cu}/\gamma_c}$ (4-17)	q_{tu}	0.22	0.25	0.27	0.29	0.31
8-	Bond strength $f_{bu} = 0.3\sqrt{f_{cu}/\gamma_c}$	f_{bu}	1.10	1.22	1.34	1.45	1.55
9-	Bearing strength = $0.67\, f_{cu}/\gamma_c$	f_{bea}	8.90	11.2	13.40	15.6	17.9
10	Modulus of elasticity E_c $E_c = 4.4\sqrt{f_{cu}}$ KN/mm^2	E_c	20.0	22.0	24.0	26.0	28.0

4-2-2 ULTIMATE LIMIT STATE – SHEAR

SHEAR EQUATIONS (Kg/cm²)

1) Nominal ult. shear stresses

Shear stress (Beams & slabs)	$q_u = Q / (bd)$	4-13
If variable depth ($\tan\beta \le \frac{1}{3}$)	$Q_{ur} = Q_u - (M \tan\beta)/d$	4-14
Max. shear stress with rft.	$q_{u\,max} = 2.2\sqrt{f_{cu}/\gamma_c} \le 30$	4-15

2) Shear strength resisted by concrete

a) Max. concrete shear strength (without rft.)	$q_{cu} = 0.75\sqrt{f_{cu}/\gamma_c}$	4-18
b) In case of axial force P_u or T_u on the section	$q_{cu} = \delta(0.75\sqrt{f_{cu}/\gamma_c})$	
For axial comp. For axial tension	$\delta_c = 1 + 0.007(P_u/A_c) \ngtr 1.5$ $\delta_t = 1 - 0.0.2(T_u/A_c)$	4-19 4-20
c) In case of slabs, footing & beams with d < 300 cm	$q_{cu} = 1.3 \times 0.75\sqrt{f_{cu}/\gamma_c}$	

3) Shear resisted by Rft. for beams

If $q_u > q_{cu}$ shear rft. should be used to resist q_{su} where	$q_{su} = q_u - 0.5\,q_{cu}$	4-21

4) Shear Reinforcenent for beams

a) V1. stirrups	$q_{sut} = A_{st}(f_y/\gamma_s)/b\,s$	4-22
b) Inclined stirrups or bent bars	$q_{sub} = \dfrac{A_{sb}(\frac{f_y}{\gamma_s})(\sin\alpha + \cos\alpha)}{b\,s}$	4-23
where	$q_{sub} = q_{su} - q_{sus}$	4-24
c) If $\alpha = 45°$	$q_{sub} = A_{sb}(f_y/\gamma_s)\sqrt{2}/b\,S$	4-25
d) For one raw of bent bars	$q_{sub} = A_{sb}(f_y/\gamma_s)\sin\alpha/b\,d$	4-26
On condition that	$q_{sub} \le 0.75\sqrt{f_{cu}/\gamma_c}$	4-27

4-2-3 ULTIMATE STRENGTH LIMIT STATE:

4.2.3.1 Critical section for torsion

a) At sec of max torsional moment.

b) In case of max M_t at support, the critical section is at distance d/2 from face of support.

4.2.3.2 Nominal Ultimate shear stresses due to torsion (q_{tu})

a- For solid R.C. Sections:

$$q_{tu} = M_{tu}/2A_o t_e \qquad (4\text{-}47)$$

Where A_0 is the area enclosed by the center- line of the shear flow zone. A_0 can be taken equal to 0.85 A_{0h}, where A_{0h} is the area enclosed by the center- line of the outer closed stirrups

t_e: is the effective thickness of the equivalent thin walled tube.

t_e can be taken equal to (A_{0h} / P_h) where P_h is the perimeter of the outer transverse torsional Rft. (Fig. 4-11-a).

b- For L & T sections, the effective part of the slab can be neglected, and the section is treated as R-section

c-If the effective part of the slab is taken into consideration in calculating the torsional stresses, the following conditions must be satisfied:

* Effective width of slab ≤ 3 times slab thickness (Fig. 4-11-b).
* Effective width of slab must be reinforced with web rft.

d- For box sections :

The shear stresses due to torsion can be calculated using eq. (4-47) where t_e is the smaller of (A_{0h}/P_h) or the smallest actual thickness of the walls of the box section.

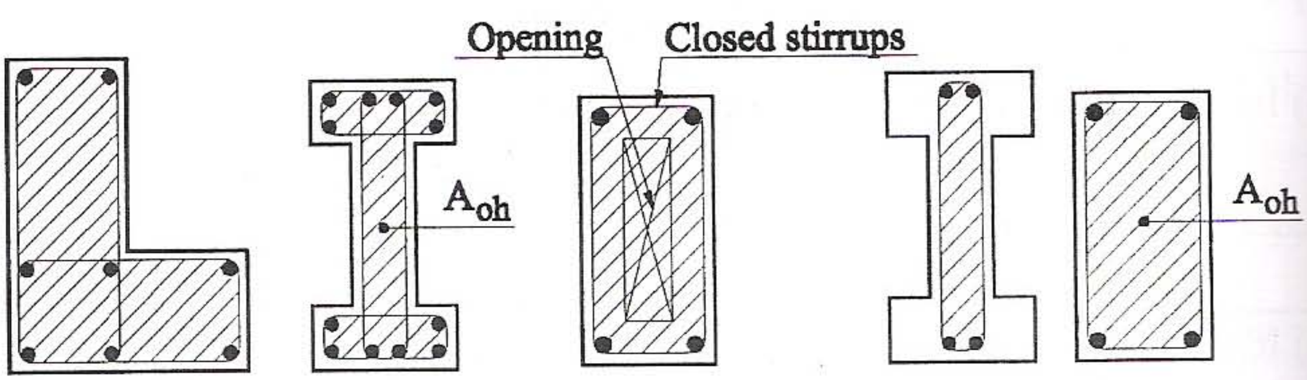


Fig. (4-11-a): Definition of A_{oh}= Hatched area

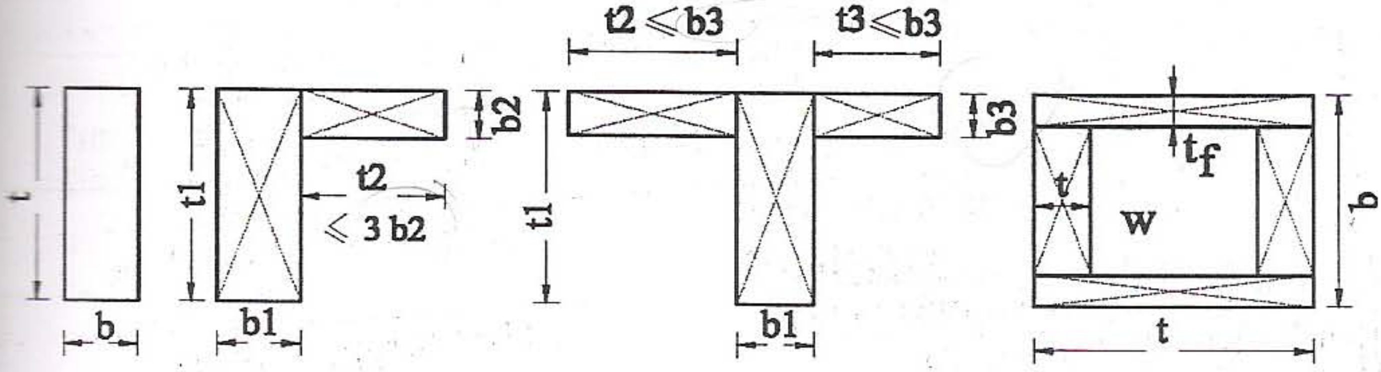


Fig. (4-11-b): Effective width of slab for torsion

4.2.3.3 Torsion effect can be neglected if: $q_{tu} < 0.06\sqrt{f_{cu}/\gamma_c}$ (4-17)

4.2.3.4 Maximum nominal shear strength:

For section subjected to torsion and shear with web & longit. Rft.:

For torsion $q_{tu\,(max)} = 0.7\delta_{ti}\sqrt{f_{cu}/\gamma_c}$ (4-48-a)

$q_{u\,(max)} = 0.7\delta_{si}\sqrt{f_{cu}/\gamma_c}$

For solid sections

$$\delta_{ti} = \frac{1}{\sqrt{(1+[\frac{q_u}{q_{tu}}]^2)}} \quad (4\text{-}49\text{-}a),$$

$$\delta_{si} = \frac{1}{\sqrt{(1+[\frac{q_{tu}}{q_u}]^2)}} \quad (4\text{-}49\text{-}b)$$

For box sections

$$\delta_{ti} = \frac{1}{(1+[\frac{q_u}{q_{tu}}])} \quad (4\text{-}49\text{-}c),$$

$$\delta_{si} = \frac{1}{(1+[\frac{q_{tu}}{q_u}])} \quad (4\text{-}49\text{-}d)$$

Table (4-4-a): Values of δ_{si} and δ_{ti} for solid sections

q_u/q_{tu}	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50
δ_{ti}	0.97	0.89	0.80	0.70	0.62	0.55	0.50	0.45	0.41	0.37
δ_{si}	0.24	0.45	0.60	0.70	0.78	0.83	0.87	0.89	0.91	0.93

Table (4-4-b): Values of δ_{si} and δ_{ti} for box sections

q_u/q_{tu}	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50
δ_{ti}	0.80	0.67	0.57	0.50	0.44	0.40	0.36	0.33	0.31	0.29
δ_{si}	0.20	0.33	0.43	0.50	0.56	0.60	0.64	0.67	0.69	0.71

4.2.3.5: Torsion and shear reinforcement:

a) The reinforcement required to resist combined torsion and shear stresses consists of closed stirrups and longitudinal bars and could be calculated as follows:

Table (4-5) Transverse rft. Resisting torsion & shear

	$q_{tu} \le 0.06\sqrt{\frac{f_{cu}}{\gamma_c}}$ N/mm^2	$q_{tu} > 0.06\sqrt{\frac{f_{cu}}{\gamma_c}}$ N/mm^2
$q_u \le q_{cu}$	Use minimum stirrup (4-2-2-1-6)	Use stirrups to resist q_{tu}
$q_u > q_{cu}$	Use stirrups to resist $(q_u - (q_{cu}/2))$	Use stirrups to resist q_{tu} & $(q_u - q_{cu}/2)$

b) The area of one leg of closed stirrups for torsion (A_{str})	$A_{str} = M_{tu} .S/[2A_0 (f_{yst}/\gamma_s)]$	(4-50)
For rectangular sections,	$A_{str} = M_{tu} .S/[1.7(x_1y_1) (f_{yst}/\gamma_s)]$	(4-51)

Where

A_{str} = area of one branch of stirrups required to resist torsion.

F_{yst} = yield stress for steel used as stirrups, but not more than 400 N/mm^2

x_1 =Width of rectangular stirrups measured between center- Lines.

y_1 =Length of rectangular stirrup measured between center- Lines.

A_o =0.85 A_{oh}.

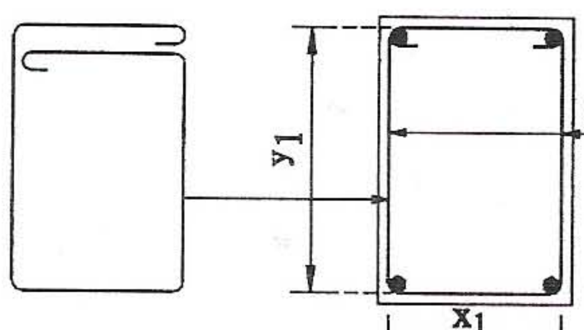


Fig. (4-12): Details of rft. for shear and torsion (stirrups with 2 branches)

Notes:

i) Cross sectional area of stirrups required for torsional moment and shear force resistance must not be less than the area calculated from the following eqn:

$$(2A_{str} + A_{st}) \ge 0.35 \,(s.b) / (f_{yst} / \gamma_s) \qquad (4\text{-}52)$$

ii) Spacing between stirrups $\not>$ the least of $(P_h/8)$ or 200 mm.

iii) For sections having stirrups with more than two branches, the external stirrup with two banches only is taken into consideration in torsional resistance. [Fig. (4-13)].

iv) For box sections, it is allowed to use the longitudinal and transverse rft. on the outer and inner section perimeter in torsional moment resistance if:

Wall thickness (t_w) $\leq$ 1/6 section breadth.

Otherwise, reinforcement on the outer perimeter only is used in resisting the torsional moment.

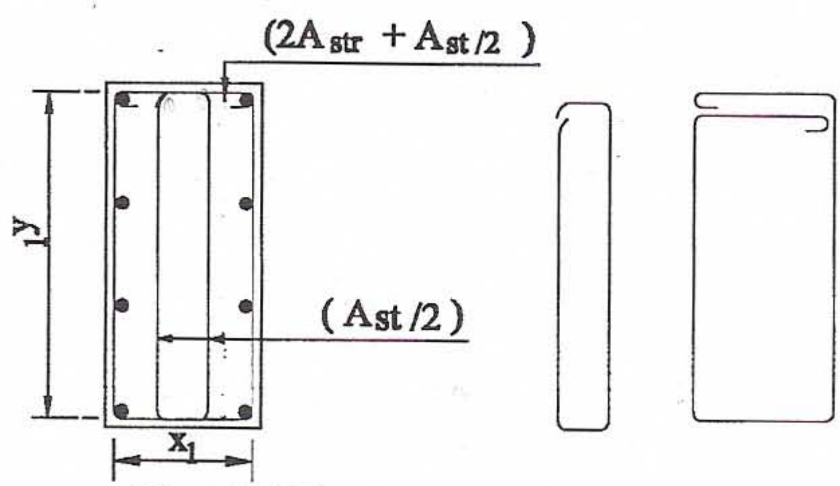


Fig. (4-13): Details of torsion and shear rft (more than 2 branches)

c) Additional long rft.	$A_{sl} = [A_{str}\, P_h / s]\,(f_{yst} / f_y)$	4-53-a
and not less than	$A_{sl\,min} = \dfrac{0.46\sqrt{\dfrac{f_{cu}}{\gamma_c}}A_{cp}}{f_y} - \left(\dfrac{A_{str}}{S}\right)P_h\left(\dfrac{f_{yst}}{f_y}\right)$	4-53-b

Where A_{cp} : The whole section area including opening,

$$\frac{A_{str}}{S} < \frac{1}{6}\frac{b}{f_{yst}}$$

This long rft. is distributed at the outer perimeters:

- min ϕ is the bigger of 12 mm or s/15 .
- max distance between long rft 300 mm.
- One additional long. bar should be put at each corner.
- This long. rft. is added to the rft from bending.

d) The torsion rft (longitudinal & transverse) must extend to a distance = ½ perimeter of stirrups after the last section of the beam required this rft.

4-2-3-6 Moment Redistribution for Statically Indeterminate Structures

Section and steel Rft must be designed as previously mentioned based on the following conditions:

i) **It is not allowed** to redistribute the moments in statically indeterminate structures for which torsional moment is essential for equilibrium (equilibrium torsion).

b) **It is allowed** to redistribute the moments in statically indeterminate structures for which torsional moment is not essential for equilibrium (compatibility torsion).

In this case the ultimate torsion moment can be reduced to

$$M_{tu} = 0.316\left(\frac{A_{cp}^{2}}{p_{cp}}\right)\sqrt{\frac{f_{cu}}{\gamma_c}} \quad (4\text{-}54)$$

Where: P_{cp} is the outer perimeter of the section and in this case flexure and shear forces must be redistributed in the adjacent spans.

4-2-3-7 Torsional rigidity of concrete sections

i) For rectangular section:

torsional rigidity = (G.C) where: $G = 0.42\,E_c$, $C = \beta\ \eta\ t\ b^3$ (4-55)

$\eta = 0.7$ for uncracked rectangular sections Where $q_{tu} < 0.316\sqrt{f_{cu}/\gamma_c}$

$\eta = 0.2$ for cracked rectangular sections.

β = Coefficient depending on $(\frac{t}{b})$ ratio given in table (4-6)

Table (4-6): Values of coeff. β for calculating the torsional rigidity

t/b	1.0	1.5	2.0	3.0	5.0	> 5.0
β	0.14	0.20	0.23	0.26	0.29	0.33

Torsional rigidity of T-,L- & box sections can be obtained by dividing them into rectangles according to article (4-2-3-2).

b) For cases of accurate calculations, the torsional rigidity can be detemined using the space truss theory.

Torsion Eqns (N/mm^2)		SUMMARY
1) Ultimate shear stresses due to torsion For sections of any shope sec.	$q_{tu} = M_{tu} / 2A_o t_e$	4-47
2) Max. shear strength for sec. subjected to torsion & shear		
with web & long. rft	(torsion) $q_{tu(max)} = 0.7\delta_{ti} \sqrt{f_{cu} / \gamma_c}$ N/mm^2	4-48-a
	(shear) $q_{u(max)} = 0.7\delta_{si} \sqrt{f_{cu} / \gamma_c}$ N/mm^2	4-48-b
For solid sections	$\delta_{ti} = 1/\sqrt{[1 + (q_u / q_{tu})^2]}$ $\delta_{si} = 1/\sqrt{[1 + (q_{tu} / q_u)^2]}$	4-49-a,b
For box sections	$\delta_{ti} = 1/[1 + (q_u / q_{tu})]$ $\delta_{si} = 1/[1 + (q_{tu} / q_u)]$	4-49-c,d
3) Torsion should be considered if	$q_{cu} \geq 0.06 \sqrt{f_{cu} / \gamma_c}$	4-17
4) Reinforcement (Torsion and shear):		
i - Colsed stirrups or welded wire mesh	$Astr = Mu \cdot S / [2A_0 * (f_{yst} / \gamma_s)]$	4-50
	For Rec. Section $Astr = Mtu \cdot S / [1.7(x_1 \cdot y_1) * (f_{yst} / \gamma_s)]$	4-51
	Check that $(2\,Astr + Ast) \geq 0.35 \dfrac{(S.b)}{(f_{ydt} / \gamma_s)}$	4-52
ii – longitudinal Rft.	$A_{sl} = (\dfrac{Astr \cdot Ph}{S})(\dfrac{f_{yst}}{f_y})$	4-53-a
	$A_{sl\,min} = \dfrac{0.46 \sqrt{(f_{cu}/\gamma_c)}\, A_{cp}}{f_y} - (\dfrac{Astr}{S}) Ph (\dfrac{f_{yst}}{f_y})$	4-53-b
5) Moment re0distribution for statically indeterminate structures:	$M_{tu} = 0.316 \left(\dfrac{A^2_{cp}}{P_{cp}} \right) \sqrt{\dfrac{f_{cu}}{\gamma_c}}$	4-54
6)Torsional rigidity of concrete sections Torsional rigidity = G.C Where: $G = 0.42\, E_c$, and	$C = \beta\, b^3\, t\, \eta$	4-55

El-Behairy Design Handbook Code 2001 Chapter (4) Torsion Equations

Torsion Eqns (kg/cm^2) SUMMARY		
1) Ultimate shear stresses due to torsion		
For sections of any shape. $q_{tu} = M_{tu} / 2A_o t_e$		4-47
2) Max. shear strength for sec. subjected to torsion & shear		
with web & long. rft	(torsion) $q_{tu(max)} = 2.2\delta_{ti} \sqrt{f_{cu}/\gamma_c}$ kg/cm^2	4-48-a
	(shear) $q_{u(max)} = 2.2\delta_{si} \sqrt{f_{cu}/\gamma_c}$ kg/cm^2	4-48-b
For solid sections	$\delta_{ti} = 1/\sqrt{[1+(q_u/q_{tu})^2]}$ $\delta_{si} = 1/\sqrt{[1+(q_{tu}/q_u)^2]}$	4-49-a,b
For box sections	$\delta_{ti} = 1/[1+(q_u/q_{tu})]$ $\delta_{si} = 1/[1+(q_{tu}/q_u)]$	4-49-c,d
3) Torsion should be considered if ≥ $q_{cu} = 0.06\sqrt{f_{cu}/\gamma_c}$		4-17
4) Reinforcement (Torsion and shear):		
i- Colsed stirrups or welded wire mesh	$A_{str} = \frac{M_{tu} . S}{2 A_0} / (f_{yst}/\gamma_s)$	4-50
	For Rec. Section $A_{str} = \frac{M_{tu} . S}{1.7(x_1 . y_1)} / (f_{yst}/\gamma_s)$	4-51
	Check that $(2 A_{str} + A_{st}) \geq 0.35 \frac{(S.b)}{(f_{yst}/\gamma_s)}$	4-52
ii -	Long Rft. $A_{sl} = (\frac{A_{str} \cdot P_h}{S})(\frac{f_{yst}}{f_y})$	4-53-a
	$A_{sl\,min} = \frac{1.455\sqrt{(f_{cu}/\gamma_c)}\,A_{cp}}{f_y} - (\frac{A_{str}}{S}) P_h (\frac{f_{yst}}{f_y})$	4-53-b
5) Moment redistribution for statically indeterminate structures:	$M_{tu} = \left(\frac{A^2_{cp}}{P_{cp}}\right)\sqrt{\frac{f_{cu}}{\gamma_c}}$	4-54
6)Torsional rigidity of concrete sections Torsional rigidity = G.C Where: G = 0.42 E_c	$C = \beta\, b^3\, t\, \eta$	4-55
El-Behairy Design Handbook Code 2001 Chapter (4) Torsion Equations		

4-2-4 ULTIMATE STRENGTH LIMIT STATE:

4-2-4-1 Max. concrete bearing strength $= (0.67\ f_{cu} / \gamma_c)\ A_1$

4-2-4-2 If $A_2 > A_1$ " " " $= (0.67\ f_{cu} / \gamma_c)\ A_1 \sqrt{A_2 / A_1}$

where : $\sqrt{A_2 / A_1} \leq 2.0$

A_1 = loaded area.

A_2 = the biggest area that coincide with A_1 and symmetric with it. figure (4-14)

4-2-4-3 If the resisting surface has inclined surface or pyramidical shape, area A_2 is taken equale to bottom base area for the biggest cone inside the incomplete pyramidical shape which its upper base is the loading surface and has a side slope 2 : 1.

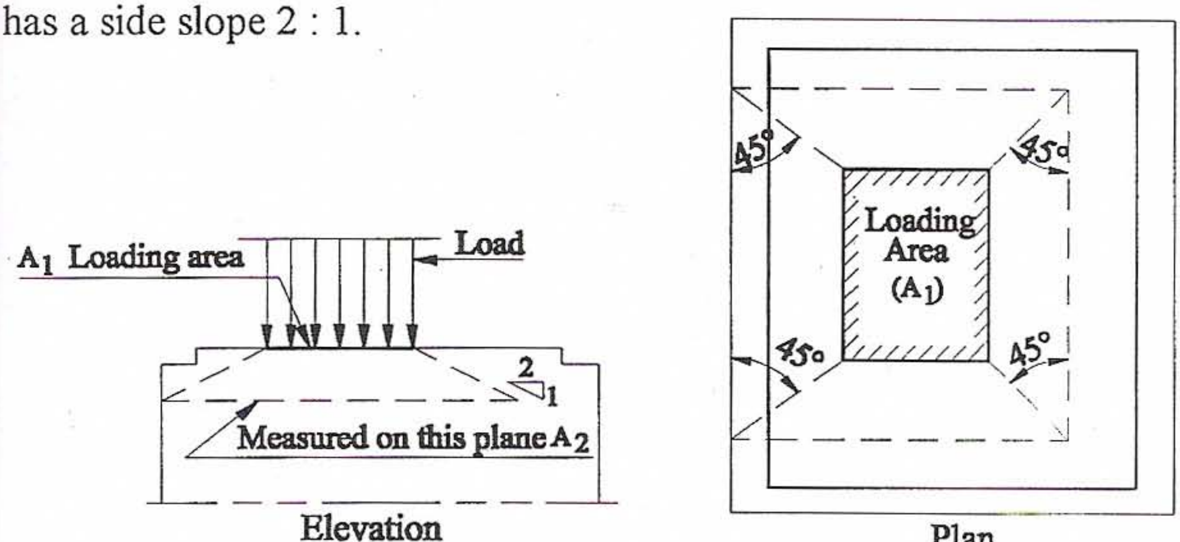


Fig. (4-14): Determination of area A_2 for bearing zones with side slope

4-2-5 BOND, DEVELOPMENT LENGTH AND SPLICIES

4-2-5-1 Development length

i) Steel bars must extend beyond the critical sections for tension, comp., and S.F.

by development length $L_d = \phi\ \alpha\ \beta\ \eta\ (f_y / \gamma_s) / (4\ f_{bu})$ (4-56)

ii) Ultimate conc. bond stress : $f_{bu} = 0.30 \sqrt{f_{cu} / \gamma_c}$ N/mm^2 (4-57)

where :

ϕ = nominal diameter of bar.

η = 1.3 for top rft. with concrete below it more than 300 mm.

= 1 for other cases.

α = Correction factor for shape of bar ends (with or without hooks), table (4-7).

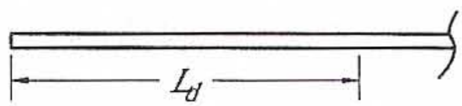
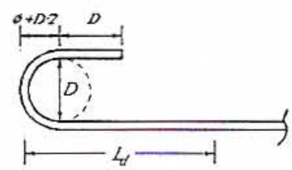
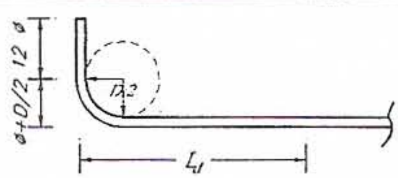
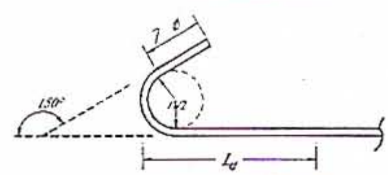
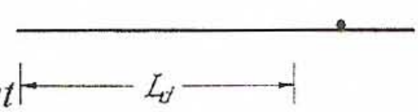
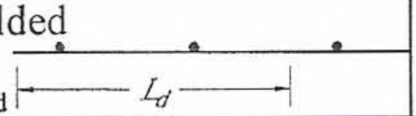
β = Correction factor for kind of bar surface (smooth or deformed), table (4-8).

c) Development length can be taken from table (4-9) for $\eta = 1.0$ & $f_{cu} >$ 20 N/mm^2.

d) L_d should not be less than 400 mm for smooth bars, and 300mm for bars with indentations.

e) Distance between bars and concrete cover must be taken according to chopter (4) and chopter (7).

Table (4-7) Values of coefficient " α"

Steel type and shape of bar end	Coefficient α	
	tension	compression
A- Bars 1- bars with straight ends 	1.00	1.00
2- U- hook (normal mild steel) 	0.75	1.00
3- L – hook (high grade steel) 	0.75	1.00
4- V- hook (high grade steel) 	0.75	1.00
B- Welded wire mesh 1- Straight bars end *without* any transverse bars within length 	1.00	1.00
2- Straight end *with one* welded cross bar within length L_d 	0.70	0.70
3- Straight end *with two* welded cross bar within length L_d 	0.50	0.50

For mild steel D = 4 ϕ

For high tensile Steel D = 6 Ф for 6mm through 25mm.

D = 8 Ф for more than 25mm.

Table (4-8) Values of correction factor (β)

Type of reinforcing bars	bars in tension	bars in compression
1- plain bar	1.00	0.70
2- deformed bar	0.75	0.50
3- deformed bar withen 2 bar bundle	1.10	0.75
4- deformed bar withen 3 bar bundle	1.20	0.80

Table (4-9) Development length L_d as multiplier of ϕ ($\eta = 1.0$)

Type of steel	bars in tension		bars in compression	Min. L_d
	straight	with hook		
Smooth bars 240/350	-	40 Φ	40 Φ	400 mm
deformed bars 280/450	50 Φ	40 Φ	40 Φ	300 mm
deformed bars 360/520	60 Φ	45 Φ	40 Φ	300 mm
deformed bars 400/600	60 Φ	45 Φ	40 Φ	300 mm

* Above values should be multiplied by $\eta = 1.3$ for top rft. so placed that more than 300 mm concrete is casted below it .

Values of ultimate bond strength of concrete $f_{bu} = 0.30\sqrt{f_{cu}/\gamma_c}$

f_{cu} (N/mm^2)	17.5	20.0	22.5	25.0	27.5	30.0
f_{bu} (N/mm^2)	1.025	1.10	1.16	1.22	1.28	1.34

4-2-5-2 Development length for shear rft.

a) Bent bars must extend beyond the inclined length by the development length in tension or compresion according to the zone at which the bars ends, and calculated according to art. (4-2-5-1-b).

b) Stirrups should be anchored in comp. zone (fig. 4-15).

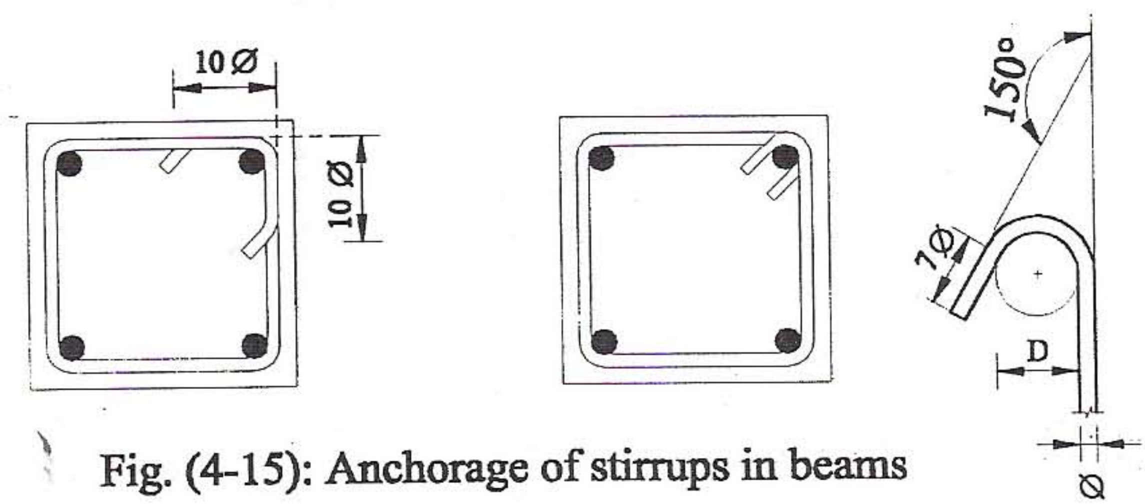


Fig. (4-15): Anchorage of stirrups in beams

4-2-5-3 Development of flexural rft.

a) If the curtailment is at tension or compression zone, steel bars must be extended to a distance not less than (L_d+0.3d) measured from the critical section at which the stresses is max in steel bars. Anchorage length (distance between bars end and the section at which this bars is not necessary for flexure resistance)must not be less than the bigger of d or (0.3d + 10ϕ) measured from bending moment diagram (Fig. 4-16).

b) If the curtailment is at tension zone Either

$$q_u \leq 2/3\ (0.5\ q_{cu} + q_{su}) \quad (4\text{-}58)$$

or $A_{str.}$ at this sec. = $A_{str.}$ for shear and torsion + A_{st}

where : $A_{st} = 0.40\ b\ s / f_y$

This additional stirrups distributed at 0.75 d from point of end of bars (Fig 4-17) taking $S \ngtr (d / 8\beta)$, where :

S = spacing between stirrups.

β = area of stopped rft. / total area of rft. at this sec

c) For simply supported and continous elements, 1/3 of postive moment rft. must be extended inside the support. For beams, distance between support center-line and bar end must not be less than 150 mm taking into consideration development length conditions in art. (4-2-5-3-d).

d) At simple supports and points of zero moment, development length at sections given in art. (4-2-5-2-b) must be checked be the following relation (Fig. 4-18):

$$\propto (M_u/Q_u) + L_a \geqslant L_d + 0.3\ d \quad (4\text{-}59)$$

Where: M_u= Ultimate moment for section reinforced with steel Rft. extended inside supports, considering that stresses in steel Rft. equal f_y/γ_s.

Q_u= Ultimate shear force calculated at the same section at which Mu was calculated.

L_a= bar extension length after support center-line or bar extension length after point of zero moment (point of inflection), and the max. value of L_a in this case is the bigger of d or 12 ϕ.

$\propto$ =1.3 For simply supported ends when loads produce compression perpendicular to beam lower edge.

$\propto$ =1.0 For other cases.

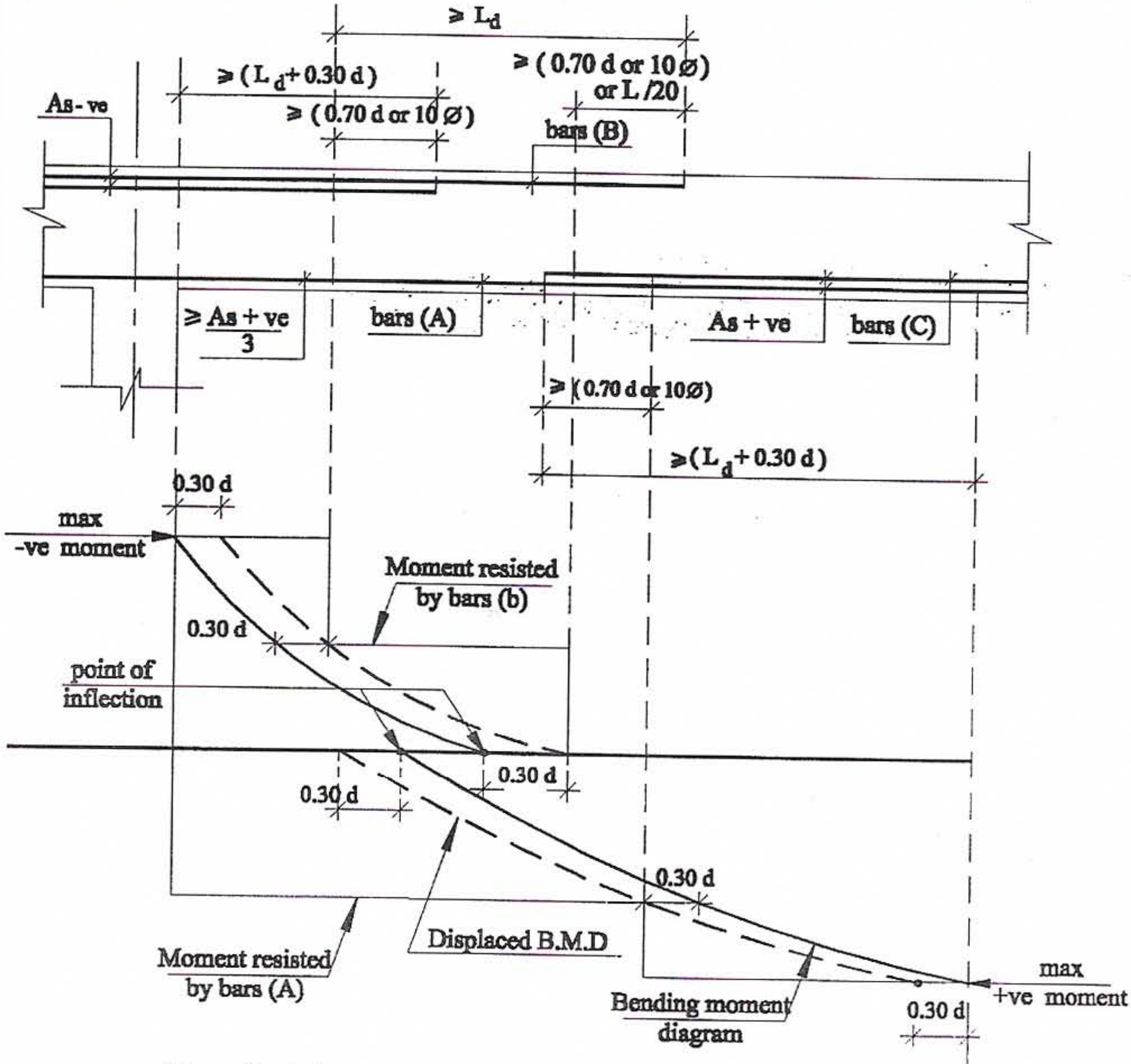


Fig. (4-16): Development of flexure reinforcement

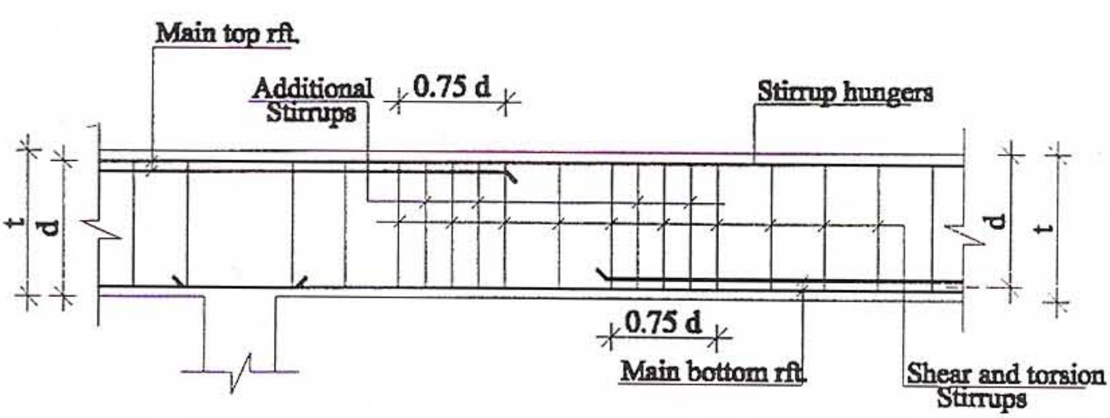


Fig. (4-17): Curtailment of bars at tension side

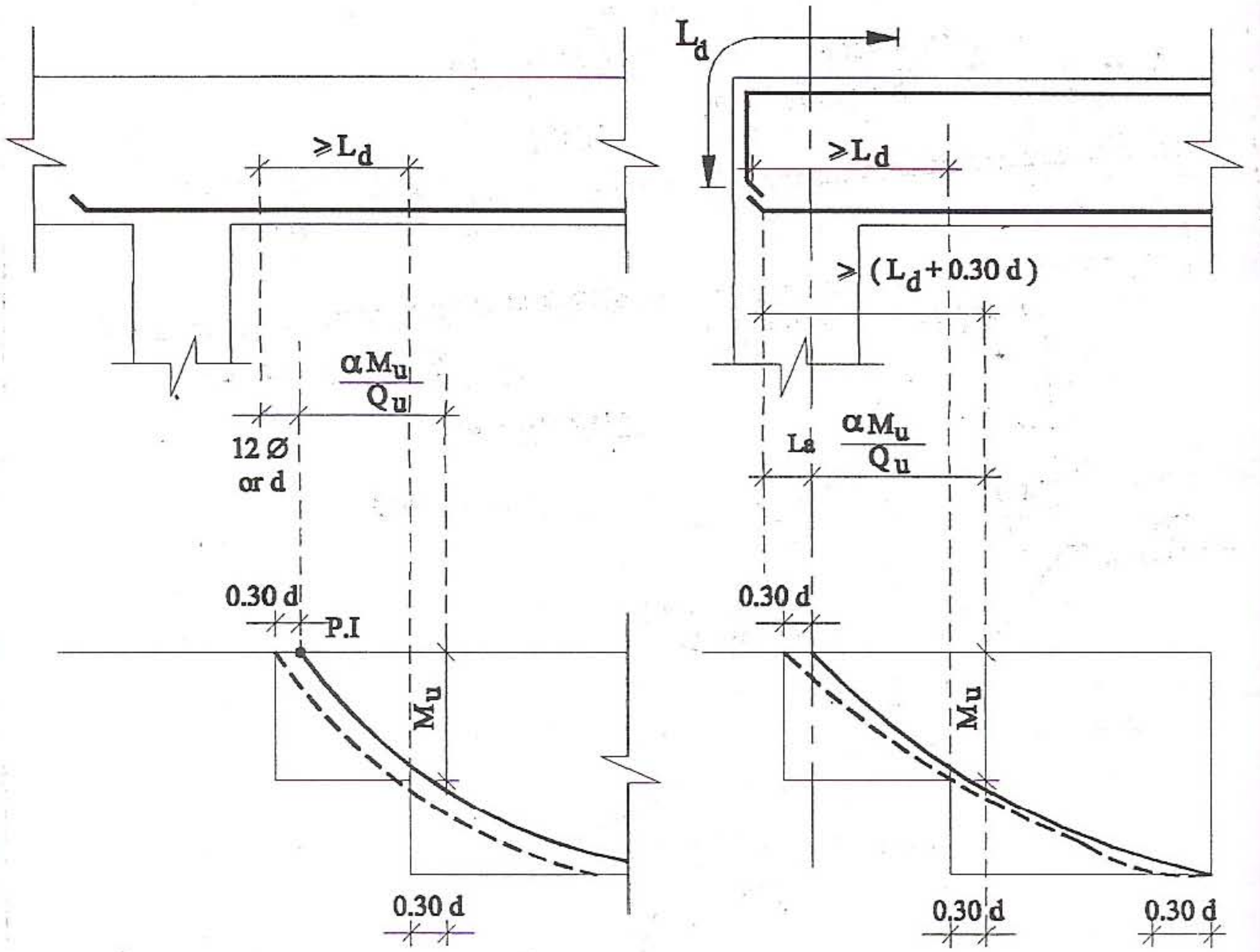


Fig. (4-18): Curtailment of bars at points of inflection and simple supports

e) 1/3 of tension rft. which resist (-ve) moment must extend after point of inflection (P.I.) by anchorage length equals to the biggest of $(0.3d + 10\phi)$, or $(0.3d + L/20)$ or (d) measured from bending moment distribution curve (Fig. 4-16).

f) All Rft. resisting (–ve) moment must be extended inside the end support by distance not less than (L_d) measured from support inner face.

g) Conditions for structures subjected to earthquake loads art. (6-7) must be taken into consideration in bars development length calculation.

4-2-5-4 Reinforcement splices:

4-2-5-4-1 Reinforcement splices is not preferable, and must be executed according to workshop drawing or under engineering supervision. Splices can be obtained by over lap between bars or weld (according to steel type) or mechanical splice. Bars must not spliced in high stresses zones.

4-2-5-4-2 Lap splices:

a- In lap splices, bars may be contact each other (fig. 4-19-a) or not (fig. 4-19-b).

Distance between the axes of any two spliced bars must be less than.	150 mm
	1/5 splice length

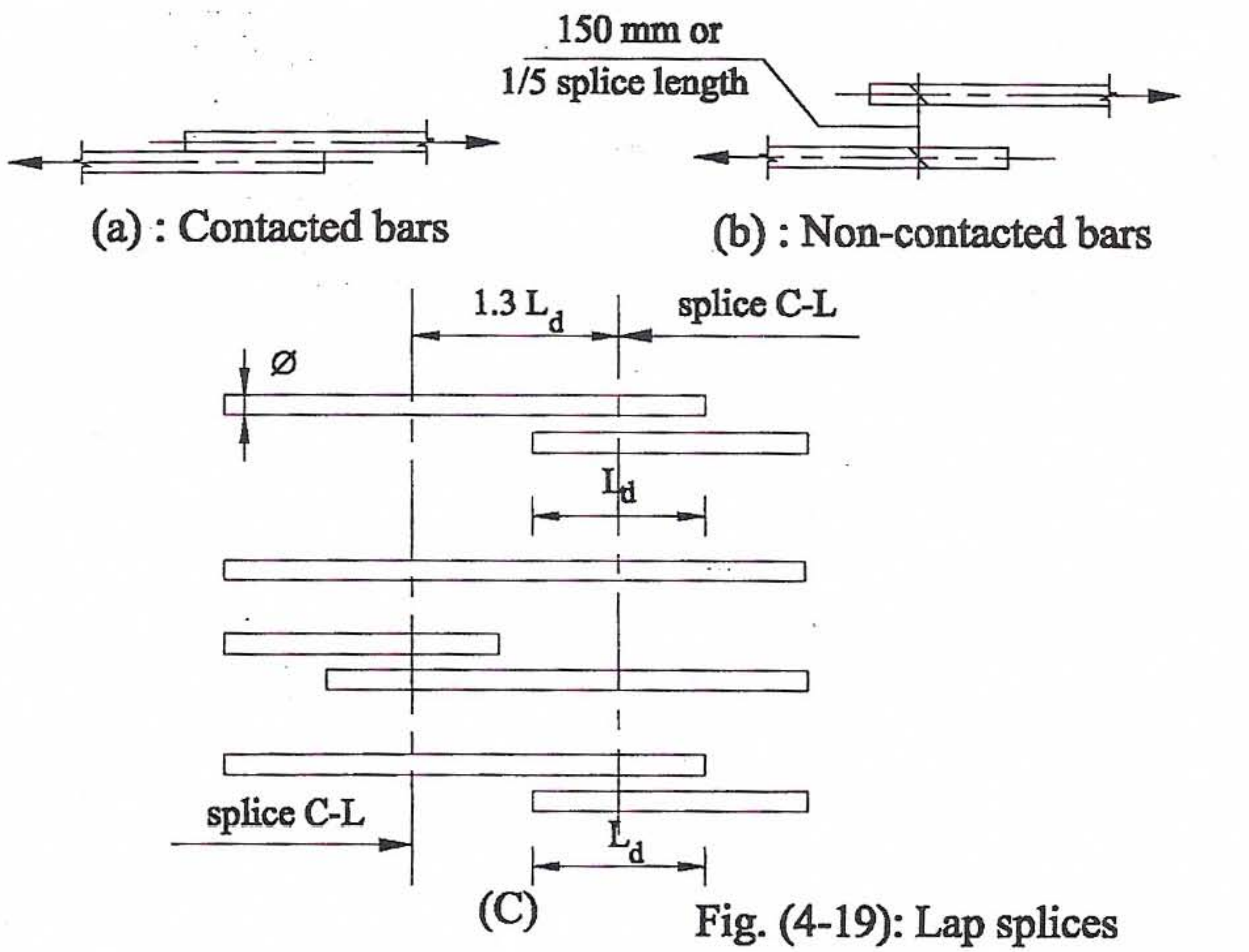


Fig. (4-19): Lap splices

b- For bars subjected to tensile forces in flexure, Lap splices must be staggered where the area of spliced bars at any section must be less than 25% of the total area of bars at this section and the distance between the axes of lap splices must not be less than 1.3 splice length as in fig. (4-19-c). It is preferable that rft. area at splice section equals to twice the required area, and the details in fig. (4-19-c) should be considered.

c- Length of lap splices (for bars subjected to tension or compression) equals to development length (L_d), but if rft. at splice section is less than twice of the required rft. (according to art. 4-2-1), in this case length of lap splice is taken (1.3 L_d).

d- Its not preferable to use lap splices for elements subjected to axial tension or eccentric tension produces tensile stresses allover the section, in this case it is preferable to splice rft. by using weld (if it is allowed) or mechanical splices.

e- It is not allowed to use lap splices for bars with diameter greater than 32mm.

f- When splicing bars with different diameter, splice length is calculated according to the greatest diameter.

g- When splicing a group of bars consists of three bars, splice length (art. 4-2-5-4-2-c) must be increased by 20 %.

h- When splicing a welded wire meshes subjected to tensile stresses, splice length must not be less than the greatest of:

Deformed bars	1.7 L_d
	200 mm
Smooth bars	1.5 L_d
	150 mm

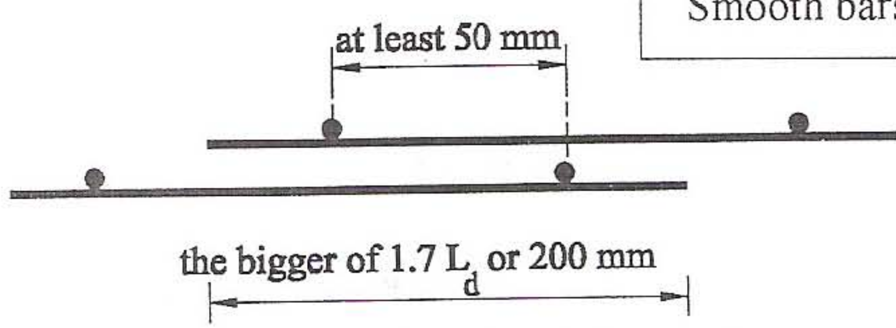


Fig. (4-20-a): Lap splices for deformed bars

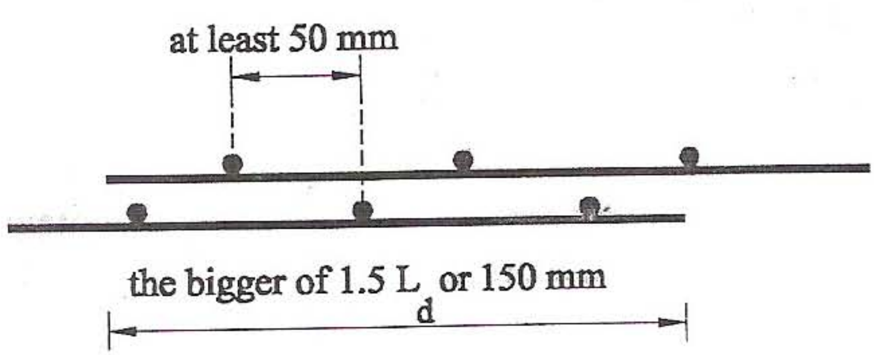


Fig. (4-20-b): Lap splices for smooth bars

4-2-5-4-3 Welded and mechanical splices:

a- It is allowed to splice bars by using weld according to standard specifications for weld at points of interlocking between any two bars taking into consideration that the axes of the two bars must be on the same line.

b- Weld section (and also mechanical splice) must resist tensile or compressive stress not less than 125 % yeild stress of the spliced bars.

c- If the previous condition is not satisfied, the splice can be accepted if:

Distance between splices ≥	600 mm
Splice resistance stress in tension or compression ≥	f_y

d- Electrical weld is only used in welding the splice.

e- It is not allowed to use weld in a distance less than 100 mm from the start point of bar rotation, and the inner diameter of rotation must not be less than (12 times bar diameter).

f- It is not allowed to use weld splice for cold treated bars before using a thermal curing for weld zone.

g- It is not allowed to use weld splice in structures subjected to frequant dynamic load.

h- For tensioned bars, weld splices or mechanical splices must be staggered where the area of the spliced bars at any section must be less than 25 % of the total area of bars at that section.

4-3 SERVICEABILITY LIMIT STATES

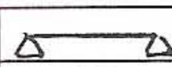
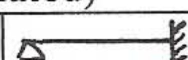
4-3-1 Deformation and Deflection Limit States

4-3-1-1 Max. Values Of Deflection (Δ) For Ordinarg Structures Subjected To Flexure;

Δ (Dead + live + temp + shrinkage + creep) loads	L / 250	One way slabs & beams	4-60-a
	L` / 450	Cantievers	4-60-b
Δ (Dead + finishes + temp + shrinkage + creep) loads	L / 350 or 20mm	One way slabs & beams	4-60-c
	L` / 600	Cantilevers	4-60-d

4-3-1-2 Effective span to total depth ratio:

Table (4-10) max effective span / depth ratio (L/d) for beams of rec. sec. with span < 10 m (if deflection is not calculated)

(L/d) min					
High tensile steel	Onyway Solid Slabs	20	24	28	10
	Ribbed slabs or beams	16	18	21	8
Normal mild steel	Onyway Solid Slabs	25	30	35	12
	Ribbed slabs or beams	20	22	26	10

For spans > 10 m check deflection

For beams with T - sec. reduce above values by coeff δ

b/B	≤ 0.3	0.4	0.5	0.6	0.7	0.8	0.9
δ	0.80	0.825	0.850	0.875	0.920	0.975	1.0

4-3-1-2-d For two way slabs with span < 10.0 m and supported on rigid beams in ordinary buildings, spans deflection Ratio is acceptable for sections subjected to bending moment and with thickness greater than 100 mm or t.

Where:

$$t = \frac{L_n \left(0.8 + f_y / 1500\right)}{36 + 9\beta} \quad (4\text{-}61)$$

L_n = Longest effective span length.

β = Long span / short span, f_y in N/mm^2.

4-3-1-3 CALCULATION OF DEFLECTION AND DEFORMATION:

Modulus of elasticity of concrete $E_c = 4400\sqrt{f_{cu}}$ N/mm^2

Effective moment of inertia $I_e = (M_{cr}/M_a)^3 I_g + [1-(M_{cr}/M_a)^3] I_{cr}$	4-62
Cracking moment $M_{cr} = (f_{ctr} I_g)/y_t$	4-63
Tensile concrete strength $f_{ctr} = 0.6\sqrt{f_{cu}}$ N/mm^2	4-64
$\Delta_{creep} = \alpha\, \Delta_{D.L}$ where $\alpha = 2 - 1.2\,(A'_s/A_s) \geq 0.6$ (with comp RFT) $\alpha = 2.0$ (without comp. RFT)	4-65

Where: I_g = gross moment of inertia neglecting cracks and steel Rft.

I_{cr} = moment of inertia of cracked section.

M_a = max. bending moment for the element.

y_t = distance between neutral axis and the extreme.

- For continuous elements, effective moment of inertia is taken the average of the effective moment of inertia of max. +ve and –ve sections.

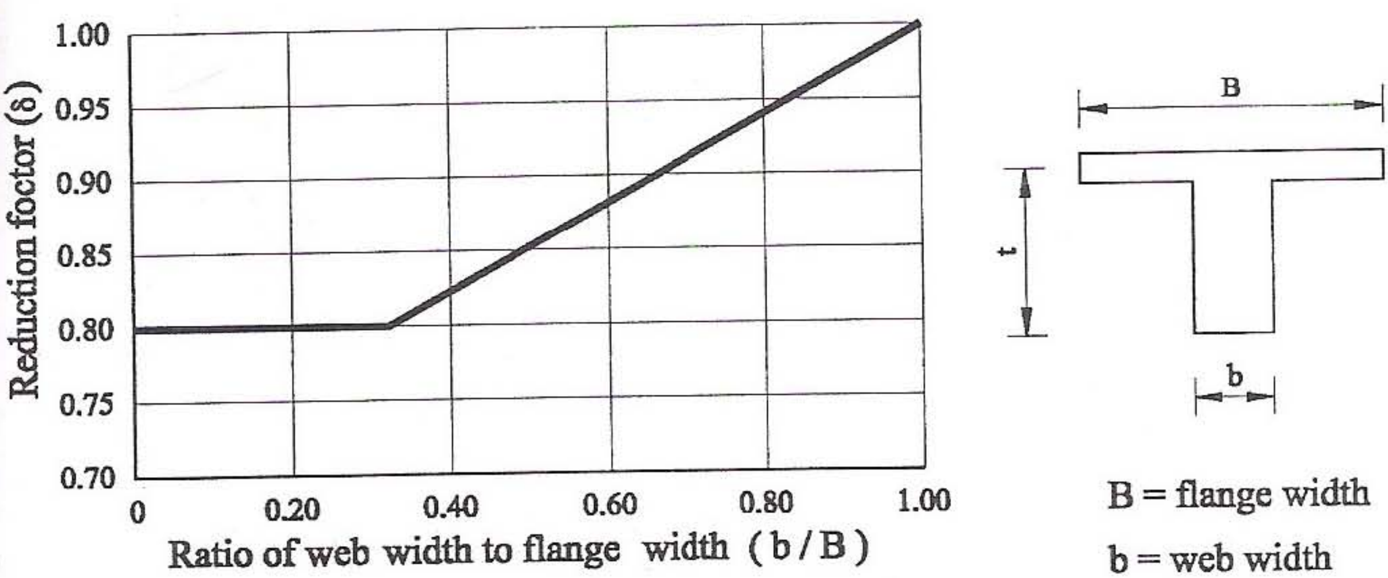


Fig. (4-21): Correction of L/t ratio for T sections

4-3-2 LIMIT STATES OF CRACKING

4-3-2-1 To protect concrete structures from severe cracking that adversely affect the durability and efficiency of these structures, the factors that affect the crack width must be considered. These factors are : concrete cover, the distribution, type, diameter and max. stresses in the steel bars subjected to tension.

4-3-2-2 To satisfy cracking limit state, the structures are classified according to the degree of exposure of their tension surfaces to the environmental effects which negatively affect the strength of the structure as given in table (4-11).

Table (4-11) Categories of structures divided according to the degree of exposure of concrete tension surface to the environmental effects

Category	Degree of exposure to environmental conditions
One	Structure with *protected* tension sides such as : a) All protected internal members of ordinary buildings b) Members permenantly submerged in water (without harmful materials) or members permenantly dry. c) Roofs well isolated against moisture and rains.
Two	Structures with *unprotected* tensile side, such as: a)Structures in open air, e.g. bridges and roofs without good isolation. b)Structures of category one built nearby sea shores . c)Structures subjected to humidity such as open halls, sheds and garages.
Three	Structures with *severly exposed* tension side, such as : a) Members with high degree of exposure to humidity. b) Members exposed to repeated moisture saturation. c) Water tanks. d) Structures subjected to vapour, gases or weak chemical attack.
Four	Structures with tension side that are *very severly* exposed to corrosive influences of strong chemical attacks which cause rusting of steel a) Structures subjected to conditions resulting in the rust of steel, such as gases, and vapours including chemicals. b) Other tanks, sewage and structures subjected to seawater.

4-3-2-3 CONDITIONS TO BE SATISFIED FOR CRACK-CONTROL :

a-1) Satisfaction of the Coefficient W_k

For reinforced concrete structurture with cracks perpendicular to steel rft.

The values of W_k should be less than or equal to the values given in Table (4-12):

$$W_k = \beta . S_{rm} . \varepsilon_{sm} \qquad (4\text{-}66)$$

$$S_{rm} = \left(50 + 0.25\, K_1\, K_2\, \frac{\phi}{\rho_r}\right) \text{mm}$$

$$\varepsilon_{sm} = \frac{f_s}{E_s}\left(1 - \beta_1\,\beta_2\left(\frac{f_{sr}}{f_s}\right)^2\right)$$

Table (4-12) Values of $W_{k\,max}$ (mm)

Category	One	Two	Three	Four
$W_{k\,max}$	0.3	0.2	0.15	0.1

Where:

ϕ = Bar diameter in mm. In case of using more than one diameter in the cross section, the average diameter shall be used.

β	**Coefficient that relates the average crack width to the design crack width. It shall be taken as follows:**	
β	For cracks induced due to loading.	1.7
	For cracks induced due to restraining the deformation in cross sections having a width or depth (whichever smaller) more than 800 mm.	1.7
	less than 300mm.	1.3
	between value, of 300 mm and 800mm, the coefficient β shall be proportionally calculated.	

β_1	Coefficient that reflects the bond properties of the reinforcing steel:	For deformed bars	0.8
		For smooth bars	0.5

β_2	Coefficient that takes into account the duration of loading:	For short term loading	1.0
		For long term loading or cyclic loading	0.5

K_1	Coefficient that reflects the type of steel of the reinforcing bars:	For deformed bars.	0.8
		For smooth bars.	1.6

In the cases of members subjected to imposed deformation, the values of k_1 should be modified to kk_1 wheres the values of k are taken as follows:

K	K
For the case in which the tensile stresses are induced due to restraining the deformation.	0.8
For rectangular section having thickness ≤ 300 mm.	0.8
For rectangular sections having thickness ≤ 800 mm.	0.5
For cases in which the tensile stresses are induced due to the restraint of extrinsic deformation.	1.0

K_2 Coefficient that reflects the effect of strain distribution shape on the distance between cracks.	K_2
for a section subjected to pure bending.	0.5
1.0 for a section subjected to pure axial tension.	1.0
For sections subjected to combined bending and axial tension, K_2 shall be calculated from. $K_2 = \frac{\varepsilon_1 + \varepsilon_2}{2\varepsilon_2}$ (4-67)	

Where ε_1 and ε_2 are the maximum and minimum strain values to which the section is subjected, and shall be calculated based on the analysis of a cracked section.

ρ_r = Ratio of effective tension reinforcement. $\rho_r = A_s / A_{cef}$

A_s = area of longitudinal tension steel.

A_{cef} = area of effective concrete section in tension.
= width of the section * t_{cef}

t_{cef} can be calculated according to Fig (4-22).

f_s = Stress in longitudinal steel the tension zone, calculated based on the analysis of the cracked section under permanent loads.

f_{sr} = Stress in longitudinal steel of the tension zone, calculated based on the analysis of the cracked section due to loads causing first cracking.

a-2) For cases in which the element is subjected to stresses due to intrinsic imposed deformations for example: restrainning against shrinkage, f_s can be taken equal to f_{sr}.

a-3) For walls subjected to shrinkage due to early thermal contraction where the lower part of wall is fixed in the wall base which has been casted previously, In this case the term s_{rm} in eqn. (4-66) can be replaced by the height of the wall in mms.

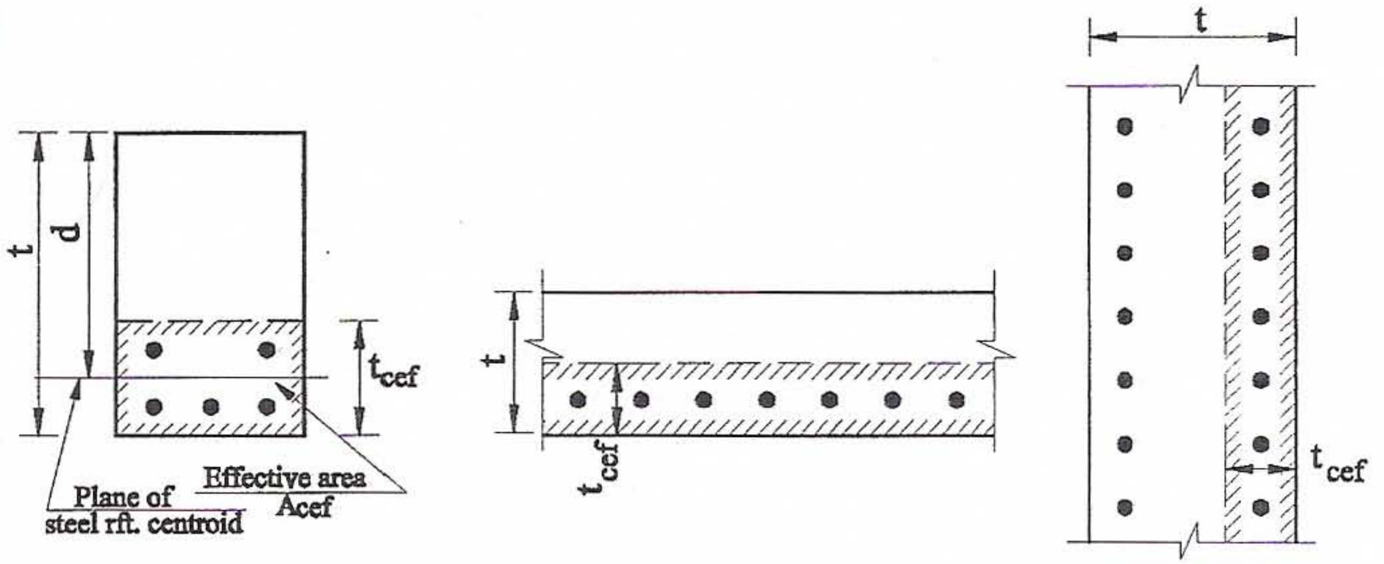
a-4) For elements having steel rft. in X & Y directions and with crack slope angle greater than 15° with the steel rft. direction, eqn. (4-66) can be obtained by replacing the term s_{rm} by the value $\dfrac{1}{\cos\theta / s_{rmx} + \sin\theta / s_{rmy}}$

Where:

θ = Slope angle between steel rft. in X-axis direction and the direction of the main tensile stresses.

S_{rmx} , s_{rmy} = the value of $[50+0.25\ k_1\ k_2\ \varnothing\ /\rho_r]$ calculated in X & Y directions, respectivly.

b) The concrete cover for steel rft. in tension should not be less than the values given in table (4-13), provided that it must be bigger than the max. diameter of the reinforcing steel. The concrete cover should be increased for the cases mentioned in article (9-7).



$t_{cef} = 2.5\,(t-d)$

t_{cef} is The least of: $(\phi/2 + \text{concrete cover})2.5$ or $(t-c)/3$

t_{cef} is The least of: $(\phi/2 + \text{concrete cover})2.5$ or $(t/2)$

a- Beams b- Slabs C- Element in tension

Fig. (4-22): Effective concrete area in tension

Table (4-13) Minimum concrete cover (mm)

Category	All element except solid slabs & walls		Walls & Solid slabs	
	$f_{cu} \leq 25$	$f_{cu} > 25$	$f_{cu} \leq 25$	$f_{cu} > 25$
One	25	20	20	15
Two	30	25	25	20
Three	35	30	30	35
Four	45	40	40	35

*concrete cover must be bigger than the max. diameter of reinforcinge steel.

c) The max. & min bar spacing according to chapters (6&7) must be satisfied.

d) Check of the allowable working concrete tensile stresses for structures of categories 3& 4, according to article (4-3-2-7).

4-3-2-4 Cases where crack width calculations are not necessary

Crack width calculations according to article (4-3-2-3-a) are not necessary if any one of the following conditions is satisfied:

a) For ordinary buildings of category 1&2 with live load not more than 5 KN/m^2 in the following two cases :

1-Solid slabs of thickness not more than 160 mm.

2-T & L - beams with flange on tension side and with B > 3b

b) For members subjected to flexure and compression force more than ($0.2\ f_{cu}\ A_c$) at working loads level.

c) If the working tensile steel stresses are equal to or less than the values given in tables (4-14) & (4-15), but Rft. ratio must be less than values in article (4-2-1-2-c).

d) When using the ultimate limit state, the yield stress is multiplied by the reduction factor β_{cr} given in tables (4-14) & (4-15).

e) Check of allowable working concrete tensile stresses at tension side for structures of categories 3 & 4 according to article. (4-3-2-6).

Table (4-14): steel working stresses and reduction factors of steel yeild stresses (βcr) which satisfy cracking limit conditions for mild steel.

f_s (N/mm^2) W.S.D	Reduction factor $\beta_{cr\,(U.L.D)}$	Category One	Category Two	Category Three & Four
		Largest Bar diameter (ϕ_{max}) in mm		
140	1.00	25	18	12
120	0.84	28	20	18
100	0.69	---	---	28

Table (4-15): steel working stresses and reduction factors of steel yield stresses (βcr), which satisfy cracking limit conditions for deformed high tensile steel.

$f_s (N/mm^2)$ W.S.D	Reduction factor $\beta_{cr\,(U.L.D)}$		Category One	Category Two	Categories Three & Four
	360/520	400/600	Largest Bar diameter (ϕ_{max}) in mm		
220	1.00	0.92	18	12	8
200	0.93	0.83	22	16	10
180	0.85	0.75	25	20	12
160	0.75	0.67	32	22	18
140	0.65	0.58	--	25	22
120	0.56	0.50	--	--	28

4-3-2-5 For sections subjected to axial tension or eccentric tensile forces which produce tensile stresses all over the section, cracking limit calculaıion for steel rft. stresses must be carried out according to art. (4-3-2-3-a), and also when using smooth welded wire mesh

4-3-2-6 For structures of calegories 3 & 4, which must be impermeable to liquids, tensile stresses calculated according to art. (4-3-2-7) must be less than values given in eqn (4-69).

- For T & L sections, it is better to take the effective flange width equal to ½ flange width in art. (6-3-1-9).
- For structures which must be impermeable for liquids, tensile stresses are checked by using the elastic method and steel working stresses are taken according to tables the (4-14) & (4-15), as alternative solution it could be calculated by using the limit states method taking into consideration the values of (βcr).

4-3-2-7 Tensile stresses in concrete sections :

a) Axial tension: $T = f_{cto}\,(A_c + nA_s)$ | where $A_s = [T/f_s]$, | $n = E_s/E_c = 10$ (4-68)

For rect. section $t = \left[\frac{T}{b\,f_{cto}}\right]\left(1 - n\left[\frac{f_{cto}}{f_s}\right]\right)$

| $t\,(mm) = \zeta \quad T(KN)$ |

For slab & walls (b=100cm) then Values of ζ average are given in the following table.

b) Simple bending: $f_{ct} = M/Z$

for R-section $t = \sqrt{6\,M/b\,f_{ct}}$ or | $t\,(cm) = \sqrt{\frac{M}{b\,\psi}}$ |

Values of ζ and ψ

Character conc. stress		f_{cu}	17.5	20.0	22.5	25.0	27.5	30.0
1)Axial tension $t \simeq \zeta\, T$		$f_c t_o$	1.4	1.5	1.6	1.8	1.9	2.0
		ζ	0.71	0.67	0.63	0.59	0.56	0.53
2) Flexure (Simple bending)	$t \geq 60$	f_{ct}	1.4	1.5	1.6	1.8	1.9	2.0
		ψ	23	25	27	30	32	33
$t = \sqrt{\frac{6M}{f_{ct}\,b}}$	$t = 40$	f_{ct}	1.5	1.6	1.7	1.9	2.0	2.1
		ψ	25	27	28	32	33	35
$t \simeq \sqrt{\frac{M}{b\,\psi}}$	$t = 20$	f_{ct}	1.8	2.0	2.1	2.3	2.5	2.6
		ψ	30	33	35	38	42	43
	$t \leq 10$	f_{ct}	2.3	2.6	2.8	3.0	3.2	3.4
		ψ	38	43	47	50	53	57

3) Eccentric tension : (M & N) [for N compression use - ve sign]

$$f_{ct} = [f_{ct}(N) + f_{ct}(M)] \leq 0.6\sqrt{f_{cu}} \;/\; \eta = \frac{f_{ctr}}{\eta} \qquad (4-69)$$

$$t_v = t\,[1 + f_{ct}(N)/f_{ct}(M)] \qquad (4-70)$$

table (4-16): values of coefficient η

Virtual thickness t_v	≤ 100 m	= 200 mm	= 400 mm	≥ 600 mm
η	1.0	1.30	1.60	1.70

CHAPTER (5)

ELASTIC (WORKING STRESS) DESIGN METHOD

5-1 General considerations

This chapter deals with the basis of design of R.C. sections, using the elastic method, under the effect of working loads (3-2--1-1-a) . To ensure the safety requirements when using the elastic method, the following condition must be satisfied:

a) The concrete, and steel stresses due to working loads, should not exceed the allowable stresses given in table (5-1) for sections subjected to flexure, eccentric force, shear and torsion stresses ..etc.

b) Conditions concerning cracking (4-3-2) and deflection (4-3-1) shall be satisfied. Stability requirements (buckling) (6-4), have to be checked.

Design of sections

5-3 Sections subjected to bending or eccentric forces	5-4 Sections subjected to shear forces	5-5 Sections Subjected to torsional moment

5-2 ALLOWABLE WORKING STRESSES

Table (5-1) gives the allowable working stresses for different concrete qualities and steel types.

Notes to table (5-1) :

1) f_{co} : represents the maximum allowable axial compressive stresses on any section under working loads.

2) The allowable working compressive stresses for sections subjected to eccentric compression with e/t ≥ 0.05, (small & medium eccentricities) should not exceed:

$$(0.23 + 0.32\, e/t)\, f_{cu} \leq f_c \qquad (5\text{-}1)$$

where f_c: Allowable concrete stresses defined in note (3).

3) f_c : Allowable stresses for beams and slabs (with t > 200 mm) due to bending or eccentric compression with big eccentricity .

4) f_{ct} : *The allowable tensile stresses for concrete which should be checked at the tension side for structures of categories three and four according to table (4-11), to satisfy the cracking limit state under working loads and for all other cases according to article (4-3-2-6,7).*

5) *Conditions of Articles (5-5-4) & (5-5-5) are to be taken into consideration.*

6) q_2 , q_c : *are specified for sections subjected to torsion and shear, by multiplying the given values (case of shear or torsion) by the factors* δ_{ti} , δ_{si} *as given in eq. (5-16) & (5-17).*

5-3 SECTIONS SUBJECTED TO BENDING OR ECCENTRIC FORCES

5-3-1 Basic assumptions and general considerations

1-Strain distribution across the section is linear and thus the steel and concrete strains are proportional to their distances from the neutral axis . This assumption is valid for all members except deep beams where the strain distribution is non-linear.

2-Concrete and steel behave as elastic materials under working loads.

3-Concrete in tension is neglected and steel rft. resists all the tensile stresses.

4-The modular ratio $n = E_s/E_c$ is taken as follows:

a) For dimensioning and calculation of stresses,

$$n = E_s/E_c = 15 \qquad (5\text{-}2\text{-}a)$$

b)For calculating elastic deformations, and for determining the statically indeterminate values and the tensile stress in concrete in members requiring a crack limit (article 4-3-2-6) :

$$n = 10 \qquad (5\text{-}2\text{-}b)$$

The whole concrete section is considered to be effective.

Table (5-1-a) Allowable working stresses for concrete (N/mm^2)
(NORMAL STRESSES)

	Characteristic strength	f_{cu}	15.0	17.5	20.0	22.5	25.0	27.5	30.0
1)	Axial comp.($e = e_{min}$)	f_{co}	4.0	4.5	5.0	5.5	6.0	6.5	7.0
2)	Comp. with small eccentricity								
	e/t = 0.1	f_c	4.0	4.6	5.2	5.9	6.5	7.2	7.9
	e/t = 0.2	f_c	4.4	5.1	5.9	6.6	7.3	8.1	8.8
	e/t = 0.3	f_c	4.9	5.7	6.5	7.3	8.1	9.0	9.8
	e/t = 0.4	f_c	5.4	6.3	7.2	8.1	9.0	9.8	10.5
	e/t = 0.5	f_c	5.9	6.8	7.8	8.8	9.5	10.0	10.5
3)	Bending or comp. with big eccentricity								
	Beams& Slabs: $t_s > 200$mm	f_c	6.5	7.0	8.0	9.0	9.5	10.0	10.5
	Slabs $120 < t_s \le 200$mm	f_c	5.0	5.5	6.5	7.5	8.0	8.5	9.0
	Slabs $100 < t_s \le 120$mm	f_c	4.5	5.0	6.0	7.0	7.5	8.0	8.5
	Slabs $80 < t_s \le 100$mm	f_c	4.0	4.5	5.5	6.5	7.0	7.5	8.0
	Slabs $t_s \le 80$mm	f_c	3.5	4.0	5.0	6.0	6.5	7.0	7.5

4) Allowable working conc. tensile stresses for crack control

				f_{cu}	17.5	20.0	22.5	25.0	27.5	30.0
a-	Cracking tensile strength $f_{ctr} = 0.6\sqrt{f_{cu}}$			f_{ctr}	2.5	2.68	2.84	3.0	3.15	3.3
b-	Axial tension $f_{cto} = f_{ctr}/1.70$			f_{ct}	1.47	1.57	1.67	1.76	1.85	1.94
c-	Simple bending $f_{ct}(M) = \frac{f_{ctr}}{\eta}$			f_{ct}	17.5	20.0	22.5	25.0	27.5	30.0
		$t_v \le 100$mm	1.0	f_{ct}	2.5	2.68	2.84	3.0	3.15	3.3
		$t_v = 200$mm	1.3	f_{ct}	1.9	2.0	2.2	23	2.4	2.5
		$t_v = 400$mm	1.6	f_{ct}	1.6	1.7	1.8	1.9	2.0	2.1
		$t_v \ge 600$mm	1.7	f_{ct}	1.47	1.58	1.67	1.76	1.85	1.94

d- Eccentric tensile force

$$f_{ct} = [f_{ct}(N) + f_{ct}(M)] \le \frac{f_{ctr}}{\eta}$$

t_v(mm)	≤100	200	400	≥ 600
η	1.0	1.30	1.60	1.70

$$t_v = t[1 + f_{ct}(N)/f_{ct}(M)]$$

Table (5-1-b) Allowable working stresses for concrete (N/mm^2)

SHEAR - TORSION - BEARING - BOND : STRESSES

Characteristic strength		f_{cu}	15.0	17.5	20.0	22.5	25.0	27.5	30.0
5) Shear or torsion (concrete strength)									
without rft. (slabs)		q_c	0.7	0.7	0.8	0.8	0.9	0.9	0.9
without rft.other members		q_c	0.5	0.5	0.6	0.6	0.7	0.7	0.7
with rft.in all members		q_2	1.5	1.5	1.7	1.8	1.9	2.0	2.1
6) Shear and torsion: Multiply q_c & q_2 by δ_{si} & δ_{ti}									
7) Torsion is neglected if: $q_t = 0.04\sqrt{f_{cu}/\gamma_c}$		q_t	0.13	0.14	0.14	0.15	0.16	0.17	0.18
8) Punching shear	a/b								
$q_p = (0.5 + \frac{a}{b})\, q_{cp}$	0.5	q_{cp}	0.7	0.7	0.8	0.8	0.9	0.9	1.0
	0.4	q_p	0.63	0.63	0.72	0.72	0.81	0.81	0.9
	0.3	q_p	0.56	0.56	0.64	0.64	0.72	0.72	0.8
	0.2	q_p	0.5	0.5	0.56	0.56	0.63	0.63	0.7
	0.1	q_p	0.42	0.42	0.48	0.48	0.54	0.54	0.6
9) Bond strength		f_b	0.58	0.63	0.67	0.71	0.75	0.8	0.82
10) Bearing strength = 0.3 f_{cu}		f_{bea}	4.5	5.2	6.0	6.7	7.5	8.2	9.0

Table (5-1-c) Allowable working stresses for steel types

Steel type	Mild steel 240/350	Steel 280/450	Steel 360/520	steel 400/600	Welded mesh 450/520	
					smooth	deformed
f_s (N/mm^2)	140	160	200	220	160	220

Note : Steel stresses (f_s) are reduced (if necessary) to satisfy the cracking limit states (article 4-3-2).

5-The allowable working stresses in the steel should be reduced to satisfy the cracking limit state conditions, article (4-3-2) .

6-If it is proved by tests that the yield stress f_y of normal mild steel round bars is *higher than 280 N/mm²* , the allowable steel stress should not be taken more than $f_y/2$ with max value of *160 N/mm²*

7-The allowable stresses of *high tensile* smooth bars, should not exceed 160 *N/mm²*

8-It is not recommended to use high tensile steel with concrete of characteristic strength after 28 days less than 20 N/mm²

9-The stresses resulting from the effect of wind, shrinkage, seismic load, change of temperature, friction at supports, or unequal settlement, should be taken into consideration, if these stresses are expected to be higher than 15% of the stresses resulting from the initial loads. The allowable stresses can be increased by 15% for each factor with maximum of 25% for all these factors. The effect of wind should not be added to the seismic effect.

10-For rectangular sections subjected to biaxial moments, the maximum allowable compressive stresses at the corner of cross section exposed to max. compressive stresses can be increased by 1 N/mm² more than the values given in table (5-1).

5-3-2 Sections subjected to flexure

1-The sections subjected to single or double moments are designed according to the general assumptions in article (5-3-1). The working stresses in concrete and steel, should not be higher than the allowable working stresses according to table (5-1) .

2-The percentage of steel rft. for sections subjected to flexure, should not be less than the values given in article (4-2-1-2-g).

3-The maximum percentage of steel for sections with rft. in the tension zone only, should not be more than the values given in table (4-2) article (4-2-1-2-c).

4-It is not allowed to redistribute the bending moments in statically indetrminate structures by more than ± 10 %, taking into consideration all the necessary requirements given in article (4-2-1-2-c).

5-Steel in the concrete side can be used to increase the capacity of sections subjected to flexure, taking into consideration the requirements in article (4-2-1-2-d).

6-The allowable concrete stresses for T-sec, should be reduced to a max value of $\frac{2}{3}$ the values given in table (5-1).

5-3-3 Sections subjected to bending and axial forces

1-Sections subjected to eccentric forces, are to be designed according to the basic assumptions in article (5-3-1), with allowable stresses as given in article (5-2) . *Also they can be designed according to the ultimate limit state given in articles* (4-2-1-3) & (4-2-1-4).

2-The minimum eccentricity to be taken in the design of sections subjected to bending moment and axial force should not be less than :

The bigger of $e_{min} = M/P = 0.05\,t$ or $e_{min} = 20mm$ (5-3)

3-Sections subjected mainly to compression force and small value of B.M. ($< P\,e_{min}$) *which may be neglected and the sections (columns) can be approximately designed as follows :*

a) For tied column	$P_w = f_{co} A_c + 0.44 f_y A_{sc}$	5-4-a
b) For spiral column	$P_w = 1.14 f_{co} A_c + 0.51 f_y A_{sc}$	5-4-b
the least of	$P_w = f_{co} A_k + 0.44 f_y A_{sc} + 0.92 f_{yp} V_{sp}$	5-4-c

Where: P_w = working loads.

- A_c, A_{sc}, A_k, V_{sp} and f_{yp} are defined in article (4-2-1-3-c-2).
- For spiral columns the conditions given in article 6-4-7 and the min. ratio of μ_{sp} (equation 4-12-a) should be satisfied.

4-Sections subjected to single B.M. and small axial compressive force with a value

$$(P_w) \leq 0.026 f_{cu} A_c \qquad (5\text{-}5)$$

The small compressive axial force may be neglected, and the section can be designed under simple bending according to article (5-3-2).

5-4 SECTIONS SUBJECTED TO SHEARING FORCES

5-4-1 Beams

5-4-1-1 The critical sections for shear are to be taken according to article (4-2-2-1-1).

5-4-1-2 Calculation of the nominal shear stresses in beams

For beams with constant depth, shear stress

$$q = \frac{Q}{b\,d} \qquad (5\text{-}6)$$

where : Q = shearing force

b = breadth of rectangular section, or breadth of the web for T - sections.

For beams with variable depth, the force Q is replaced by :

$$Q_r = Q - (M \tan \beta) / d \qquad (5\text{-}7)$$

where (tan β) is the slope of change of depth and should not be more than 0.33. It is assumed in equation (5-7), that the depth increases with the increase of the bending moment, else the - ve sign in eq. (5-7) is replaced by a +ve sign.

5-4-1-3 The shear stress (q) should not be more than the value given in table (5-1), taking into consideration article (5-5-4) for sections subjected to shear and torsion.

5-4-1-4 The max. allowable shear stresses carried by the concrete only should not be more than (q_c) given in table (5-1), for sections which are not subjected to torsional stresses.

For sections subjected to shear and tensile forces, the value (q_c) is neglected (equal to zero).

5-4-1-5 If the shear stress (q) is more than the concrete strength (q_c), web reinforcement should be used according to art. (5-4-1-7) by using one or more of the following types :

1- Stirrups perpendicular to the axis of the member.
2- Inclined stirrups.
3- Bent bars making an angle not less than 30° with the axis of the member, and stirrups perpendicular to the axis.

5-4-1-6 The web reinforcement should resist the value q_s

where $$q_s = q - 0.5\, q_c \qquad (5\text{-}8)$$

Figure (4-7) shows the zones requiring web reinforcement, with attention to article (4-2-2-1-6) of the minimum percentage of the web reinforcement in the other parts.

5-4-1-7 Calculation of the web reinforcement

	Type of shear reinforcement	Shear stress carried by rft.	
a	Stirrups perpend. to the axis	$q_{st} = \dfrac{A_{st}\, f_s}{b\, s}$	5-9
b	Inclined stirrups or bent bars with angle α to the axis	$q_{sb} = \dfrac{A_{sb}\, f_s\, (\sin\alpha + \cos\alpha)}{b\, s}$	5-10
c	Bent bars + perpend. stirrups	$q_s = q_{st} + q_{sb}$	5-11
d	If α = 45° then	$q_{sb} = \dfrac{\sqrt{2}\, A_{sb}\, f_s}{b\, s}$	5-12

Where: s = Spacing between stirrups.
A_{st} = Cross sectional area of stirrups.
A_{sb} = Cross sectional area of inclined stirrups or bent bars.

5-4-1-8 General requirements for the choice and arrangement of web reinforcement

All requirements related to the minimum percentages, and the detailing of the web reinforcement given in article (4-2-2-1-6) should be taken into consideration.

5-4-2 Slabs and footings

1-The shear stresses in slabs and footings are calculated in the longitudinal and transverse directions, same as for beams according to articles (5-4-1-2) to (5-4-1-4), taking into consideration that nominal shear stress calculated from eqn (5-6) shall be less than 0.5 q_c given in table (5-1).

2-Punching shear stresses are calculated according to article (5-4-3).

5-4-3 Punching shear

5-4-3-1 The critical section for punching shear stresses is at distance (d/2) from the perimeter of the concentrated load (figure 4-9).

5-4-3-2 Punching shear stress:

$$q_p = Q_p / (b_o\, d) \qquad (5\text{-}13)$$

Where b_o is the length of the perimeter of the critical section.

5-4-3-3 The effect of the moment transmitted from flat slabs to columns, should be added to the punching shear according to article (6-2-6-7).

5-4-3-4 The thickness of the slabs and footings necessary to resist the punching shear, should be calculated on the condition that the punching shear is resisted by concrete only without contribution of the shear reinforcement, and it should satisfy the following eqns:

Column position	interior	edge	Corner		
				$q_b = 2.5\left(\frac{\alpha \cdot d}{b_o} + 0.2\right) q_{cp} \leq q_{cp}$	5-14-a
α	4	3	2	$q_p = [\,0.5 + (a/b)\,]\, q_{cp} \leq q_{cp}$	5-14-b

Where: value of q_{cp} is given in table (5-1), (a, b) are shown in figure (4-9), and article (4-2-2-3-d).

5-5 DESIGN OF SECTIONS SUBJECTED TO TORSION

5-5-1 The critical sections are taken according to article (4-2-3-1).

5-5-2 Nominal shear stresses due to torsional moment:

a)For solid R.C sections

$$q_t = \frac{M_t}{(2A_0 \cdot t_e)} \quad ..(5\text{-}15)$$

Where A_0 , te are defined in art (4-2-3-2).

b) For T & L - sections, the effective part of the slabe can be neglected and eqn. (5-15) can be used.
If the effective part of the slab is considered, refer to article (4-2-3-2) and figure (4-11-b).

c)For box sections: refer to art. (4-2-3-2-d).

5-5-3 Effect of torsional moment may be neglected, if the resulting value of q_t is less than. $0.04\sqrt{f_{cu}/\gamma_c}$

5-5-4 Shear stresses resulting from torsional moment should not be more than (q_2). This should be multiplied by the factor (δ_{ti}), if the section is accompanied by shearing force. Also, the shear stresses resulting from shearing forces should not be more than (q_2) multiplied by (δ_{si}), if it is accompanied by torsional moment.

$q_{t_2} = \delta_{ti}\, q_2$	(5-16)
$q_{s_2} = \delta_{si}\, q_2$	(5-17)

Where δ_{ti} & δ_{si} from table (4-4-a), (4-4-b) for solid & box sections respectivity.

5-5-5 Steel rft. required to resist combiend shear and torsion:

If $q_{t2} > q_t > 0.04\sqrt{f_{cu}/\gamma_c}$, web and longitudinal rft. should be used to resist torsional moment. This rft. is to be added to any rft. required to resist flexure and shear stresses.

a) Area of transverse rft. A_{str} which consists of closed stirrups or welded wire mesh [fig. (4-12)]:

For rectangular section

$A_{str} = M_t.s / 2A_0.f_s$	(5-18-a)
$A_{str} = M_t.s / 1.7\, X_1.y_1.f_s$	(5-18-b)

A_{str}, X_1, y_1, s are defined in article (4-2-3-5)

Cross sectional area of stirrups required to resist shear and torsion shall not be less than the value given in eqn (4-52).

b- Area of additional longitudinal rft.: $A_{sl} = (A_{str}.P_h / s)(f_{yst} / f_y)$ (5-19)

Where P_h is defined in article (4-2-3-2)

This rft. is distributed along the outside perimeter. The longitudinal rft. should not be less than the value given in eqn (4-53-b).

- Spacing between stirrups should not be more than the smaller of $P_h / 8$ or 200 mm.

- The minimum bar diameter used in longitudinal rft. is the greater of (1/15) the spacing between stirrups or 12 mm.

Table (5-2) Transverse rft. to resist the torsional moment and shearing force

	$q_t \le 0.04\sqrt{f_{cu}/\gamma_c}$	$q_t > 0.04\sqrt{f_{cu}/\gamma_c}$
$q \le q_c$	The minimum % of steel rft., article (4-2-2-1-6)	Rft. to resist the torsion (q_t)
$q > q_c$	Rft. to resist the shear $(q - q_c / 2)$	Rft. to resist both of $(q - q_c/2), (q_t)$

- The additional longitudinal rft. is distributed on the perimeter of the section. Spacing between bars should not be more than 300 mm. One bar is placed at each corner.
- The additional rft. is added to the longitudinal rft. required for resisting the bending moment.
- The transverse and longitudinal rft. required to resist the torsional moment, should be extended by a distance equal 1/2 the perimeter of the stirrup after the theoretical end point of this rft.

- It is not allowed to redistribute the torsional moment in the statically indeterminate structures in which torsional moment is essential for equilibrium.

5-5-6 The torsional rigidity is calculated as shown in article (4-2-3-4-7).

5-6 BEARING

5-6-1 Bearing resistance should not be more than ($0.3\ f_{cu}\ A_1$).

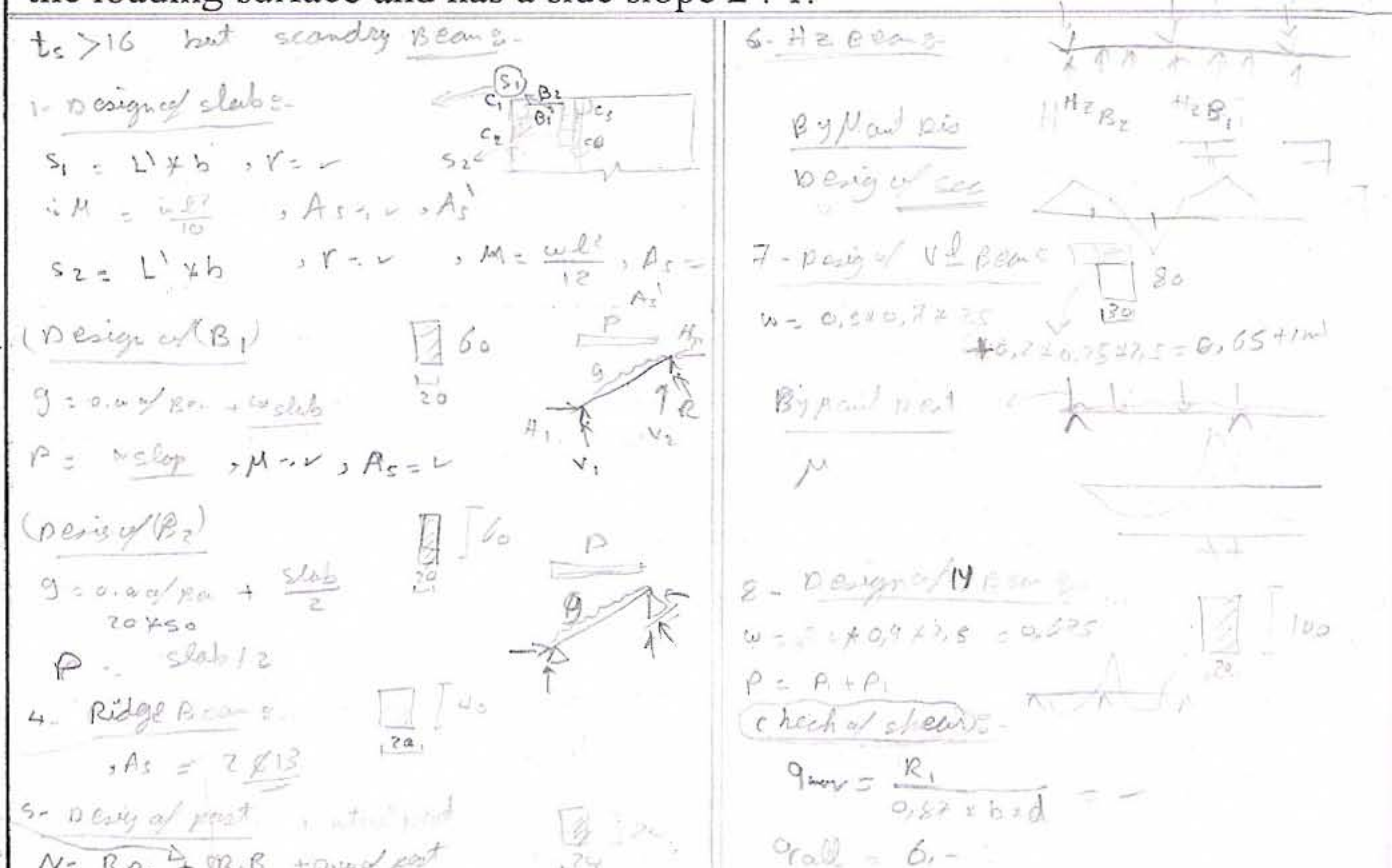
A_1 = Bearing area

5-6-2 If the resisting surface is bigger than loading surface, the bearing resistance will be equal to the value given in (5-6-1) multiplied by the factor,

$$\sqrt{A_2/A_1} \leq 2$$

A_2: The biggest area of the resisting surface, identical in shape and centric with loading surface, figure (4-14).

5-6-3 If the resisting surface has inclined surface or pyramidical shape, area A_2 is taken equals to bottom base area for the biggest cone inside this pyramid or inconplete cone inside the pyramidical shape that its upper base is the loading surface and has a side slope 2 : 1.



Chapter (6)

CONTINUE

CHAPTER (6)

STRUCTURAL ANALYSIS OF STRUCTURAL ELEMENTS

6-1 General Assumptions

a) Any method of structural analysis can be used if it achieves the structure equilibrium and strains compatibility.

b) The different elements of a structure are to be designed for the maximum internal forces that are liable to take place due to dead and live loads placed in the worst position.

c) Ordinary buildings subject to uniform loads may be calculated for full panels loaded, i.e without considering partial loading of any span.

d) When calculating the reactions of the slabs and beams it is allowed to neglect the effect the effect of continuity for approximately equal spans and loads, provided that the reaction of the first internal support is increased by 10% and the shear at the face of the first internal support in the edge spans is increased by 20 %.

e) Effect of continuity should be considered when calculating the reactions of beams on columns or girders, if the spans differ by more than 20 % . In this case, the reactions may be calculated for all panels loaded.

f) In case of cantilever slabs or beams, the reactions at external supports should be increased due to the cantilever moment .

g) In ordinary buildings, it is not necessary to consider the effect of temperature and shrinkage in the statical calculations except in buildings in which the stresses produced by temperature have a considerable effect. Expansion joints should however be arranged in long buildings to reduce the effect of temperature and shrinkage according to sections (9-5-7) and (9-5-8).

h) Long term strains may not be taken into consideration in the calculations of the internal forces in ordinary buildings except when these strains have a considerable effect.

6-2 SLABS

This part includes the following types of slabs :

1- One way solid slabs.
2- Two way solid slabs.
3- Ribbed and hollow block slabs.
4- Panelled beams.
5- Flat slabs.

6-2-1 ONE WAY SOLID SLABS

Definition

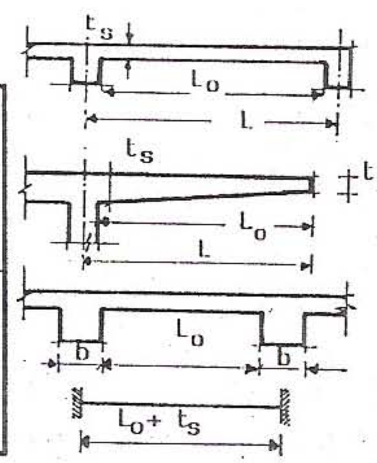
1) One way solid slabs are slabs supported in one direction on two opposite beams or walls

2) Slabs supported on four sides with effective length greater than twice its effective breadth $(L/L_s \geq 2)$.

3) One way slabs are to be calculated based on strips of unit length in the short diection.

6-2-1-1 Effective span The least of

Simple slabs	$C_L - C_L$ of beams or greatest of	$\left[\begin{array}{l} 1.05\ L_o \\ L_o + t_s \end{array}\right.$
Contiliever slabs	C_L till end or	$L_0 + t_s$

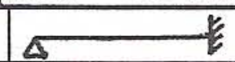
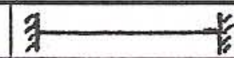
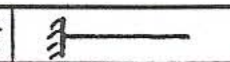


Continuous slabs with b > 20 % L_o can be designed as fixed slabs with separate bays.

6-2-1-2 Minimum thickness

1) Minimum thickness must satisfy the deflection condition according to article (4-3) and there may be no need for check of deflection if the slab thickness is not less than the value given in table (4-10).

Table (4-10) Max span / depth ratio (L/d)
(Slabs with spans < 10 ms,)

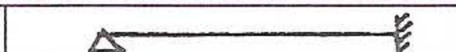
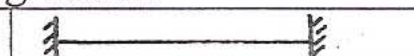
(L/d) min.				
High tensile steel	20	24	28	10
Normal mild steel	25	30	35	12

where L = Effective span

4-3-1 Max. allowed deflection (Δ)

Case of loading	Cantil. slabs	other slabs
All loads, taking temp., creep and shrinkage effect into consideration	$\frac{L'}{450}$	$\frac{L}{250}$
Loads including finishes, temp, creep, shrinkage but without L.L.	$\frac{L'}{600}$	$\frac{L}{350}$ or 20 mm

2) Slab thickness must not be less than the following values:

		
t_{min}=L/30	t_{min}=L/35	t_{min}=L/40

Where L is the effective span for the oneway slab.

3) For ordinary buildings the slab thickness should not be less than :

$t_s \nless 80$ mm	under static loads
$t_s \nless 120$ mm	under dynamic loads

4) For pre-cast slabs, the thickness can be taken less than the above values.

<u>**6-2-1-3 Bending moments:**</u>

1)Continuous slabs can be calculated as continuous beams over rigid supports on the condition that the top steel is kept in the right position.

2)Negative moments can be reduced as shown in figure (6-1).

3)Positive bending moments in any intermediate span should not be less than $(+wL^2/16)$ taking into considration art. (4-2-1-2-g).

4)Min - ve B.M. at exterior supports for slabs supported over walls, masonry or plain concrete (with partial fixation), is given in eq. (6-1)

5)For slabs supported over R.C. beams monolithically casted with the slab is given in eq.(6-2). The positive bending moments are to be calculated neglecting this partial fixation.

6) Slabs can be considered fixed at their ends if they are prevented to rotate by acertain enough method at all cases of loading.

7)In cases of heavy live loads $P > 2g$, the Min. negative field moments must be calculated. If the slabs are casted monolithically with beams, the -ve field moments due to live loads only can be reduced to one half their values due to torsion rigidty of beams which carry the slabs Min-ve field moment is taken accaording to eqn (6-4).

8)In cases of equal loads (and $p \leq g$) and equal spans or spans with max. difference of 20 % from biggest span, the values of the bending moments according to equation (6-3) may be assumed .

9)In cases of design using limit state method , w_u is used instead of w.

a	Min - ve B.M. over exterior supports 1)Walls, masonry or plain conc. (supports with partial fixation)	M_{-ve} min = - $wL^2/16$	6-1
	2) R.C. beams monolithically casted with slabs.	M_{-ve} min = - $wL^2/24$	6-2
b	B.M. for equal spans & loads (or difference ≤ 20 %) 1) Simple span	$M + ve = wL^2/8$	6-3-a
	2) Two spans	$M + ve = wL^2/10$ $M - ve = wL^2/8$	6-3-b 6-3-c
	3) Continuous slabs : value of K given in fig. (6-2)	$M = \pm wL^2/k$	6-3-d
c	In case of heavy L.L. (p>2g) The min.-ve field moment	$M_{min} = (g-(p/2)) \frac{L^2}{24}$	6-4

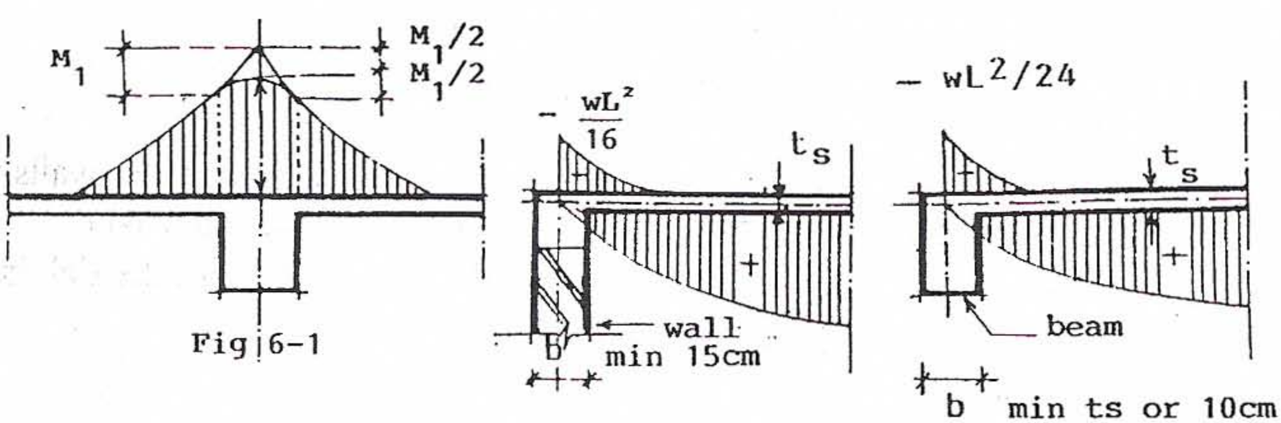


Fig 6-1

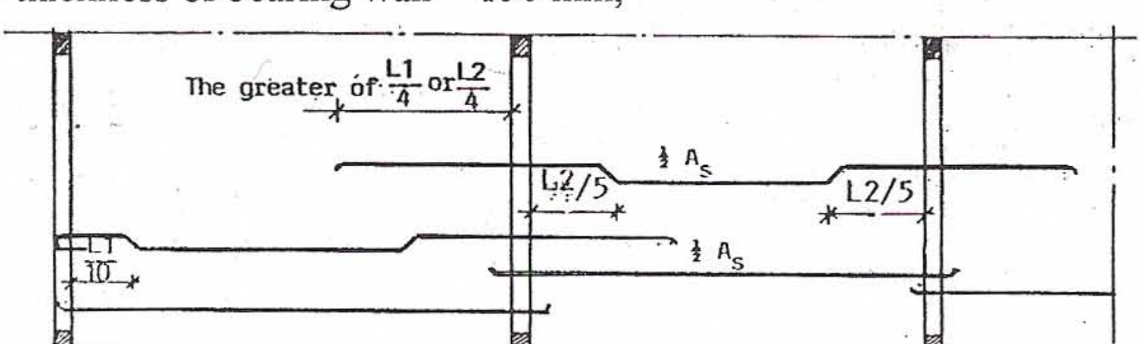
6-2-1-4 Reinforcement

1	Minimum main reinforcement :	
	normal mild steel	$A_s \geq 0.25\% A_{c\,req.}$ but $\nless 0.15\% A_c$ chosen
	high tensile steel	$A_s \geq 0.25\% \frac{f_y(\phi)}{f_y(o)} A_{creq.}$ but $\nless 0.15\% A_c$ req.
2	Rft. must cover all tension zones and extend by the required development length.	
3	For equal spans and equal loads or with variation of max 20%, the details of Rft., figure (6-2), may be adopted.	
4	Maximum spacing between main steel bars should not be more than twice slab thickness and not more than 200 mm. For $t_s \leq 100$ mm max spacing can be taken 200 mm.	
5	Min. (1/3) of bottom main rft. must be extended to supports.	
6	When using st. mesh, the above conditions must be satisfied.	
7	Secondary steel $A'_s \nless 0.2\, A_{s\,main}$ with min. 4 bars / m`	
8	Min. diameter is ϕ 6 mm for straight bars and ϕ 8 mm for bent bars. Bars with smaller diameter may be used for welded wire mesh and for precast concrete.	
9	Upper steel mesh should be added for slabs with thickness more than 160 mm with $A_s \geq$ 20 % of main steel in each direction with lower limit 5 ϕ 8/m` for mild steel and 5 ∅ 6/m` for high tensile steel.	

6-2-1-5 Supports

Breadth for slab support should not be less than slab thickness and not less than 100 m except for precast slabs.

Min. thickness of bearing wall = 150 mm,



SUMMARY: ONE WAY SOLID SLABS

6-2-1-1 Effective span :

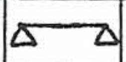
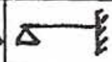
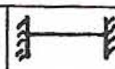
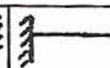
The least of :

Simple slabs	$C_L \rightarrow C_L$ of beams	$L_o + t_s$
Cantilever slab	$C_L \rightarrow$ end	$L_o + t_s$

6-2-1-2 Min. dimensions

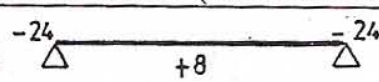
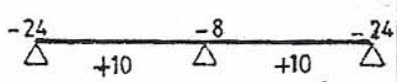
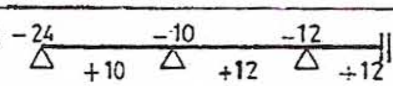
- Min. slab thickness = 80 mm static loads; 120 mm dynamic loads

- and not less than that required for max span/ depth ratios given in table (otherwise check deflection):

max. (span/depth) ratio				
				
High tensile steel	20	24	28	10
Normal mild steel	25	30	35	12

max. deflection	
Cantilever slabs	other slabs
L` / 450	L/250
L` / 600	$\frac{L}{350}$ or 20mm

6-2-1-3 Bending moments

a	Min - ve B.M. over exterior supports	
	1) Walls, masonry or plain conc. supports with partial fixation	$M_{-ve\ min} = -wL^2/16$
	2) R.C. beams monolithically casted with slabs	$M_{-ve\ min} = -wL^2/24$
b	B.M. Equal spans & loads (difference ≤ 20 %)	
	1) Simple span 	$M+ve = wL^2/8$
	2) Two spans 	$M_{+ve} = \frac{wL^2}{10}$, $M_{-ve} = \frac{-wL^2}{8}$
	3) Continuous slabs : 	$M = \pm w L^2/k$
c	In case of heavy L.L. (P> 2g) The min. -ve field moment	$M_{min} = [g-(p/2)] \frac{L^2}{24}$

6-2-1-4 Min. rft.

1- Min main rft should not be less than :

st 240/350	0.25 %	of A_c required	and not less than 0.15 % A_c chosen
st 360/520	0.167 %		
st 400/600	0.15 %		

2) A_s distributers ≮ 20% A_s main with min 4 φ 6/m`

3) Min φ 6 for straight bars & φ 8mm for bent bars

4) Max spacing : $2t_s$ or 200 mm.

5) Min 1/3 main steel must extend to supports.

6-2-2 TWO WAY SOLID SLABS

6-2-2-1 General

1-Rectangular slabs supported on all four sides can be considered as two way slabs if the rectangularity ratio according to sec.(6-2-2-4) is samaller than 2.

2-These slabs can be solved according to the elastic theories on condition that rft. resisting negative moments is placed in its right position during casting.

3-This code of practice is valid only for ordinary buildings with small live load up to 400kg/m2. Slabs of other structures such as bridges, liquid containers, storehouses .... etc., are to be designed according to their corresponding special codes.

6-2-2-2 Spans : Refer to sec. (6-2-1-1)

6-2-2-3 Min. thickness

For simply supported slabs	t_{min} = a/35	6-5-a
For slab continous at one end	t_{min} = a/40	6-5-b
For contin. or fixed slabs	t_{min} = a/45	6-5-c

where a = the smaller effective span of the slab.
Conditions (3&4) according to art. (6-2-1-2) should be satisfied.

6-2-2-4 Simplified method for calculating bending moments in two way slabs

Rectangular slabs monolithically casted with beams and supported on all four sides where effective length is not more than two times the effective breadth, and subjected to uniform loads can be calculated in normal cases according to the following simplified method:
Assume a and b = theoritical spans
m_a = ratio of length between points of inflection for a loaded strip in direction of span "a" , to the span.
m_b = same for span "b"
Values of m and m_1 are to be determined from theory of elasticity.

In continuous slabs, the following approximation in determining the values the of m and m_1 may be adopted:

Slab conditions	Simply supported	Contin. at one end	Contin. at both ends
m_a and m_b	1	0.87	0.76

On these assumptions the rectangularity ratio between lines of inflection (r) can be determined as follows:

$$r = b_{1/} a_1 = (m_b\, b)/(m_a.a) \qquad ..\ (6\text{-}6)$$

Table (6-1) gives the values of α amd β, which are used in calculating the bending moments in two way slabs, in the two directions a and b respectively, according to the corresponding value of r, and these values for live load < 5 KN/m².

Where $\propto = 0.5\, r - 0.15$ & $\beta = 0.35\, /r^2$ (6-7)

If live load > 5 KN/m², load is distributed according to table (6-3).

B.M. value for slabs in both directions can be taken as following:

	M_a	M_b	
Continous from one side	$\pm \propto wa^2/_{10}$	$\pm \beta\, wb^2/_{10}$	6-8-a
Continous from the two sides	$\pm \propto wa^2/_{12}$	$\pm \beta\, wb^2/_{12}$	6-8-b
Simply supported	$\pm \propto wa^2/_8$	$\pm \beta\, wb^2/_8$	6-8-c

6-2-2-5 Reinforcement details:

1-	Max. spacing of main reinforcement in middle spans is twice the total thickness of slab, but not more than 200 mm . However, for slabs smaller than 100 mm, 5 bars per meter may be allowed.
2-	Reinforcement in secondary direction should not be less than 0.25 of the main reinforcement and not less than 5 bars per meter.
3-	Positive reinforcement adjacent and parallel to a continuous edge may be reduced by 25% for a width not exceeding ¼ of the shorter dimension of the panel.
4-	For other constructional datails, refer to one way slabs (art. 6-2-1-4).

6-2-2-6 Load distribution for slabs rested on masonry walls:

In this case loads is distributed according to table (6-2) but live load must be less than 5 KN/m². If live load > 5 KN/m², values in table (6-3) must be used.

Table (6-1) α and β values
for solid slabs casted monolithically with beams & subjected to uniform L.L. < 5 KN/m²

r	1.00	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.00
α	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
β	0.35	0.29	0.25	0.21	0.18	0.16	0.14	0.12	0.11	0.09	0.08

whrer : $\beta = 0.35/r^2$ & $\alpha = 0.5\,r - 0.15$

Table (6-2) (Marcus) : α & β values
and for two way ribbed slabs with complete comp. flange

r	1.00	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.00
α	.396	.473	.543	.606	.660	.706	.746	.778	.806	.830	.849
β	.396	.333	.262	.212	.172	.140	.113	.093	.077	.063	.053

Table (6-3) (Grashoff) Values of α and β
for ribbed slabs with non - complete compression flange (cover slab partially omitted)

r	1.00	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.00
α	.500	.595	.672	.742	.797	.834	.867	.893	.914	.928	.914
β	.500	.405	.328	.258	.203	.166	.133	.107	.086	.072	.059

Where: $\beta = 1/(1 + r^4)$ & $\alpha = r^4/(1 + r^4)$

6-2-3 DESIGN OF SLABS BY THE YIELD LINE THEORY

Slabs may be designed according to the yield line theory which depends on the behaviour of slabs at failure stage.

When using this method in the design, the minimum slab thickness must be checked . It must be regarded that this method doesn't satisfy the conditions of crack width control for slabs exposed to environmental conditions of category three & four according to sec. (4-3-2-4-e). Thus it must not be used in these cases. The ratio of the max. negative moment (M'_u) to the max. positive moment (M_u) must lie between 1 & 1.5.

$$M'_u / M_u = 1.00 \rightarrow 1.50 \qquad .. (6\text{-}9)$$

6-2-4 CONCENTRATED LOADS ON SLABS

Concentrated loads on slabs may be :

a) Concentrated single loads.

b) Concentrated line loads (such as walls)

Slabs subjected to concentrated loads must be calculated according to the theory of elasticity. Also the following method for analyzing such slabs, may be followed :

6-2-4-1 One way slabs :

6-2-4-1-1 Max. breadth for concentrated load distribution

For : t_1 = breadth of load perpendicular to the main steel .

t_2 = breadth of load parallel to the main steel.

c = thickness of cohesive flooring.

t = slab thickness .

s_1 = initial breadth for load distribution perpendiclar to the main steel

s_2 = initial breadth for load distribution parallel to the main steel.

Then :

$$s_1 = t_1 + 2c + t \quad ..(6\text{-}10)$$

$$s_2 = t_2 + 2c + t \quad ..(6\text{-}11)$$

The load breadth will be equal to S_1 at the top of support and increases gradually until it reaches the max. breadth of distribution indicated latter.

The increase in breadth follows lines inclined with an angle α with the direction of main reinforcement as shown in fig (6-3) in which :

$\tan\alpha = 1$ for calculation of B.M.

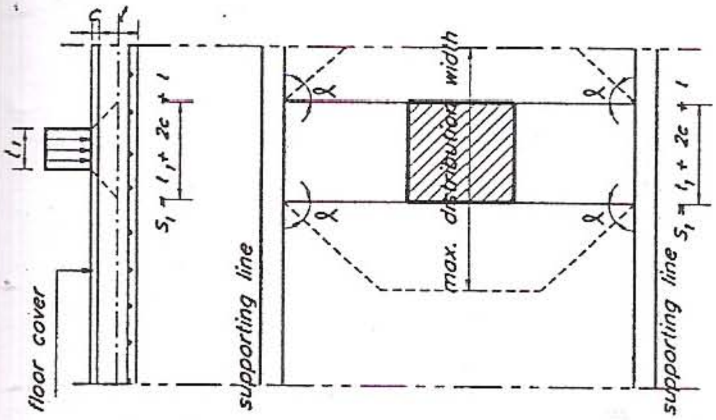
$\tan\alpha = 1/2$ for calculation of S.F.

Therefore, the maximum breadth of distribution in the perpendicular direction to the main steel for calculating the bending moments :

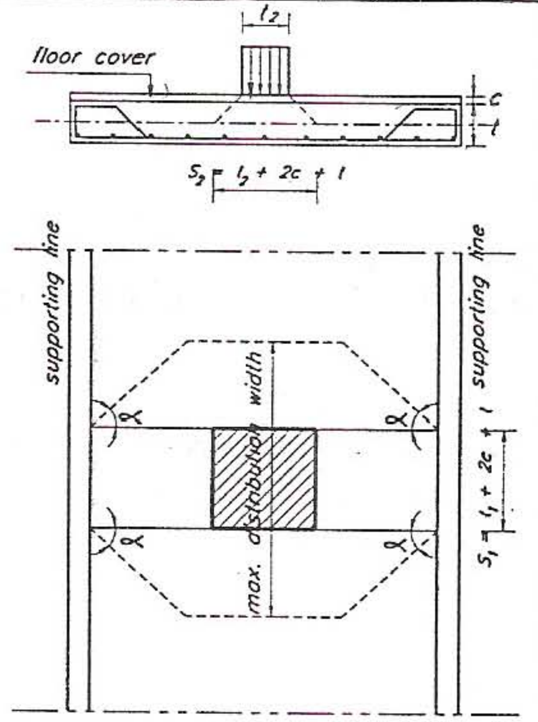
$$= s_1 + \frac{A_s(\text{sec})}{A_s(\text{main})} \times L \quad ..(6\text{-}12)$$

The max. ratio of $A_s(\text{sec}) / A_s(\text{main})$ to be taken in the equation is 2/3.

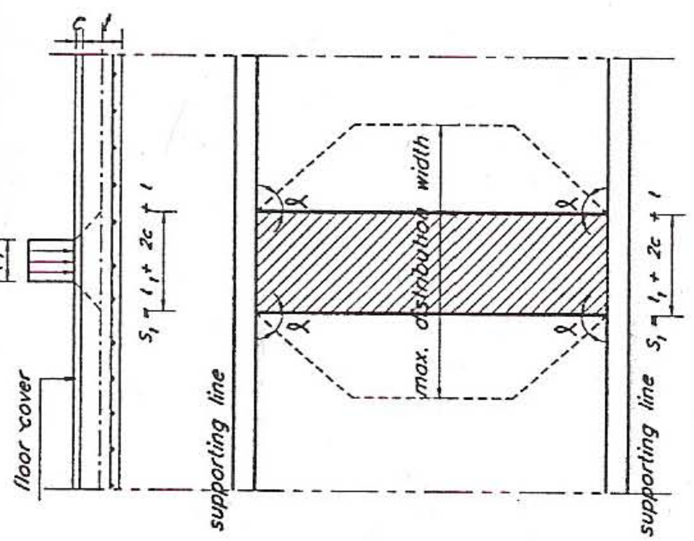
The max. breadth is not to exceed s_1 + 2.00 meters or the length of slab in the perpendicular direction to the main reinforcement.



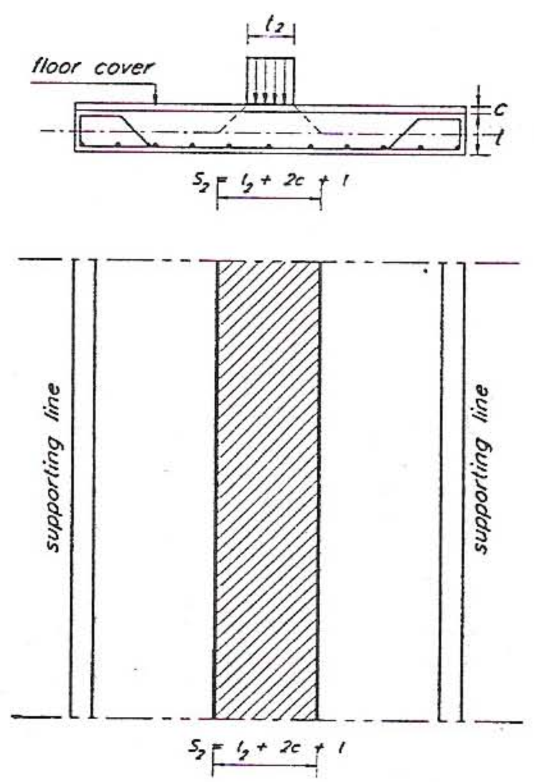
(6-3-b) Concentrated load near the edge of slab



(6-3-a) Concentrated load at the middle of slab



(6-3-d) Line load perpindicular to the supporting axis of slab



(6-3-c) Line load parolell to the supporting axis of slab

Fig. (6-3) : Distribution of concentrated and line loads on one way slabs

where L = effective span for simply supported slabs
& L = distance between inflection lines in continuous slabs.

If the concentrated load is near to the unsupported side of the slab or near to the beams of shorter sides, the effective length is taken for distribution and perpendicular to the main rft equal to half the mentioned values plus the distance between the center of load & the unsupported side or the edge of the beam on the short direction of the slab.

The maximum breadth of distribution for calculation of shearing forces in the perpendicular direction to the main reinforcement is given by

$$= s_1 + \frac{A'_s\,(sec)}{A_s\,(main)} \times L$$

That value is not to exceed $s_1 + L/3$ or $s_1 + 1.00$ meter or length of slab in direction perpendicular to the main rft.

When the concentrated load is near to the beam at the short direction of the slab, then max. distribution length for calculating S.F. is $s_1 + 4\,t$.

6-2-4-1-2 Bending moments and design:

a) In calculating the additional bending moment due to the concentrated load, it is taken into consideration that the concentrated load is distributed on a length (s_2) of the effective span of the slab. The breadth affected by that load in the perpendicular direction and which is considered in the design of the slab is the maximum length of distribution in the perpendicular direction on the main reinforcement as mentioned before.

b) The bending moment considered for design inside the maximum breadth of distribution = the sum of bending moments due to dead and live loads plus the additional bending moment due to the concentrated load.

c) The main reinforcement is calculated and placed according to what have been mentioned. The additional secondary rft. due to concentrated load to be extended a length at least, equal to the breadth of distribution considered.

6-2-4-2 Concentrated loads on two-way solid slabs

If a_1 and b_1 are the short and long suspended spans respectively and $b_1/a_1 \leq 1.5$, it is allowed to use the following load distribution in two directions. For values of b_1/a_1 exceeding the given value, the slab may be considered one way.

Concentrated load distribution in 2 directions:

Load in dir ection a_1 ,	$P_{a_1} = P \dfrac{b_1}{a_1 + b_1}$	.. (6-13-a)
Load in direction b,	$P_{b_1} = P \dfrac{a_1}{a_1 + b_1}$	. . (6-13-b)
Max. breadth of distribution in direction of $a_1 = s_2 + 4\ a_1$		..(6-14)
Max. breadth of distribution in direction of $b_1 = S_1 + 0.4\ a_1\ (2 - \dfrac{a_1}{b_1})$		..(6-15)

Calculation of bending moment due to the concentrated load in 2 directions:

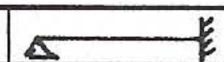
In direction a_1: P_{a1} is considered distributed on a length ($= S_2 + 0.4\ a_1$) of the effective span (a) , and a breadth $= S_1 + 0.4\ a_1\ (2- a_1/b_1)$ in the direction perpendicular to direction (a_1) . That breadth is the breadth considered in the design of the slab.
In direction b_1 : p_{b1} is considered distributed on a length $=S_1 + 0.4\ a_1(2- a_1/b_1)$ of the effective span (b) and a breadth $= S_2 + 0.4\ a_1$ in the direction $\perp$ to direction (b_1).

These additional bending moments have to be added to the bending moments due to dead and live loads. The total reinforcement in each direction is to be calculated and placed in the lengths affected by the concentrated load.

Min. thickness:
where a : short span, b : long span

simply sup. slabs	$t_{min} = a/35$
continuous slabs	$t_{min} = a/45$

Ratios of rectangularity:
$r = m_b\, b / m_a . \alpha$

			
m_a and m_b	1.0	0.87	0.76

	M_a	M_b	
Continous from one side	$\pm \alpha\, wa^2/_{10}$	$\pm \beta\, wb^2/_{10}$	6-8-a
Continous from the two sides	$\pm \alpha\, wa^2/_{12}$	$\pm \beta\, wb^2/_{12}$	6-8-b
Simply supported	$\pm \alpha\, wa^2/_{8}$	$\pm \beta\, wb^2/_{8}$	6-8-c

Table (6-1) α and β values for solid Slabs casted monolithically with beams ($\beta = 0.35/r^2$) (L.L < 5 KN/m^2)

r	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.00
α	.35	.40	.45	.50	.55	.60	.65	.70	.75	.80	.85
β	.35	.29	.25	.21	.18	.16	.14	.12	.11	.09	.08

Table (6-2) (Marcus) : α & β values for slabs resting on masonry walls and for two way ribbed slabs with complete comp. flange

r	1.00	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	00
α	.396	.473	.543	.606	.660	.706	.746	.778	.806	.830	49
β	.396	.333	.262	.212	.172	.140	.113	.093	.077	.063	53

Table (6-3) (Grashoff) Values of α and β for ribbed slabs with non - complete compression flange (cover slab partially omitted)

r	1.00	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.00
α	.500	.595	.672	.742	.797	.834	.867	.893	.914	.928	.914
β	.500	.405	.328	.258	.203	.166	.133	.107	.086	.072	.059

6-2-5 RIBBED AND HOLLOW BLOCK SLABS

6-2-5-1 General

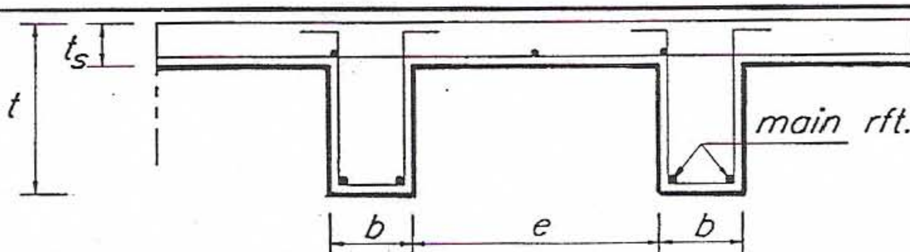
* In designing hollow - block slabs, the hollow blocks are considered statically not acting.
* The following conditions regarding dimensions should be complied with (figure 6-4) :

1	Max. clear spacing between ribs : e_{max} = 700 mm.
2	Min width of ribs : b = 100 mm . or 1/3 the depth (the bigger).
3	Min thickness of acting compression slab:50mm or e/10(the bigger)
4	The slab should be able to carry concentrated loads that may act between ribs.

6-2-5-2 One way ribbed slabs (Ribs in one direction)

conditions :

1	Distributing bars normal to ribs should not be less than the values given in art. (6-3-1-10). Bars parallel to ribs are min. 3 ϕ 6 mm/m` and min. one bar ϕ 6mm is to be placed between each 2 ribs.
2	For L.L. $\leq$ 3 KN/m^2 and spans over 5.0 m , at least one cross rib is to be provided in the middle of the span, of a cross section and bottom reinforcement not less than those of the main ribs, and with a top reinforcement of at least 1/2 the bottom reinforcement .
3	For L.L. > 3 KN/m^2, and span ranging from 4.0 to 7.0 m, one cross rib is to be provided as above . For spans above 7 m, three cross ribs are to be provided, with the same dimensions and reinforcement as above.



e = max. 70 cm

b = bigger value of 5 cm and $t/3$

t_s = bigger value of 10 cm and $e/10$

Figure (6-4) Cross section and dimensions of ribbed slab

6-2-5-3 Two-way ribbed slabs (Ribs in two directions)

There are two cases:

i) *Beams having the same depth of the slab* (embedded beams), can be designed as flat slabs (or using the method given for case (b)).

ii) *Beams having depth greater than slab depth:*
These slabs are of two types:

1- Slabs with ribs having a complete compression flange :
a) If L.L. ⩽ 5 KN/m^2: loads are distributed in both directions using the factors given in tabe (6-2). The rft. in the compression slab must not be less than 3 ϕ 6 mm/m` in each direction.
b) If L.L. > 5 KN/m^2: loads are distributed by using values in tabe (6-3).

2- Slabs with ribs having non complete compression flange :
i.e. the ribs have the shape of T sec. with a limited width of the compression flange or without. In this type the loads are distributed in both directions using factors given in table (6-3) (Grashoff).

6-2-5-4 General remaks

The following notes are applied to both one - way and two-way ribbed slabs

1	Min . distribution bars in the compression slab = 3 ϕ 6 mm/m`
2	Shearing forces in ribs are to be treated according to article (6-3-1-7) and (6-2-7-7) when design two way ribbed slab as flat slab.
3	Solid concrete part is to be provided at internal supports for continuous slabs, to resist negative bending moments and shearing forces.
4	For effective spans and bending moments in continuous slabs refer to article (6-2-1-1) and (6-2-1-3).
5	The breadth of the support over brick wall must not be less than 15 mm.
6	In case of simple spans, hollow-blocks should not be extended over supports to provide solid parts to resist shearing forces.

6-2-6 Panelled Beams:-

a) when the total dimensions for two way slabs is relativily big so that it is not suitable to design it as solid slab or ribbed slab or hollow block slab, a statical system consists of panelled beams forming gid sapporting a group of solid slabs (or hollow blocks) with relativily small dimension can be used for such case.

b) Panelled beams are regrally arranged in two perpenduclar directions forming rectanggular grid (or square), also it can be arranged in diagonals direction to form skew grid, or in three directions to form triangular grid, or in four directions to form quadruple grid.

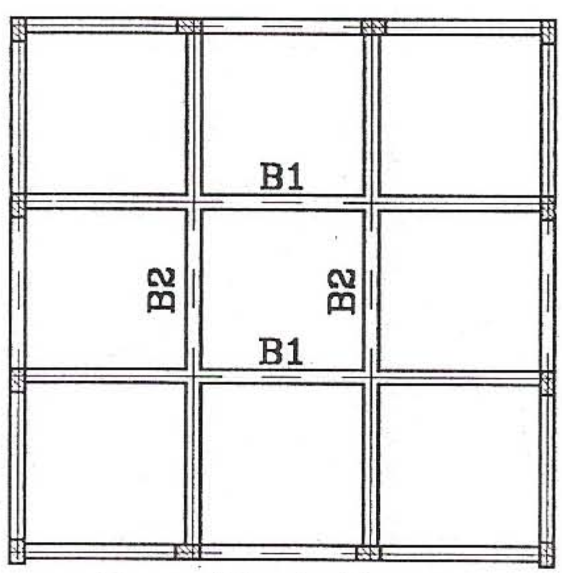
c) This system is structurally suitable when sections of panelled beams are equal also when rectangularity ratio for slabs total dimensions equals 1.00 → 1.50.

d) Internal forces calculation and design of slabs between panelled beams is according to art. (6-2-2) and art. (6-2-5).

e) Internal forces in panelled beams are calculated by using theory of elasticity which includes satisfaction of stability and strain compatability conditions.

One of the simplified method can be used with condition that this solution must be compatable with the actual behavior of panelled beams statical system.

f) Article (6-3) must be satisfied.



Panelled Beams

6-2-7- FLAT SLABS (BEAMLESS SLABS)

6-2-7-1 General

It is generally meant by the term flat slab, reinforced concrete beamless solid slabs with or without drops, directly supported by columns with or without flared column heads, (figure 6-5). These slabs may be with or without drop panel.

Flat slabs include solid slabs, two way ribbed and hollow slabs with or without blocks.

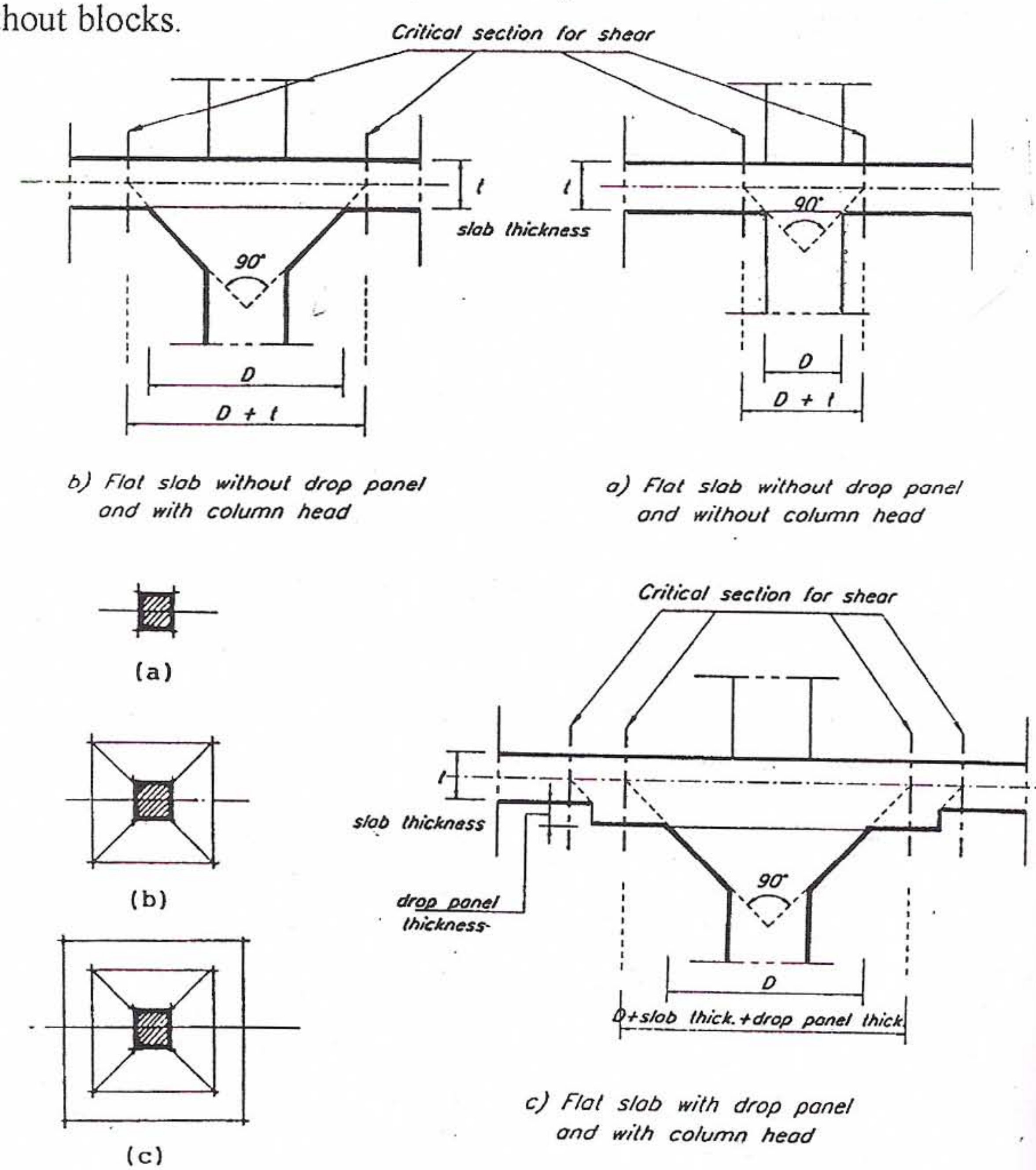
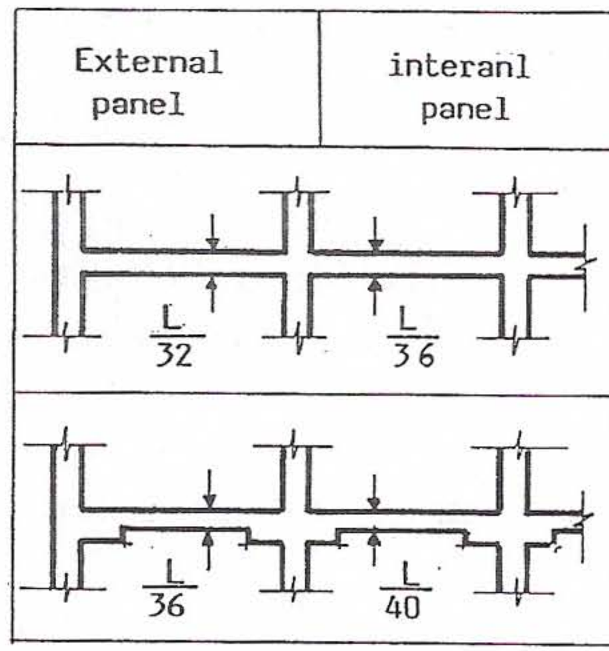


Figure (6-5) Types of flat slabs & Critical sections for shear

Notations :

L_1 : Length of the panel in the direction of the span.
L_2 : Width of the panel in the perpendicular direction.
L_1 & L_2 are measured from the centers of the columns .
D : Diameter of the column or column head, or the largest circle drawn inside its sec.
w : Total load per unit area of the panal = working load when designing by W.S.D method or the ultimate load when designing by limit state design method.
t : Total thickness of slab.
d : Effective depth.



6-2-7-2 Minimum dimensions

a) Minimum slab thickness :

Min flat slab thickness is 15 cm and should not be less than :

	External panel	internal panel
Without drop	L/32	L/36
With drop	L/36	L/40

where $L = (L_1 + L_2)/2$

b) Minimum column dimensions :

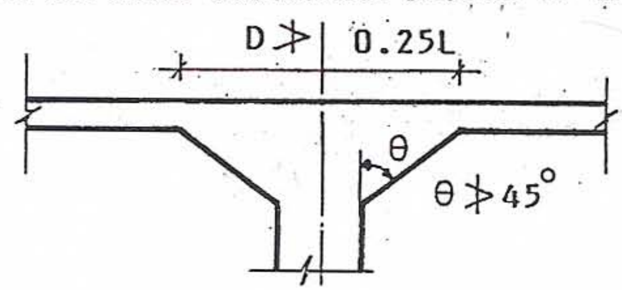
The diameter of a column (if circular), or the side length of a rectangular column should in no case be less than the greatest of the following values:

* D_{min} = 30 cms	30 cms
* 1/20 of the panel length in the same direction	L/20
* 1/15 of the height of the floor (H)	H/15

c) Dimensions of column head :

Where column heads are provided, the heads of interior columns, and such portions of the heads of exterior columns which lie within the building, should satisfy the following requirements :

* The angle of the greatest slope of the head should not exceed 45° from the vertical.



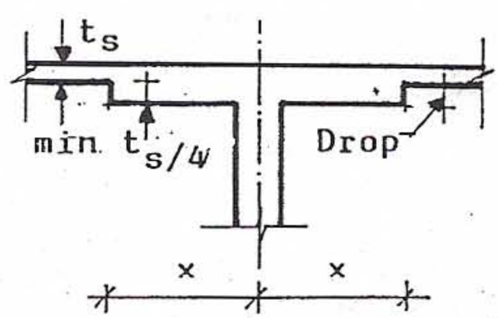
* The effective diameter D to be included in the design should not be more than 0.25 L.

Where the column and column head are not of circular cross section the term diameter used in this article should be deemed to mean the diameter of the largest circle which can be drawn within the section.

d) Drop panels determination :

In cases when it is required to increase slab thickness over column head to resist negative bending moments and to reduce steel reinforcement, drop panels may be used with the following conditions :

1- Thickness of the drop below slab should be at least = 1/4 the slab thickness

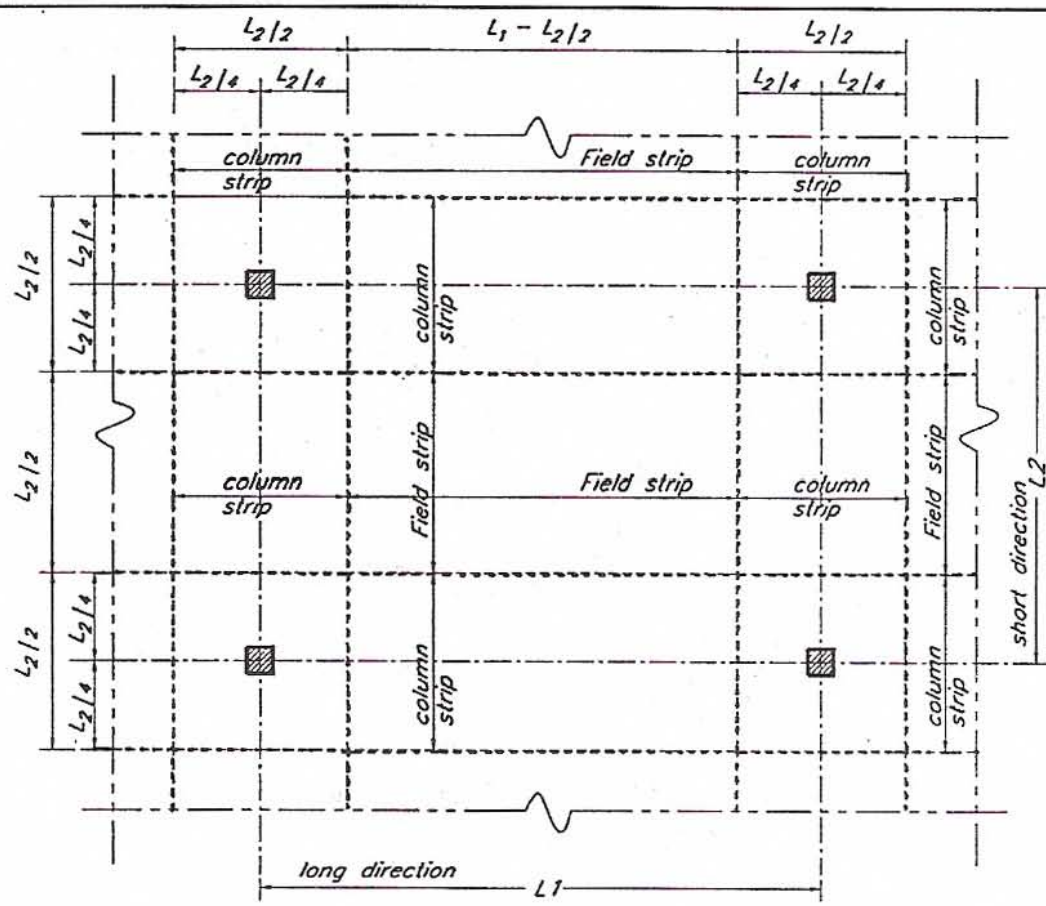


2- Drop panel should extend at least $\frac{1}{6}$ of the span in the direction considered, but not more than 1/4 of the shorter span. $\frac{L_1}{6} \leq x \leq \frac{L_{min}}{4}$

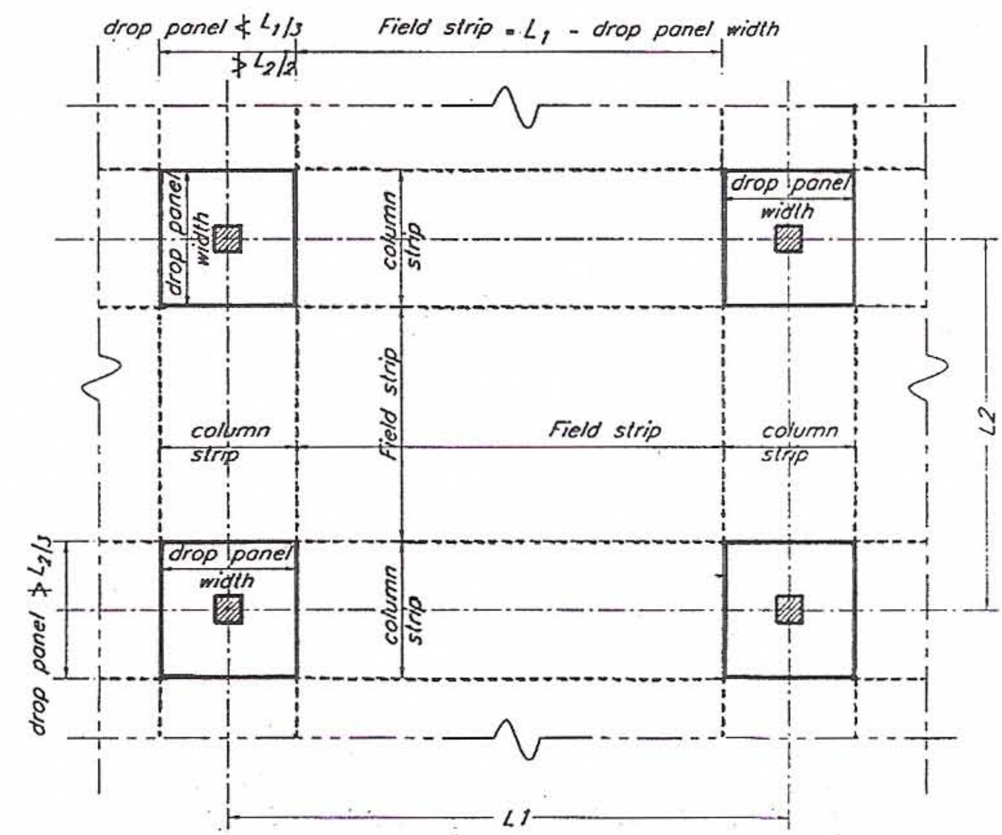
e)Panels ar assumed to be divided into the following strips (figure 6-6)

1-Column strip with width = half shorter span ($L_{min}/2$) except in case of drop panel where the column strip width = drop panel width.

2-Field strip with width = difference between span and column strip width (or drop panel width).



(6-6-a) Flat slab without drop panel



(6-6-b) Flat slab with drop panel

FIG.(6-6): DEVISION OF FLAT SLAB SPANS TO COLUMN STRIP & FIELD STRIP

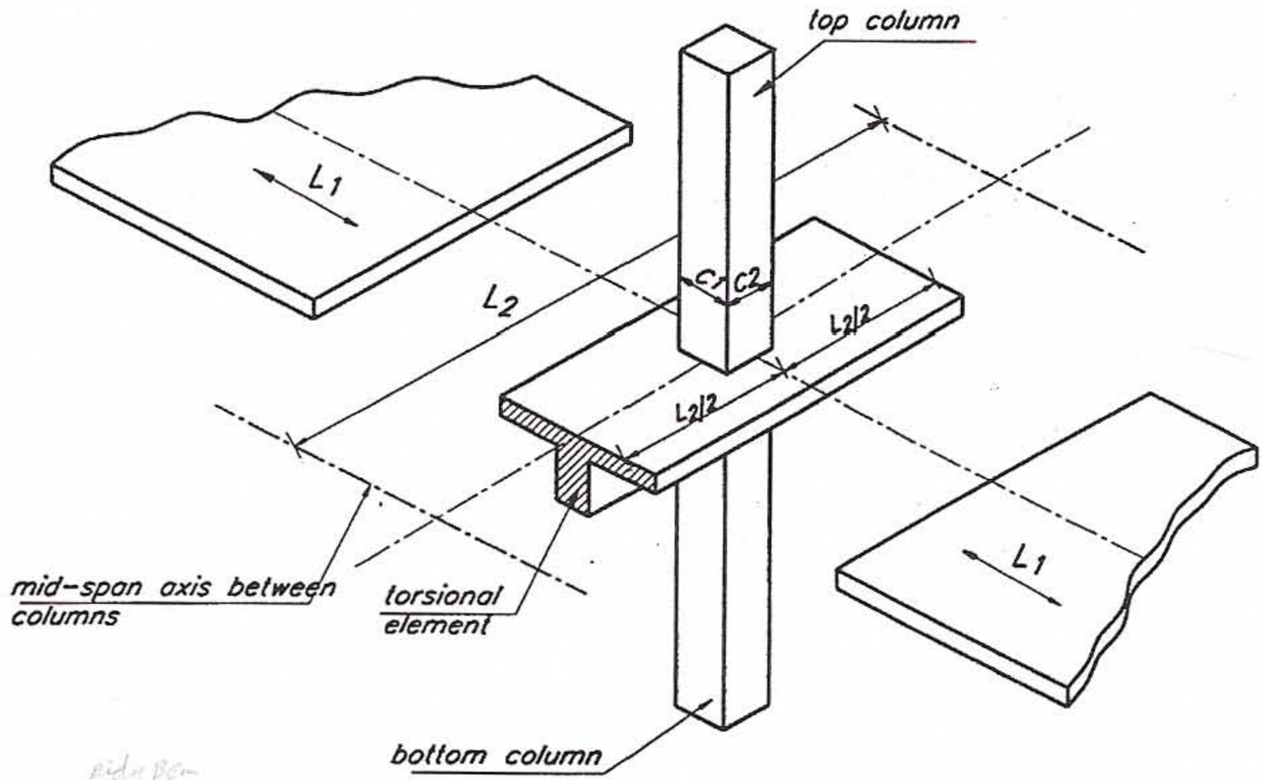


Figure (6-7): Equivalent column (columns and torsional elements)

Saw Tooth Slab $t_s \leq 16$ cm, $l_{max} = 10$ m

- $t_s = \frac{L}{35}$, - Dis. bet. post = 2-5 m
- $\theta = 20^\circ - 30^\circ$

Design :- 1- Design slab

2. Design of Ridg Beam

w = o.w of Beam + R_1

3- Design of post

$P = F_{co} A_c + 0.44 f_y A_{sc}$

4- Design of H2 Beam :-

w = H2

5. Design of Tie :-

$A_s = \frac{N}{F_s}$

6- Design of VL Beam :-

w = o.w of VL Beam + o.w of H2 + V2 = t/m

7- Design of Y Beam

Reaction of Ridg Beam

w = o.w + V_1

8- Design of Colm

$N_1 = A \times \gamma \times h + R$ From VL Beam

$N_2 = A \times \gamma \times h + R$ From Y Beam

→ Effect of wind :-

$w_2 = 0.8 \times 100 \times 5.5 = 0.44$ t/m

by Force method

Design as Colm (M, N)

6-2-7-3 Structural analysis

a) Flat slabs can be analyzed according to theory of elasticity. Yield line method can also be applied on the condition that the ratio between the-ve and +ve and moments according to article (6-2-3) is satisfied . It is to be noticed that the last method does not satisfy the crack width control on the tension side for slabs subjected to environment of the third or fourth categories according to article (4-3-2-4-e), so it must not be used in these cases.

b) Flat slabs which have columns on straight lines with difference not exceeding 10% of panel length and perpendicular to the direction can be designed according to one of the following two methods:

1- As *continuous frames* as given in article (6-2-7-4).
2- Using *empirical method* explained in article (6-2-7-5).

6-2-7-4 FRAME ANALYSIS

Design of flat slabs as continuous frames

Flat slabs if not designed exactly according to theory of elasticity may be designed as follows :

a) The bending moments and shearing forces may be determined by analyzing the structure as a continuous frame and the following assumptions may be followed :

* The structure is considered to be divided longitudinally and transversely into frames consisting of a row of columns and strips of slab with a width equal to the distance between the center lines of panels on each side of the row of columns.

* Each frame may be analyzed as a separate frame with columnsabove and below assumed to be fixed at their extremities under full dead and live load in each direction, and in position giving max. internal forces. The spans used in the analysis should be equal to the distances between the centers of supports and difference of moment of inertia should be taken into consideration.

During calculating the flexure stiffness of columns one of the following methods can be used:

a-1) the summtion effect of each flexure stiffness of column and torsion stiffness of torsional elements which are connected with the column such as beams and effective torsional parts of the slab in direction ⊥ axis of the

frame considering that torsional element in flat slab equal to (c_1+3t_s) according to articale (4-2-3-2) and fig. (4-11-b) should be taken into consideration.

The equivalent flexure stiffness of column is calculated according to the following relation (fig. 6-7).

$$K_{ec} = \Sigma\, k_c\,[1+(\Sigma\, k_c / k_t)] \qquad (6\text{-}16\text{-}a)$$

where

$\Sigma\, k_c$ = Summtion of stiffnesses of the column above and below level of the slab considering fixation of the column at its upper and lower ends

$$k_c = 4\, E_c\, I_g / h^3 \qquad (6\text{-}16\text{-}b)$$

h = Column height.

Ig = Moment of inertia outside the connection for the whole comcrete section of column neglecting cracks and steel reinforcement.

Ec = Concrete modululus of elasticity according to art. (2-3-3-1).

In case of slabs with drop panel or column head or non-prismatic columns, it is preferable to calculate columns stiffness (K_c) considering its actual stiffness distribution.

k_t = Stiffness of torsional elements and it can be calculated from the following relation:

$$k_t = \Sigma[9.E_c.C / (L_2\,(1-\frac{c_2}{L_2})^3)] \qquad (6\text{-}16\text{-}c)$$

Where: c_2 and L_2 are column dimension and panel length (respectivily) in the direction perpendicular to analysis direction according to fig. (6-7), and C is section constant and it can be calculated from the following relation:

$$C = \Sigma\,[\,1-0.63\, b/t\,]\,(b^3\, t/3\,) \qquad (6\text{-}16\text{-}d)$$

Where: b is the short dimension and t is the other dimension of torsional element, and value of (C) can be calculated for T & L- section by dividing the section into rectangles.

a-2) Equivalent moment of inertia I_{ec} can be calculated according to the following relation :

	$I_{ec} = \psi\, I_g$	6-17-a
For extérior columns	$\psi = [0.6 + 0.4\,(\alpha\, L_{2a} / L_{1a})]\,(L_{2a} / L_{1a})^2$	6-17-b
For interior columns	$\psi = [0.3 + 0.7\,(\alpha\, L_{2a} / L_{1a})]\,(L_{2a} / L_{1a})^2$	6-17-c

With condition that $1.0 \geq \psi \geq 0.3$, and the ratio $(\alpha\, L_{2a} / L_{1a}) \leq 1.0$

Where: α = the ratio between moment of inertia of torsional beam and mement of inertia of slab strip.

L_{1a} = average length of the two spans at column sides in analysis direction.

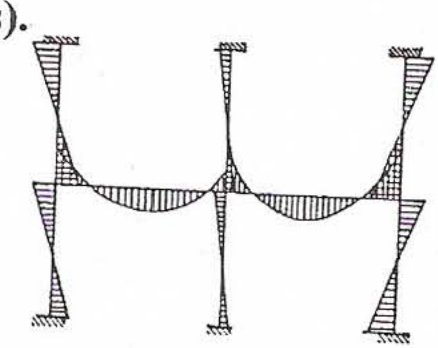
L_{2a} = average length of the two spans at column sides in the direction perpendicular to analysis direction.

b) The slab should be designed for the bending moments calculated at any section, and the negative moments should not exceed values of negative moment at column face.

The calculated bending moments should be divided between the column and the field strips in the proportion given in table (6-4).

Table 6-4 Distribution of B.M. between col. strips and field strips (Flat slab designed as cont. frames).

Type of moment		Column strip	Field Strip
- ve moment	Internal span	75 %	25 %
	External span	80 %	20 %
+ ve moment		55 %	45 %

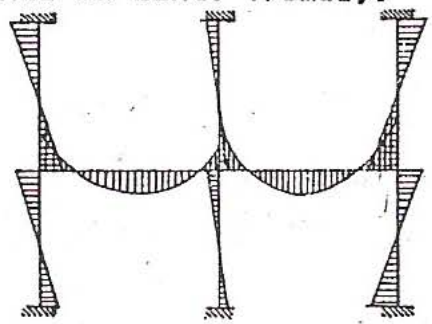


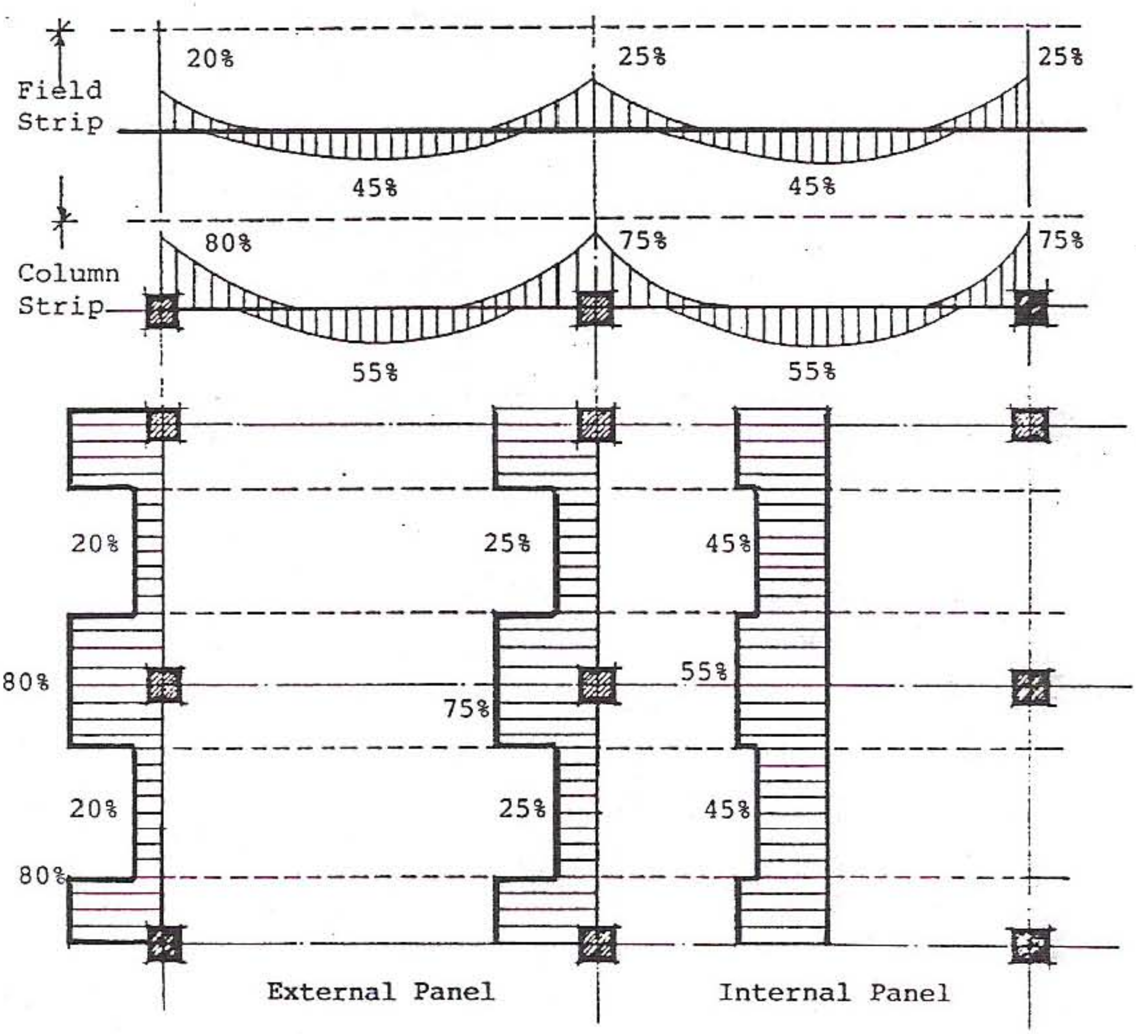
c)When the col. strip is taken equal to the width of the drop, the width of field strip is increased to a value greater than 1/2 of the breadth of the panel. The B.M. in the field strip (tables 6-4 & 6-5) should be increased by the same ratio and the B.M. in the col. strip is to be decreased. Anyhow the sum of both B.M. in column and field strip should not be less than the sum of B.M. (+ve & -ve) which is resisted by both the column and field strip together.

d)The effective width is taken equal to the total width of the slab (distance between axis of columns) for calcolating the rigidity of the slab under vertical loads while under transverse loads it is taken equal to the width of column plus three times the slab thickness from each side of column under the condition that it should not be greater than 1/3 distance between axis of columns.

Table 6-4 Distribution of B.M. between col. and field strips (Flat slab designed as cont. frames).

Type of moment		Column strip	Field strip
-ve moment	Internal span	75%	25%
	External span	80%	20%
+ve moment		55%	45%





6-2-7-5 EMPIRICAL METHOD

For design of flat slabs under uniformly distributed loads

a) This method can be applied only when the following conditions are satisfied :

1-The slabs should comprise a series of rectangular panels of approximately constant thickness arranged in at least three rows in two directions at right angles, and the ratio of the length of a panel to its width should not exceed 4 : 3 (i.e.min. 3 spans ; $\frac{L_1}{L_2} \ngtr 4:3$)

2-The length or width of any two adjacent panels in a series should not differ by more than 10 % of the greater length or width, and any two spans should not differ by more than 20 %. End spans may be shorter, but not longer than interior spans . Where adjacent spans differ, the length should always be taken as that of the longer span in calculating the bending moments.

3-The drops, if any, must have a length in each direction of not less than one-third of the panel length in the same direction. For exterior panles the width of the drop at right angles to the edge, and measured from the center line of the column, should be equal to 1/2 the width of the drop in interior panles. Depth of drop should not be more than 1/2 and not less than 1/4 the slab thickness.

b) Critical sections for B.M. in flat slabs:

For interior panels, fully continuous, critical sections are as follows:

* For positive moments: along center lines of the panels .

* For negative moments: along the edges of the panel on the line joining the center of columns and around the perimeter of the column heads.

c) Bending moment in flat slabs :

Moments in each direction of the panels are calculated according to the following equation :

$$M_o = \frac{wL_2}{8}[L_1 - \frac{2D}{3}]^2 \qquad ..(6\text{-}18)$$

where L_1 = Length of panel in the direction considered

L_2 = Length of panel in the perpendicular direction.

This moment should be divided between the column and field strips as shown in table (6-5) .

d) In case of heavy L.L. the -ve B.M. at the mid spans of interior panels should not be less than :

Column strip	$M - ve = (g - \frac{2}{3}p)\frac{L_2}{40}[L_1 - \frac{2}{3}D]^2$	(6-19-a)
Field strip	$M - ve = (g - \frac{2}{3}p)\frac{L_2}{100}[L_1 - \frac{2}{3}D]^2$	(6-19-b)

where g and p : the distributed dead and live load .

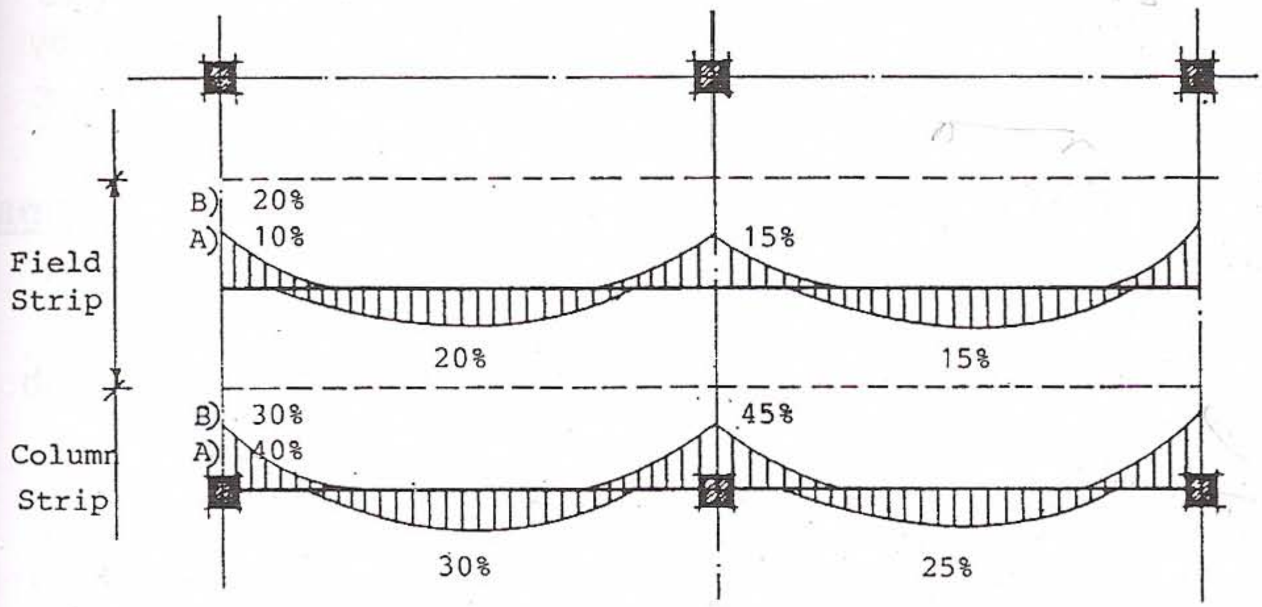
Table 6-5 Distribution of B.M. in panels of flat slabs in percentage of M_o

Strip	End support type	Exterior panel		Interior panel	
		- ve M	+ ve M	- ve M	+ ve M
Column strip	A	40	30	45	25
	B	30			
field strip	A	10	20	15	15
	B	20			

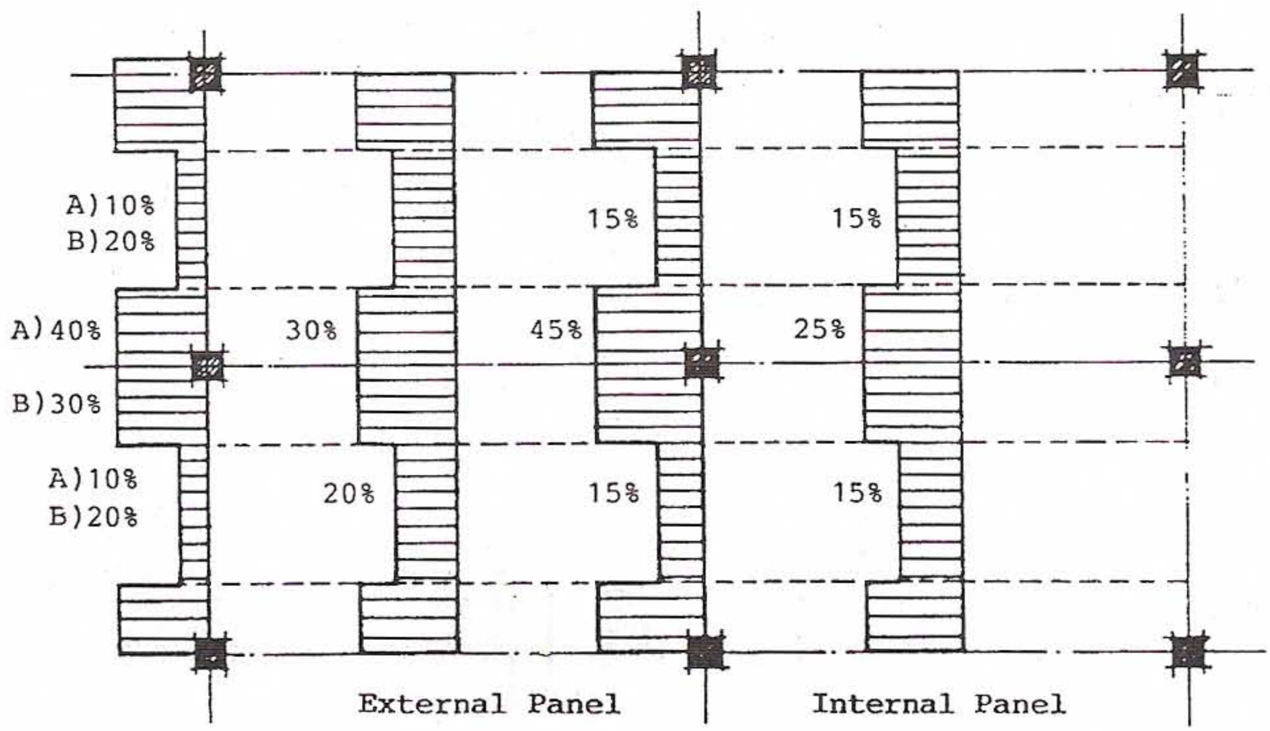
A : no beams . *B : beams with total depth equal to or more than 3 times the slab thickness.*

e) Bending moments on columns for flat slab

1-The internal and external columns should be designed to resist bending moments equal to 50 % & 90 % of the -ve beding moments in column strip respectively, according to table (5-6) . These moments are divided between the upper and lower columns according to their stiffness.



A : no beams. , B : beams with total depth equal to or more than 3 times the slab thickness.



Distribution of BM. (+ ve & - ve) between column & field Strip - Empirical method.

In this case, the direct load on the internal column, can be reduced considering that only one side of the panel is loaded with live loads.

2)If the external columns carry parts of the slabs and walls as cantilevers, the bending moment in these columns, can be reduced by the value of the bending moment due to the dead load of the cantilever.

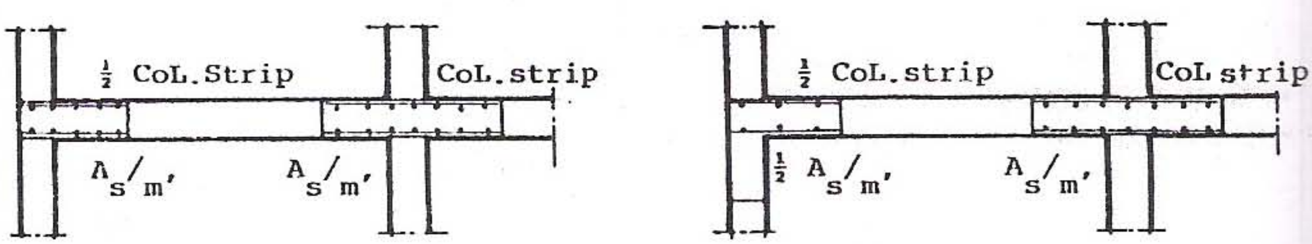
6-2-7-6 Bending moment in panels with & without marginal beams

a) For Marginal beam of total depth equals to or more than 3 times slab thickness, then :

1)The total load to be carried by the beam should be equal to the direct loads on the beam plus a uniformly distributed load equal 1/4 the total load on the panel.

2)The bending moment in the half column strip adjacent to the beam will be $\frac{1}{4}$ the values given in tables (6-4 & 6-5) .

ii)Case of no marginal beam : The B.M. in the half edge column strip will be half the values given in tables (6-4 & 6-5) .



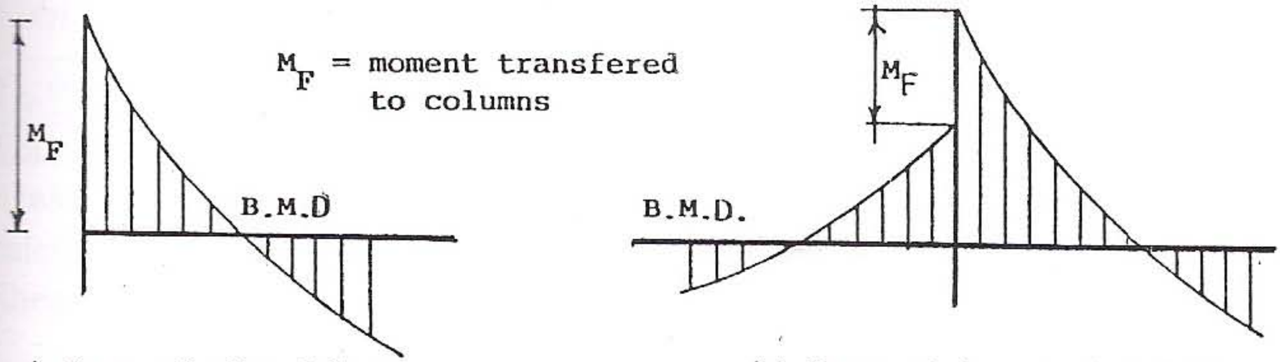
a) Without marginal beam

b) With marginal beam

Reinforcement of ½ column strip

Figure : Reinforcement of half column strip

6-2-7-7 Transfer of - ve moments from flat slab to columns



a) Case of edge Column

b) Case of internal Column.

M_F

By flexure γ_f *By torsion* $\gamma_q = 1 - \gamma_f$

Figure (6-8) transfer of moment from column strip to columns

6-2-7-7-1 The unbalanced -ve moments of the column strips are transferred to columns by flexural and torsional moments as follows :In external spans , all the -ve moments are transferred to columns (Fig. 6-8-a), while in internal spans the difference of -ve moments are transferred to columns (Fig. 6-8-b) according to the following distribution :

a) Part transferred directly to columns by bending moments according to the equation :

$$\gamma_f = \frac{1}{[1 + (2/3)\sqrt{(c_1 + d)/(c_2 + d)}]} \qquad ..(6\text{-}20)$$

where :

γ_f = Coefficient of moments transferred by flexure.

c_1 = dimension of rec. column head or the equivalent rec. to column head measured in the direction of B.M.

c_2 = dimension of rec. column head or the equivalent rectangle to column head in the ⊥ direction.

d = effective depth of the slab.

The required reinforcement to resist these moments is to be concentrated in the effective width b_e as shown in figure (6-9).

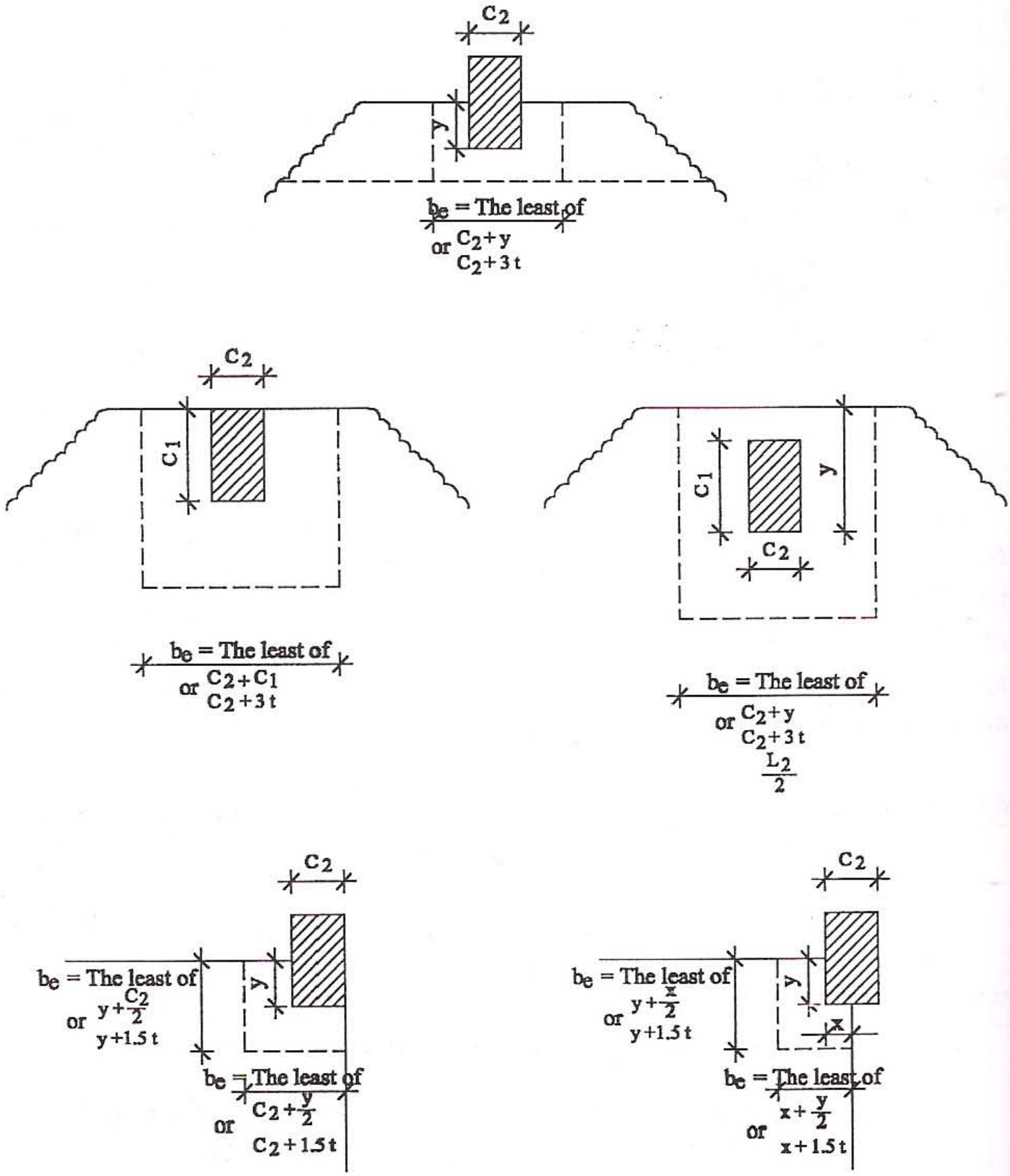


Figure (6-9): Strip width (be) which transfer bending moment

b) Part transferred to columns by torsional moment according to the equation :

$$\gamma_q = 1 - \gamma_f \quad ..(6\text{-}21)$$

γ_q = coefficient of moments transferred by torsion.

This transfer of torsional moments produces additional punching shear stresses on the critical sections shown in figure (6-12). These stresses can be calculated in both directions according to the following equations :

Shear stresses due to M_x: $q_x = M_x \cdot \gamma_{qx} \cdot C_{CB} / j_{cx}$ (6-22-a)

Shear stresses due to M_y: $q_y = M_y \cdot \gamma_{qy} \cdot C_{AB} / j_{cy}$ (6-22-b)

These stresses are added to punching shear stresses resulting from V1 loads according to equation (4-31) in art. (4-2-2-3) in case of designing by limit state method, or according to art. (5-4-3) in cases of designing by working stress method.

$$q_{total} = \frac{Q}{d \sum b_o} + q_x \; (\text{or } q_y)$$

Where j_{cx} and j_{cy} : polar moments of inertia x & y axis respectively. Figure (6-10 & 6-11) shows the shear stresses resulting from moment M_y where the value of j_{cy} are as follows:

a) Case of interior columns [fig. (6-10)]

$$j_{cy} = \frac{d(c_1+d)^3}{6} + \frac{(c_1+d)d^3}{6} + \frac{d(c_2+d)(c_1+d)^2}{2} \quad ..(6\text{-}23)$$

b) case of edge columns

$$j_{cy} = d\,(c_2+d)\,c_{AB}^2 + \frac{2}{3}\,d\,c_{cD}^3 + \frac{2}{3}\,d\,c_{AB}^3 + \frac{(c_1+0.5d)d^3}{6} \quad ..(6\text{-}24\text{-}a)$$

where:

$$c_{AB} = \frac{(c_1+0.5d)^2}{(c_2+d)+2(c_1+0.5d)} \quad ..(6\text{-}24\text{-}b)$$

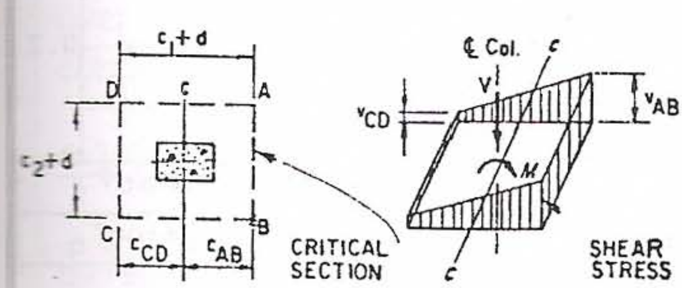


Figure (6-10) Internal column

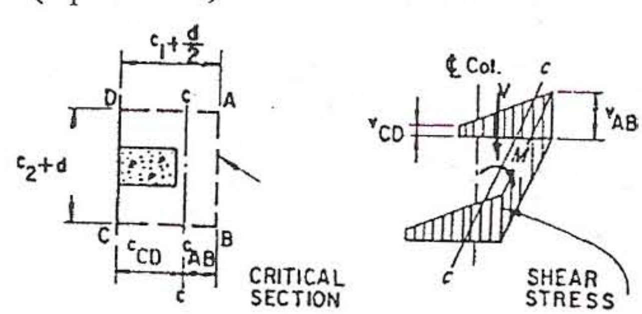
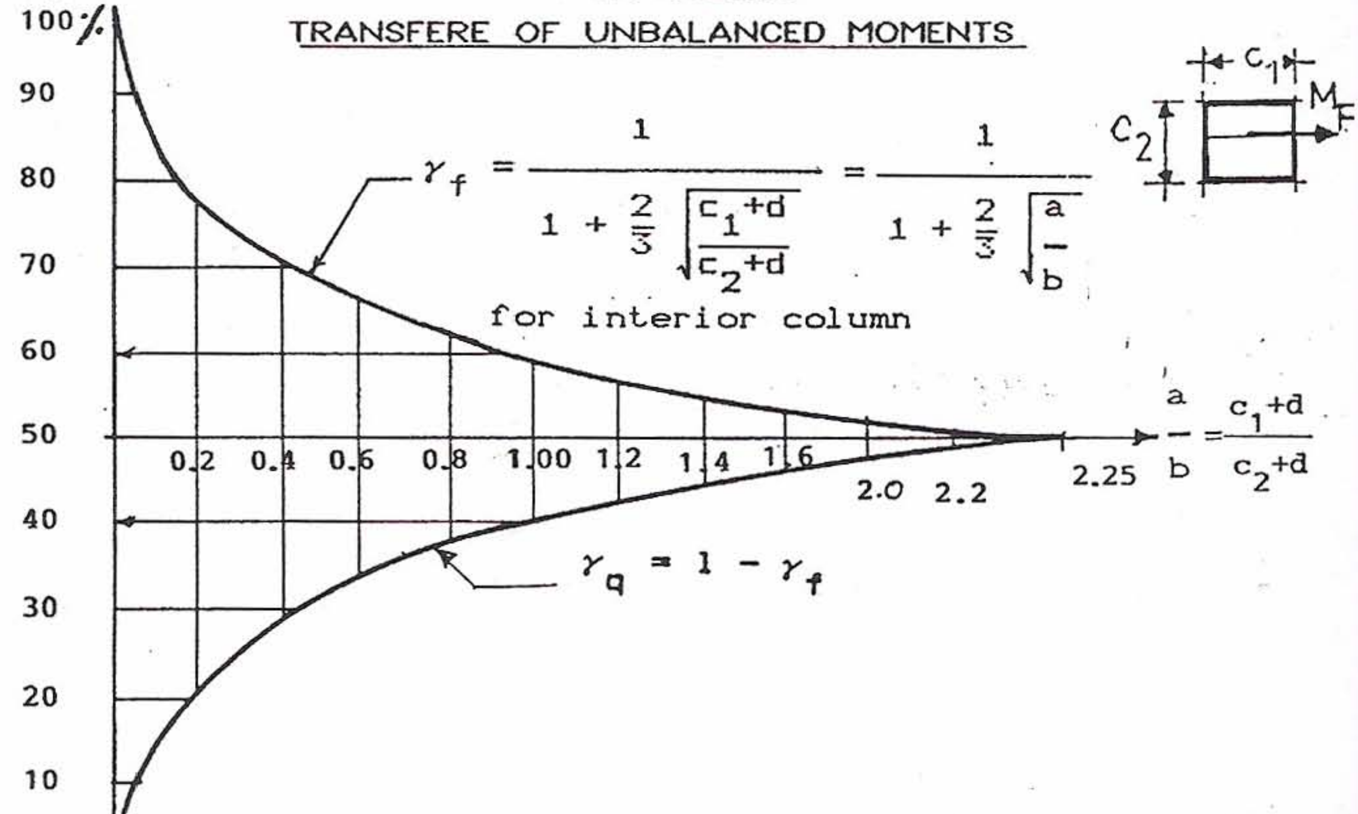


Figure (6-11) Edge column

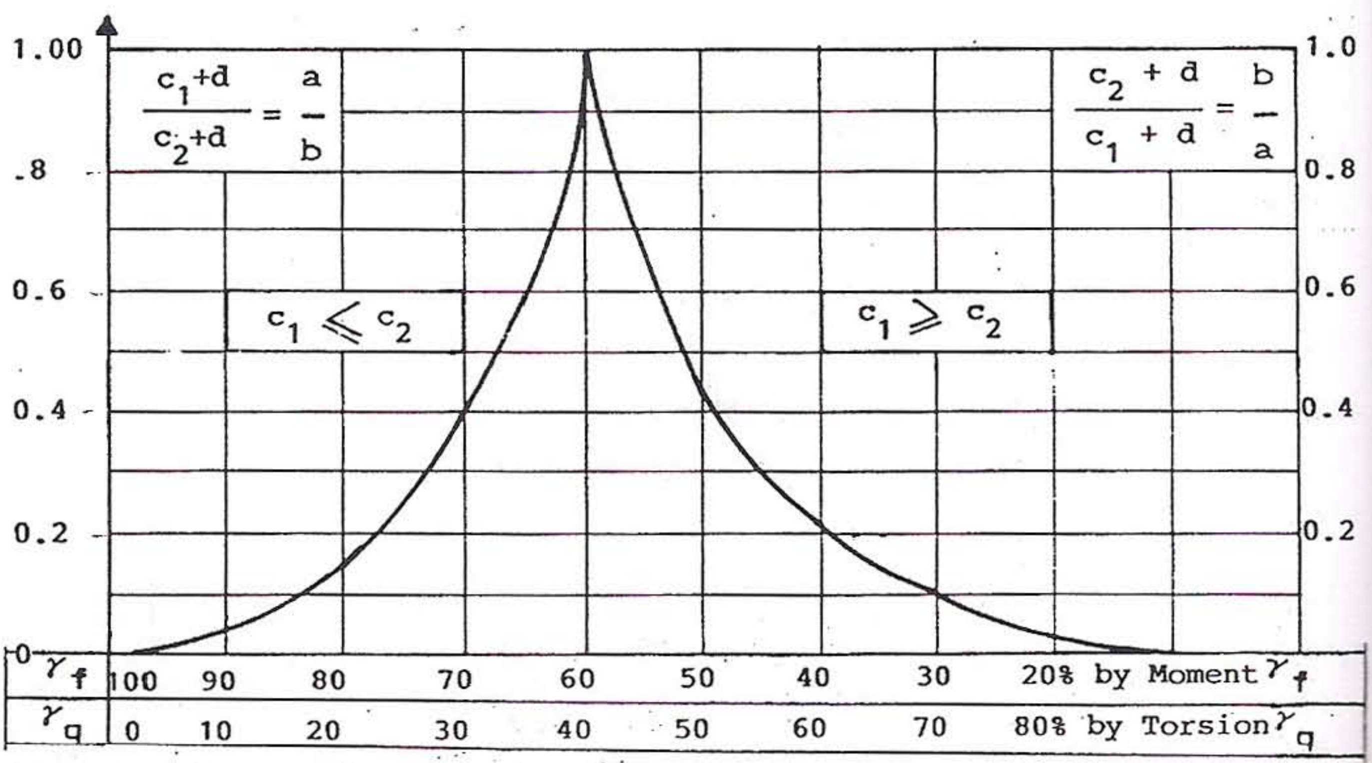
PUNCHING SHEAR STRESSES

FLAT SLABS

TRANSFERE OF UNBALANCED MOMENTS



$(c_1+d)/(c_2+d)$	0.0	0.2	0.4	0.6	0.8	1.00	1.2	1.4	1.6	1.8	2.00	2.25
γ_f	100	77	70	65	62	60	57	55	54	52	51	50
γ_q	0	23	30	35	38	40	43	45	46	48	49	50



Transfer of unbalanced Moments

6-2-7-7-2 The transfer of bending moment from slab to column (article 6-2-7-7-1) can be neglected in the following cases :

a) Internal columns :

1- In case of L.L. not more than 4 KN/m^2

2- When the adjacent spans are equal or differ by not more than 20%

b) External columns :

1- If there is a marginal beam with depth ≥ 3 times the slab thickness .

2- If there is a cantilever slab, extending outside the columns by a distance not less than 1/4 span length from the outside face of the columns, and loaded by the same load of the slab.

6-2-7-7-3 Simple method to calculate the shearing force resulting from the effect of flexure bending between the flat slab and the columns:

$$q = Q_t \beta / b_o d \qquad (6\text{-}25)$$

where

Q_t = The design shearing force transemitted to the column during loading its adjacent panel with a total load to a max. limit.

β = Factor depending on the eccentricity of shearing force and it is taken as follow :-

$\beta = 1.15$	case of interior columns
$\beta = 1.3$	case of exterior columns
$\beta = 1.5$	case of corner columns

b_o = Length of section perimeter

d = Depth of the slab.

6-2-7-8 Arrangement of Reinforcement in Flat Slabs

Slabs designed according to the previous methods are to be reinforced in two directions. The rft should be so arranged that each strip is reinforced over its full width, see figure (7-4), taking into consideration art. (7-4) and conditions of earthquake strength in art. (6-7-2-1).

6-2-7-9 Reinforcement of column head :

Column heads are to be reinforced by rft. (1 &2) which are anchored by stirrups (3) shown in figure (6-12), which must be sufficient to resist the B.M. from the worst position.

and the B.M. mentioned in article (6-2-7-4-a) and (6-2-7-5-e) . Minimum reinforcement is :

1)For rectangular column head : 1/25 of area of negative reinforcement /m` of column strip is direction considered multiplied by length of perpendicular panel to this reinforcement.

2)For circular column head The sum of rft.1 & 2 obtained as above are to be distributed along the perimter of the column head.

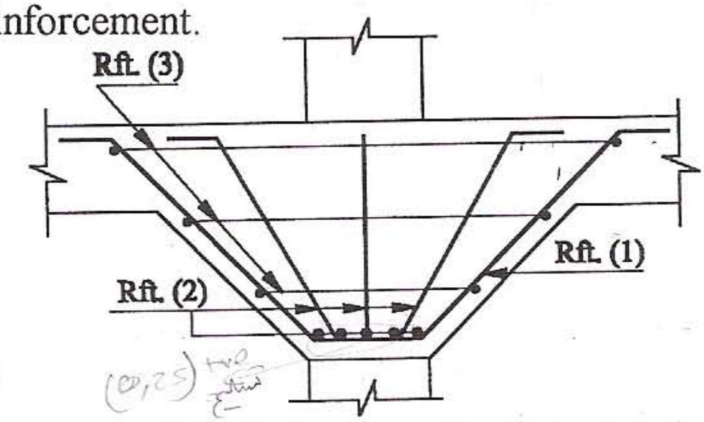
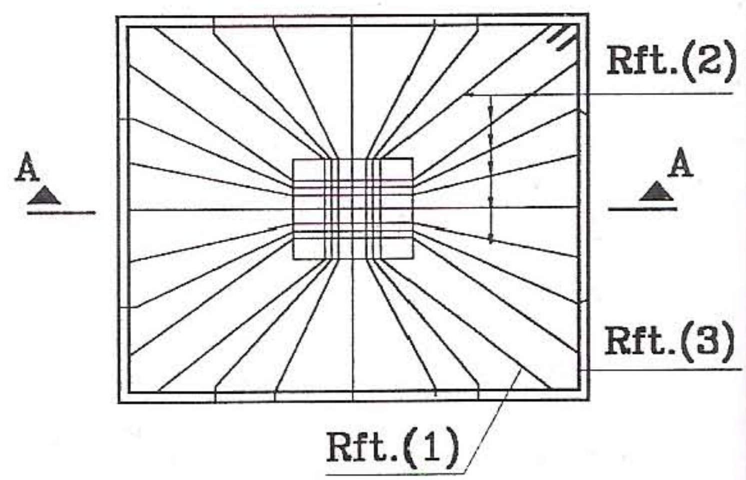
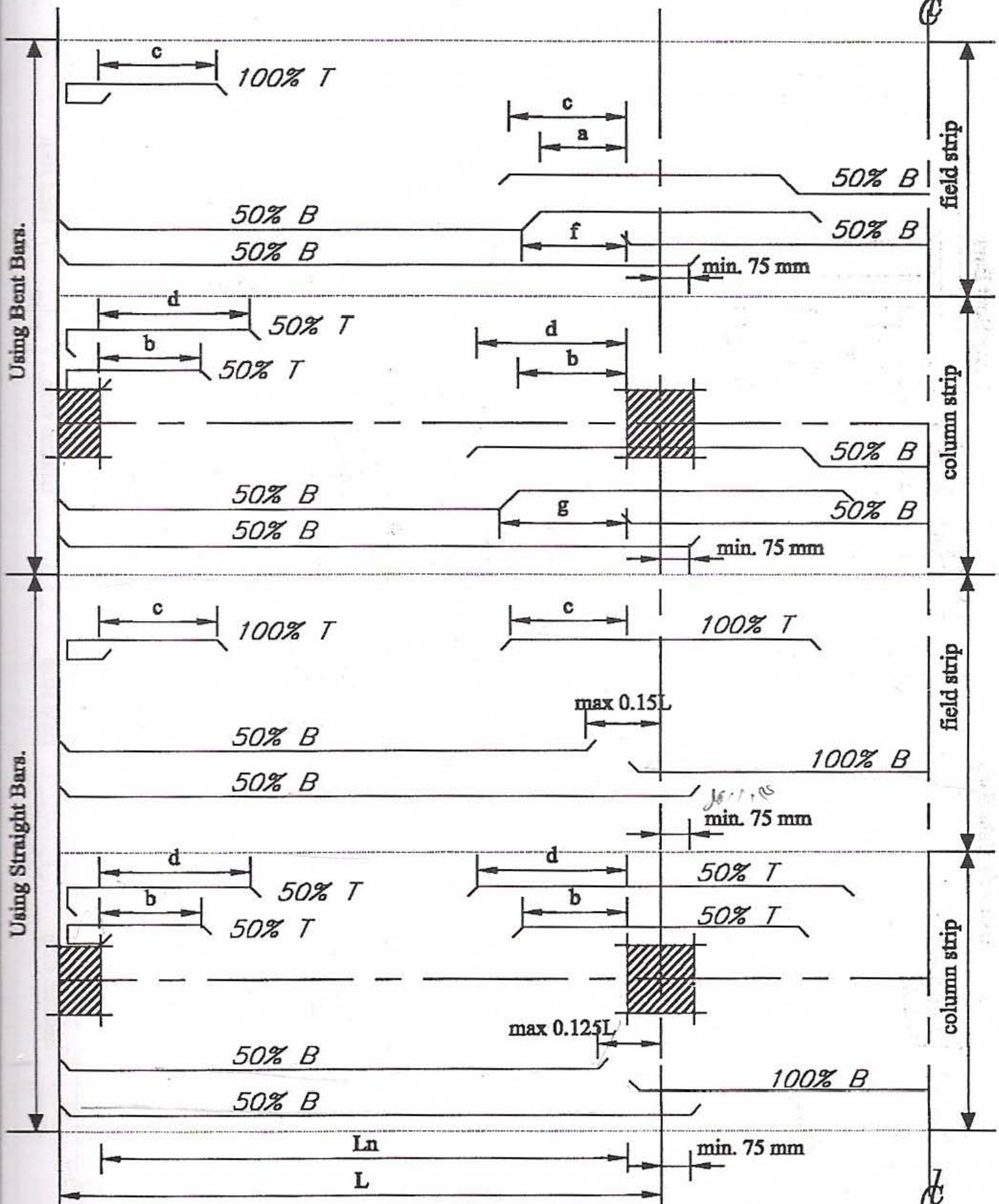


Figure (6-12) Reinforcement of column head



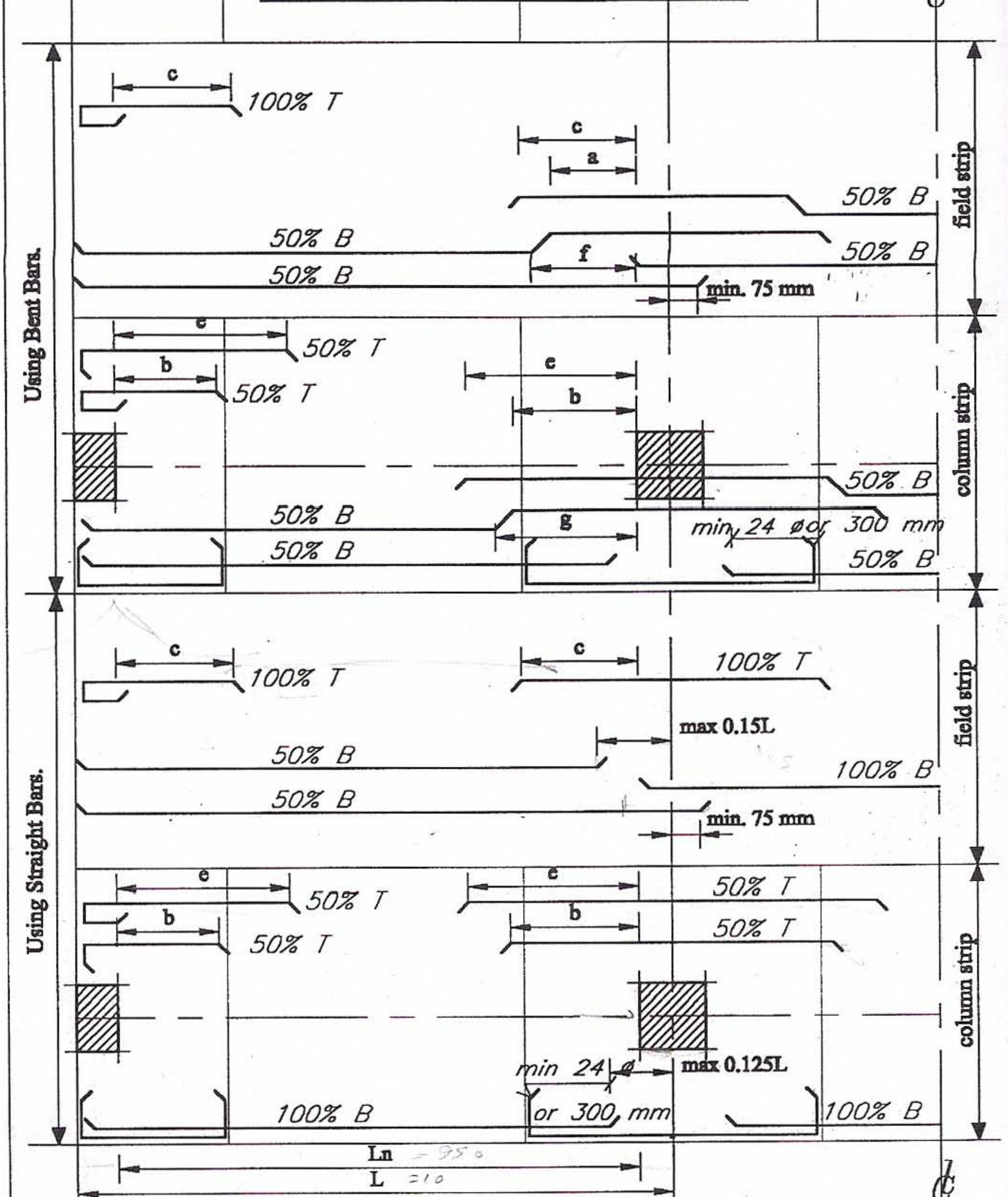
FLAT SLAB WITHOUT DROP PANEL



min. Length				max.Length		
a	b	c	d	f	g	
0.14	0.20	0.22	0.30	0.20	0.24	Ln

B : Bottom R.F.T.
T : Top R.F.T.

FLAT SLAB WITH DROP PANEL

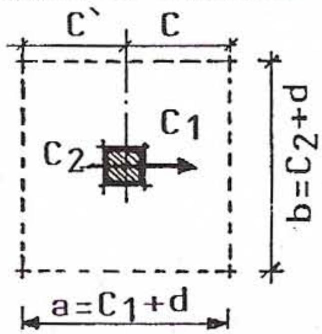


min. Length				max.Length		
a	b	c	d	f	g	
0.14	0.20	0.22	0.33	0.20	0.24	Ln

B : Bottom R.F.T.
T : Top R.F.T.

Polar Moments of Inertia $\gamma_f = 1/(1 + (2/3)\sqrt{a/b})$

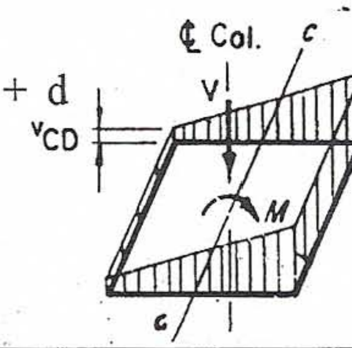
1) Interior column



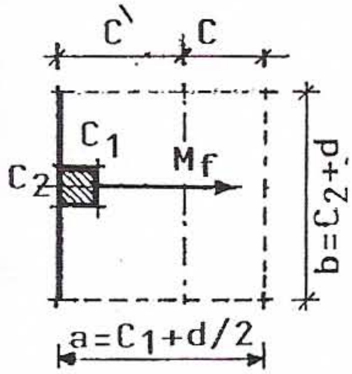
$a = c_1 + d$ & $b = c_2 + d$

$c = c` = a/2$

$\frac{j}{c} = [ad(a+3b) + d^3]/3$



2) Edge column



(bending ⊥ to edge)

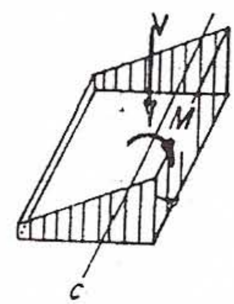
$a = c_1 + d/2$ & $b = c_2 + d$

$c = a^2/(2a+b)$

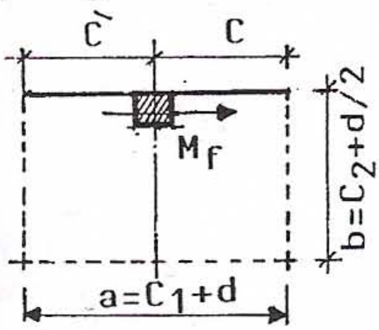
$c` = a(a+b)/(2a+b)$

$\frac{j}{c} = [2ad(a+2b) + d^3(2a+b)/a]/6$

$\frac{j}{c`} = [2a^2 d(a+2b) + d^3(2a+b)]/6(a+b)$



3) Edge column



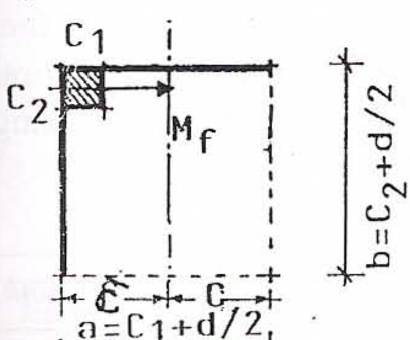
(bending // to edge)

$a = C_1 + d$ & $b = C_2 + d/2$

$C = C` = a/2$

$\frac{j}{c} = [ad(a+6b) + d^3]/6 = \frac{j}{c`}$

4)corner column



$a = C_1 + d/2$ & $b = C_2 + d/2$

$C = a^2/2(a+b)$

$C` = a(a+2b)/2(a+b)$

$\frac{j}{c}[ad(a+4b) + d^3(a+b)/a]/6$

$\frac{j}{c`} = [a^2 d(a+4b) + d^3(2a+b)]/6(a+2b)$

6-2-7-10 Openings in flat slab

a) It is not allawable to openings at regions of column head .

b) It is allawable to make openings at the sharing areas between field strips under the following conditions :

1-The max. dimension of the opening should not be more than 0.4 L in the direction pararell to the exis.

2-The +ve & - ve flexure design moments should be redistributed on the residual parts of the structure to be compatable with the variation occured due to exsistance of the opening.

c) It is allawable to make openings at the sharing areas between field strips and column strips under the following conditions :-

1- The max. dimension of the opening should not be more than $\frac{1}{4}$ width of column strip .

2- The sections of the two strips at region of the opening can resist the flexure design moments.

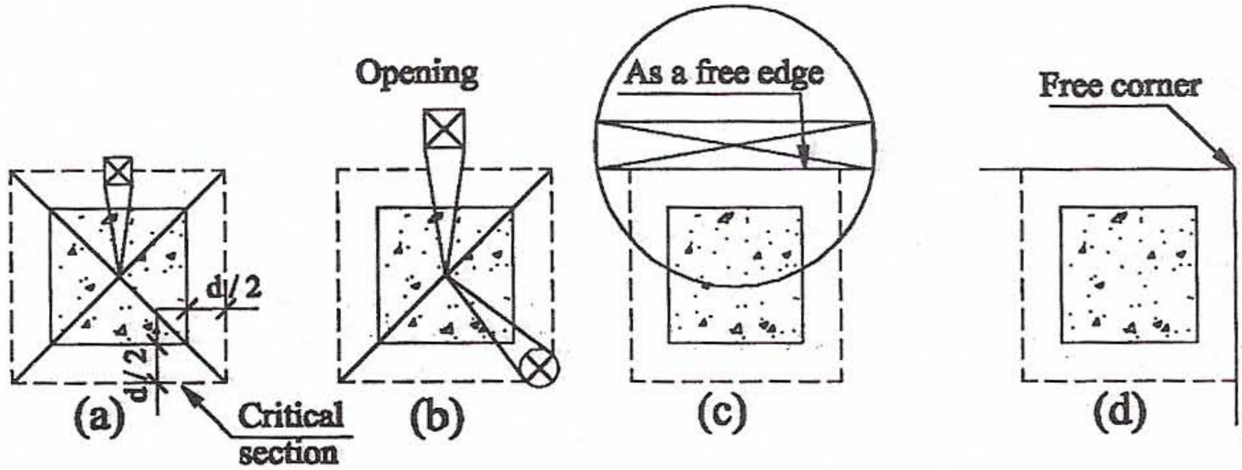
d) It is allawable to make openings at the sharing areas between two columns strips under the following condition :

1-The max. dimension of the opening should not be more than (1/10) width of the smaller column strip.

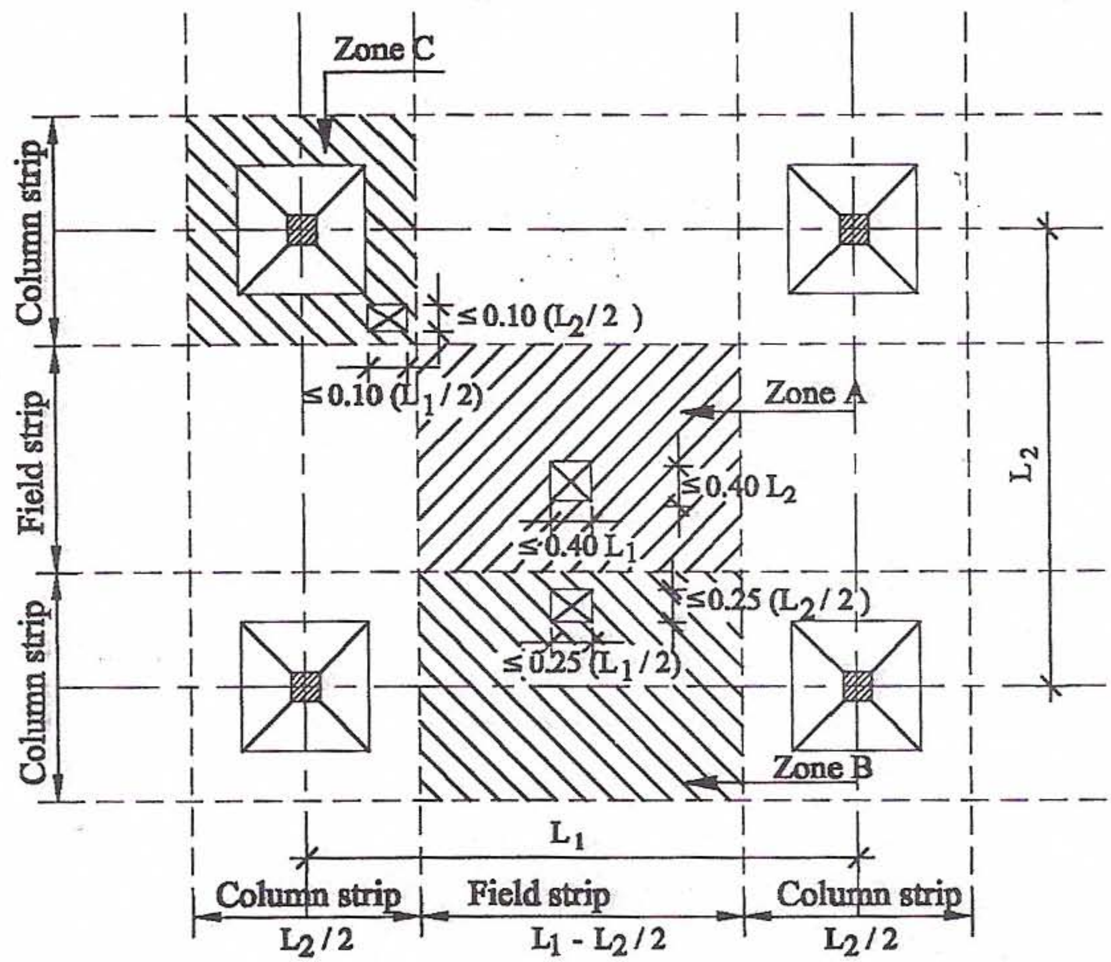
2-The sections of the two strips at region of the opening can resist flexure design moments.

3-The value of live load used to calculate shear stress could be reduced by a value equal to the effect of the opening which intersect the mentioned perimeter fig (6-13-a).

e) In case of exceedance the dimensions of openings more than the proportions mentioned befor a, b, c, d. An accurate structural calculations which achieve resistance requirments and cases of working limits should be made.



(6-13-a) Shapes of openings in flat slab



(6-13-b) Max. dimension of openings in flat slab

(6-13) Shapes of openings & its max. dimensions in flat slab

6-3 BEAMS

This part includes : 1- Ordinarybeams. 2- Deep beams.

6-3-1 Ordinary beams

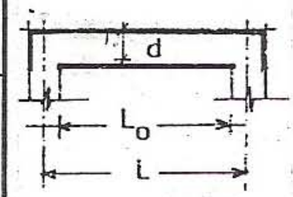
6-3-1-1 General condition

Ordinary beams are beams with

$\frac{\text{depth}}{\text{span}}$ ratio <	0.8 simple beam
	0.4 cont. beam

6-3-1-2 Effective span (L_{eff}) : The least of :

1) Simple beams	C_L - C_L	1.05 L_o	L_o + d
2) Continuous beams : Monolithic with support	C_L - C_L	1.05 L_o	-
Supported on brick wall	C_L - C_L	-	L_o + d
3) Cantilevers	L	-	L_o + d



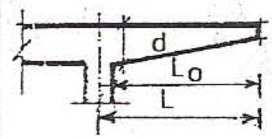
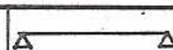
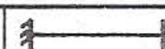
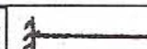


Table (4-10) Max span/depth ratio (L/d) (Beams with spans < 10m. For span > 10m check deflection)

(L/d) min					
High tensile steel	Onyway Solid Slabs	20	24	28	10
	Ribbed slabs or beams	16	18	21	8
Normal mild steel	Onyway Solid Slabs	25	30	35	12
	Ribbed slabs or beams	20	22	26	10

Reduction factor δ for (L/d) ratio for T & L-sections

b/B	≤ 0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
factor δ	0.8	.825	.85	.875	.92	.95	.975	1.0

4-3-1 Max. allowed deflection (Δ)

Case of loading	Beams	Cantilevers
All loads, taking temp., creep and shrinkage effect into consideration	$\Delta < \frac{L}{250}$	$\Delta < \frac{L`}{450}$
Loads including finisshes, temp., creep, shrinkag but without L.L.	$\Delta < \frac{L}{350} < 20mm$	$\Delta < \frac{L`}{600}$

6-3-1-3 Load distribution on beams

Slab loads transmitted to beams can be calculated from the areas limited by lines bisecting the angles at the corners of any panel (fig. 6-14). These loads can however be assumed as uniformly distributed over the the beam span (except contilever beams) in the following manner:

Assume
W: Uniform distributed load per unit area.
L: Span of beams
X: Max. distance of loading

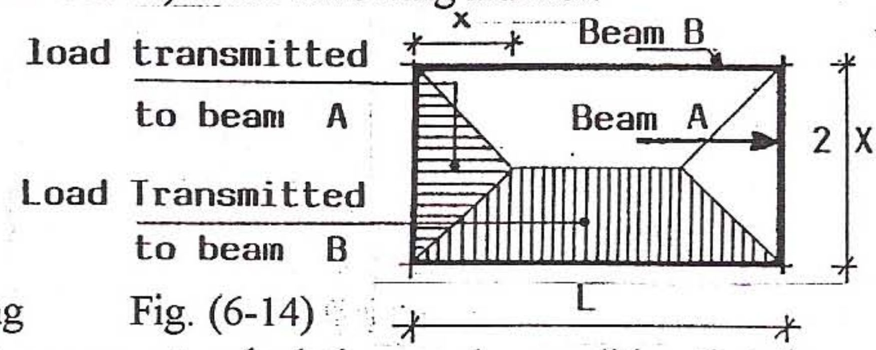


Fig. (6-14)

α wx: Equivalent Load for moment calculations under condition that the load is distributed over the total span with max. intensity at mid span.

β wx: Equivalent Load for reaction and shear calculations and also for moment calculations in cases not fulfilling the above conditions.

table (6-6) Values of α and β
Equivalent uniformly distributed loads cating on beams

L/2 x	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
α	.667	.725	.769	.803	.0.830	.853	.870	.885	.897	.908	.917
β	.500	.544	.582	.615	.642	.667	.688	.706	.722	.737	.750

Where $\alpha = 1 - 1/3\,(2x/L)^2$ & $\beta = 1 - 1/2\,(2x/L)$

6-3-1-4 Structural analysis methods

1)The internal forces and moment calculated by any method of structural analysis must be in equilibrium with the design loads.

2)Linear elastic analysis can be used to obtain the internal forces and moments in beams for both working stress and limit state design methods.

3)Redistribution of moments by not more than ± 10% may be applied according to article (4-2-1-2-c).

6-3-1-5 Flexural rigidity

1) When using the linear elastic analysis, the flexural rigidity for the total concrete section neglecting steel may be assumed (E_cI_g) where E_c is calculated according to art. (2-3-3-1).

2) It is also allowed to use other assumption taking effect of cracks into consideration such as (E_cI_g) for columns and ½ E_cI_g for beams.

3) In all cases the same calculations for the flexural rigidity should be used for all members of the structure.

In case of T & L-sections, flange width can be taken equals ½ the value obtained from art. (6-3-1-9)

6-3-1-6 Bending Moments and shearing forces in contin. beams

Bending moments and shearing forces in continuous beams can be calculated assuming that beams are supported on rigid knife edge supports. For beams with constant depth, equal spans and equal loads or with difference in spans not more than 20% the following values can be assumed:

a) Two spans	b) more than two spans	
Supports: -24, -9, -24; Spans: +11, +11	Supports: -24, -10, -12; Spans: +12, +16, +16	$M = \pm \frac{wL^2}{k_m}$
0.4, 0.6 / 0.6, 0.4	0.45, 0.5, 0.5 / 0.6, 0.5	$Q = k_q\, wL$

For -ve moment take average span and average loads around the supports.

Negative moments at mid span must be calculated for heavy live loads and it is allowed to reduce the -ve field moments at midspans due to live loads only to 2/3 their values due to the rigidity of the columns or the supporting girders.

In case of equal spans subjected to heavy live loads (p>1.5 g), the min -ve field moments at mid span is :
where L is the greater span.

$$M_{min} = (g - \frac{2}{3} p)\left(\frac{L^2}{24}\right) \qquad (6\text{-}26)$$

6-3-1-7 Critical sections for bending moments and shearing forces (cases of monolithic beams):

1) For moments: at column face

2) For shear: at column face except cases in item (3) (figure 6-16).

3) For shear : at distance $\frac{d}{2}$ when reaction produces compression in this distance (figure 6-17)

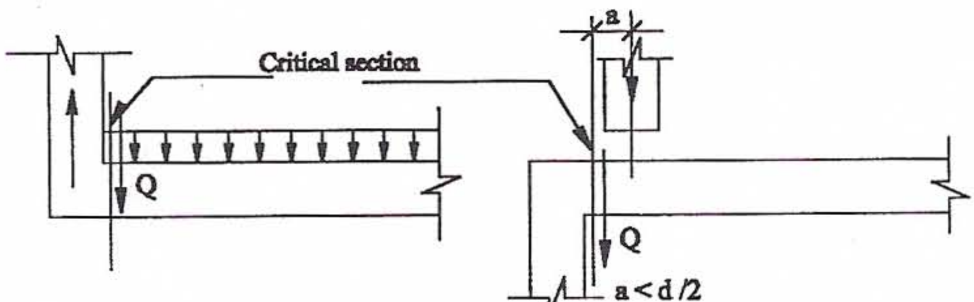


Fig. (6-16) Critical sec. for shear at face of support

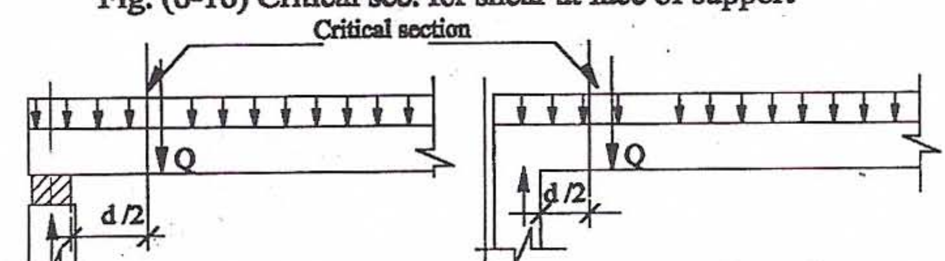


Fig. (6-17) Critical sec. at distance d/2 from face of support

6-3-1-8 Slenderness limit

The laterally unsupported distance between points of inflection must not exceed the smaller of :

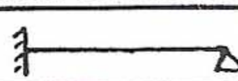
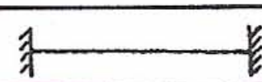
simple or continuous beams	$200\ b_c^2/d$ or	$40\ b_c$
Laterally braced cantilever beams	$80\ b_c^2/d$ or	$20\ b_c$

where b_c = beam width at compression side

6-3-1-9 Effective flange width for T and L sections

The least of	T- sec	$16\ t_s + b$	$L_2/5 + b$	(C_L to C_L of beams)	6-27-a
	L- sec	$6\ t_s + b$	$L_2/10 + b$	½ (C_L to C_L of beams)	6-27-b

where L_2 distance between points of inflection

System				
L_2/L	1.0	0.8	0.7	2.0

6-3-1-10 General conditions

1	To consider the beam as T or L-sec., it must be casted monolitically wit slab of min thickness 80 mm.
2	The top Rft. in the flange perpendicular to the web, must not be less than0.3% of the cross sectional area of the slab, and should be arranged all over the flange width (art. 6-3-1-9) at max. spacing 200 mm
3	The stirrups must extend from web to the top of the flange
4	For isolated T-sections, the thickness of slab must not be less than $\frac{1}{2}$ the web width , also total flange width should not be more than 4 times web width.
5	For beams with depth > 700 mm, shrinkage bars must be added with min. area of 8% of the tension steel. The distance between shinkage bars must not exceed 300 mm.

6-3-1-11 Minimum steel percentages			Steel 240/350	steel 360/520
Flexure	tension steel	$\mu_{min} = \frac{A_s}{bd} = \frac{1.1}{f_y}$ or $(1.3 \frac{A_s\ req}{b\ d})$	0.25 %	0.15 %
	comp. steel	$\mu`_{min} = \frac{A_s}{bd} = \not< 10\% \frac{A_s\ main}{b\ d})$		
	side Rft (shrinkage bars) $\not<$ 8% $A_{s\ main}$			
Shear	web Rft (stirrups) $\mu_{min} = \frac{0.4}{f_y}$ (5ϕ 6/m`) ($\mu_{min} = A_{str} / b\,S$)		0.15%	0.1%

* % of compression rft is preferred not be more than 0.4 %

($A_{s\ max}$ = 0.4 % bd)

Types of steel	Normal mild st.		High tensile st.		Wire mesh
	240/350	280/450	360/520	400/600	450/520
μ_{min} (flex.) $= \frac{1.1}{f_y}$	0.46%	0.40%	0.30%	0.27%	0.275%
μ_{min} (shear) $= \frac{0.4}{f_y}$	0.17%	0.14%	0.11%	0.10%	0.10%

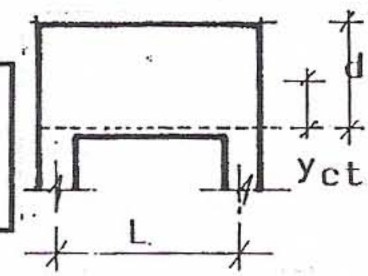
6-3-2-1 Definition

Deep beams are those which have depth to effective span ratio greater than :

$\frac{\text{depth}}{\text{span}}$ ratio	> 0.8	simple beams
	> 0.4	continuous beams

6-3-2-2 Lever arm

simple beams		$y_{ct} = 0.86\ L$	In all cses not more than 0.87 d
continuous beams	mid span	$y_{ct} = 0.43\ L$	
	at support	$y_{ct} = 0.37\ L$	



6-3-2-3 Minimum percentage of main steel

μ_{min} = The least of $\frac{1.1}{f_y}$ or $1.3\ A_{s\ req}/bd$	(6-30)

μ_{min} should not be less than	0.25 % A_c	for mild steel
	0.15 % A_c	for high tensile steel

Note: The min. steel ratio may be less than $\frac{1.1}{f_y}$ if all sections for +ve and -ve moments are reinforced by more than 30 % of the required rft. ($1.3\ A_{s\ req}$).

6-3-2-4 Minimum percentage of web rft.

	mild steel	high tensile st.	S=the least of
VL rft: $A_v/bs_v \geq$	0.2 %	0.15 %	$S_v < 450$ mm or $\frac{d}{5}$
HL rft: $A_h/bs_h \geq$	0.25 %	0.20 %	$S_h < 450$ mm or $\frac{d}{3}$

S_v = distance between VL stirrups, S_h = distance between HL stirrups

But Sv & Sh should not be more than the least of [2b, 250 mm

4-2-2-6 shear in Deep beam

a	This article applies only for deep beams loaded at top surface by comp. stresses			
b	The critical section for shear is at distance :			
	for uniform load	$0.15\ L_n$	But not more than d/2 from face of support	
	for concen. load at distance (a) from face of support	$0.5\ a$		
c	Shear stress where g = the least of the span or the effective depth	$q_u = Q_u/(b\ g)$		4-41
d	Max shear stress	$q_{u\,max} = \delta_d (0.7 \sqrt{f_{cu}/\gamma_c})$ $\delta_d = (\frac{1}{3})[2 + (0.4L_n/d)]$		4-42
e	Concrete shear strength: where	$q_{cu} = \delta_{dc}(0.24 \sqrt{f_{cu}/\gamma_c})$ $\delta_{dc} = 3.5 - 2.5(M_u/Q_u d)$		4-43
	but should not be more than	$q_{cu} = \le 0.46 \sqrt{f_{cu}/\gamma_c}$		4-44
f	If $q_u > q_{cu}$ use shaer rft.	$q_{su} = q_u - 0.5 q_{cu}$		4-45
g	<u>Calculations of shear Rft</u> Shear stress (with rft)	$q_{su} = \delta_v q_{suv} + \delta_h q_{suh}$		4-46-a
	Hl. shear stress	$q_{suh} = (A_h/S_h)(f_y/b\,\gamma_s)$		4-46-b
	V1. shear stress	$q_{suv} = (A_v/S_v)(f_y/b\,\gamma_s)$		4-46-c
	where :	$\delta_h = (11 - L_n/d)/12$		4-46-d
	and	$\delta_V = (1 + L_n/d)/12$		4-46-f

where : A_h = *HL. shear rft.* & *Av = VL. shear rft.*

L_n = *clear span for deep beam.*

h) The shear rft is used with the same value all over the span.

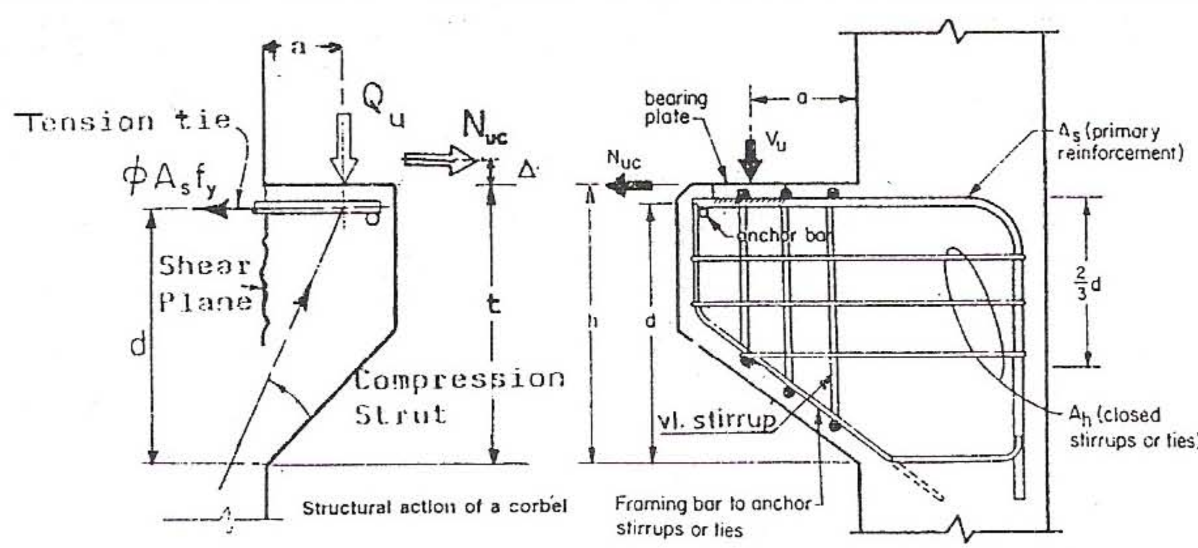
i) Main rft must extend all over beam length with enough anchorage at support.

j) Web rft. must be designed for cases of loading producing tension on loaded side and by lateral loads.

4-2-2-5 Brackets and corbles

short cantilevers with : a < d

Main Reinf. A_s: the bigger of	$(A_n + A_f)$ or $(A_n + \frac{2}{3} A_{sf})$	but not less than $\mu = A_s/bd = .03 f_{cu}/f_y$	4-37-a 4-37-b
A_f = Steel area required to resist flexure (M_u) where A_n = Steel area required to resist tension (N_u)		$M_u = Q_u a + N_u (t + \Delta\text{-}d)$ $A_f = M_u / f_y \, j \, d$ $A_n = N_u/(f_y/\gamma_s)$	4-38 4-39
A_{sf} = Steel area required to resist friction shear		$A_{sf} = \dfrac{Q_u}{\mu (f_y/\gamma_s)}$ μ =1 (monolithic)	
Q_u = max shearing force N_u = tension force (considered L.L.)			
HL. Rft. (closed stirrups) To bc arranged at ($\frac{2}{3}$ d) from top.		$A_h = 0.5 (A_s - A_n)$	6-40
For cases in which the short cantilevers is exposed to torsion moments due to eccentricity of loading or due to hrizontal forces vl. stirrups should be added to satisfy the section resistance requirements of torsion moments. In all cases the web rft. should not be less than the conditions mentioned in articale (4-2-3).			



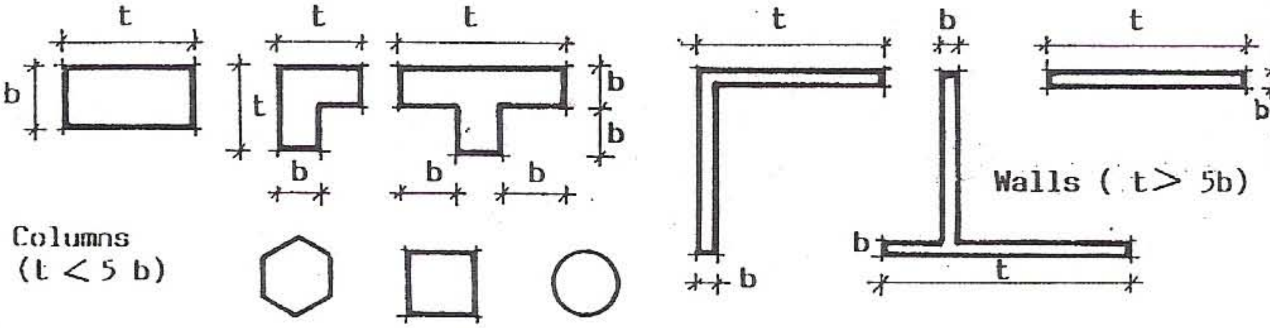
6-4 COLUMNS

6-4-1 Definitions

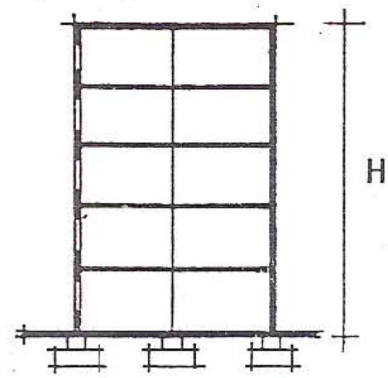
Columns are compression members which have :

1) Height greater than 5 times the smaller side (h>5b).
2) Length not more than five times the breadth (t > 5b).

Examples of column sections:



If length is greater than five times the breadth (t>5b) the member is considered as a wall.



6-4-2 Braced and unbraced columns

columns are considered braced if :

1- Horizontal loads are resisted by special members e.g. shear walls or braced columns.
2- They are supported by concrete bearings.
3- They satisfy the following 2 equations:

For no of stories ≥ 4	$\alpha = H\sqrt{N/\sum EI} \leq 0.6$	6-30-a
For no of stories ≥ 4	$\alpha = H\sqrt{N/\sum EI} \leq (0.2 + 0.1\,n)$	6-30-b

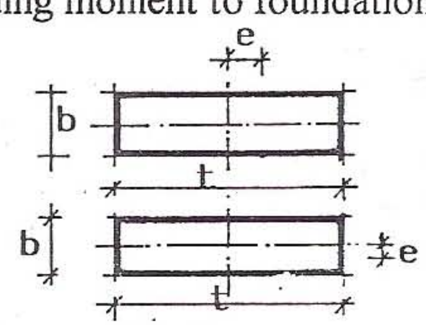
where N = Σ vertical loads & n = № of stories

H = Heigth of building above fixation level.

ΣEI = Flexural rigidity of all vertical columns.

- R.C. wall verifed by equation (6-30) should be connected to the foundation to transmit all horizontal forces and its corresponding moment to foundation.

6-4-3 Minimum eccentricity for columns:



e min ≮ 0.05 t or .05 b or ≮ 20 mm

e is to be taken in the direction with greater slenderness ratio λ_b . i.e.

If $\frac{H_e}{t} > \frac{H'_e}{b}$ take $e = 0.05\,t$, If $\frac{H_e}{t} < \frac{H'_e}{b}$ take $e = 0.05\,b$

where He and H`e = buckling lengths in t and b directions.

6-4-4 Short columns (neglecting buckling effect)

6-4-4-1 Columns are considered short if $\lambda_b=H_e/b$ (or $\lambda_D=H_e/D$ and $\lambda_i=H_e/i$ for circular section) are less than the following values:

Table (6-7) max. Slenderness ratio for short column

Column condition	λ_b rec. sec.	λ_b circular sec.	λ_i
Braced	15	12	50
Unbraced	10	8	35

where : i = radius of gyration = 0.3 b for rec. column
= 0.25 D for circular column
& He = effective height, according to eq. (6-31 & 6-32 & 6-33)

6-4-4-2 Design of short column

According to article (4-2-1-3) taking in consideration the greater of M acting on column or $P_u\, e_{min.}$.

For members subjected to axial compression and single moment of a value $\leq P_u\, e_{min.}$, it can be approximately designed according to the following equations :

a) Tied column	$P_u = 0.35\, f_{cu}\, A_c + 0.67\, f_y\, A_{sc}$	**4-12-a**
b) Spiral column	$P_u = 0.35\, f_{cu}\, A_k + 0.67\, f_y\, A_{sc} + 1.38\, f_{yp}\, v_{sp}$	**4-12-b**
the least of	$P_u = 0.4\, f_{cu}\, A_c + 0.76\, f_y\, A_{sc}$	**4-12-c**

For short columns subjected to double bending refer to article (6-4-6).

6-4-5 Slender columns (long columns)

Slender columns are those which have slenderness ratio greater than the values given in table (6-7), but in any column the slenderness ratio should not be greater than the values given in table (6-8).

Table (6-8) max. slenderness ratio for long columns

Column condition	λ_b rec. sec.	λ_b circular sec.	λ_i
Braced	30	25	100
Unbraced	23	18	70

6-4-5-1 Bucking length

The buckling length of slender column $H_e = k\ H_o$ where k is the least of :

braced columns	$k = [0.7 + 0.05(\alpha_1 + \alpha_2)] \le 1.0$	6-32-a
	$k = [0.85 + 0.05\ \alpha_{min}] \le 1.0$	6-32-b
unbraced columns	$k = [1.0 + 0.15(\alpha_1 + \alpha_2)] \ge 1.0$	6-33-a
	$k = [2.0 + 0.3\ \alpha_{min}] \ge 1.0$	6-33-b

where $$\alpha = \frac{\text{stiffness of columns}}{\text{stiffness of beams}} = \frac{\sum E_c\, I_c / H_o}{\sum E_c\, I_c / L_b} \quad .. (6\text{-}34)$$

α_1 & α_2 at lower and upper ends of the column.

α_{min} = min. of α_1 and α_2 .

I_c , I_b : moments of inertia of column and beam cross sec.

For ordinary buildings the buckling length for braced and unbraced columns can be approximately obtained from table (6-9), which depends on the degree of fixation at upper and lower ends:

Tables (6-9 &10) Buckling length for ordinary buildings
$H_e = K \times H_o$ where k values are as given below

Upper end condition	(6-9) Braced column			(6-10) Unbraced column		
	Lower end condition			Lower end condition		
	a	b	c	a	b	c
a	0.75	0.8	0.9	1.2	1.3	1.6
b	0.8	0.85	0.95	1.3	1.5	1.8
c	0.9	0.95	1.0	1.6	1.8	-
d	-	-	-	2.2	-	-

Where end conditions are defined as:



a) *fixed*

Case (a) : The end of the column is monolithically casted with beams or slabs having a depth not less than the lateral dimension of the column in the direction considered. Also for the end of the column which is connected to the foundations if the foundations are designed to carry moment.



b) *partially fixed*

Case (b) : The end of the column is monolithically casted with beams or slabs having a depth not less than the lateral dimension of the column in the direction considered.



c) *hinged*

Case (c) : The end of the column is connected to a member, unable to prevent rotation.

Case (d) : The end of the column or wall is not prevented from rotation or horizontal movement.

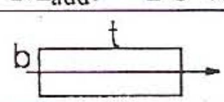
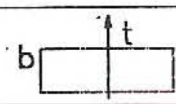
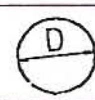
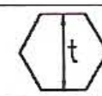
In all casse the length of column and cantilever wall should not be more than:

$$L_0 = 50\, b^2/a \leqslant 30\, b \qquad (6\text{-}35)$$

6-4-5-2 Design of long braced columns

i) Additional bending moment due to buckling:

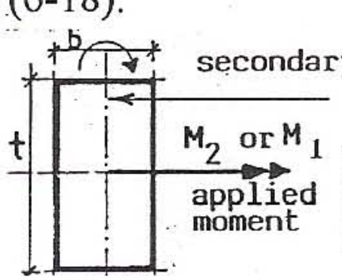
The effect of buckling can be considered as an additional moment $M_{add.} = P\delta$ where.

			
$\delta = \lambda^2_t.t/2000$	$\delta = \lambda^2_b.b/2000$	$\delta = \lambda^2_D.D/2000$	$\delta = \lambda^2_i.t`/2000$
$\lambda_t = H_e/t$	$\lambda_b = H_e/b$	$\lambda_D = H_e/D$	$\lambda_i = H_e/t`$

ii) Design of long braced columns subjected to bending moment about the secondary axis only:

Design moment is to be taken the greater of the shown in figure (6-18).

$\lambda_b = He/b \geqslant 15$



$\lambda_t = He/t < 15$

1- M_2 2- $M_i + M_{add.}$ 3- $M_1 + (M_{add.}/2)$ 4- $P.e_{min}$	6-37
Where: $M_i = 0.4\, M_1 + 0.6\, M_2 \geqslant 0.4\, M_2$	6-38

M_1 is to be considered (–ve) for columns with double bending.

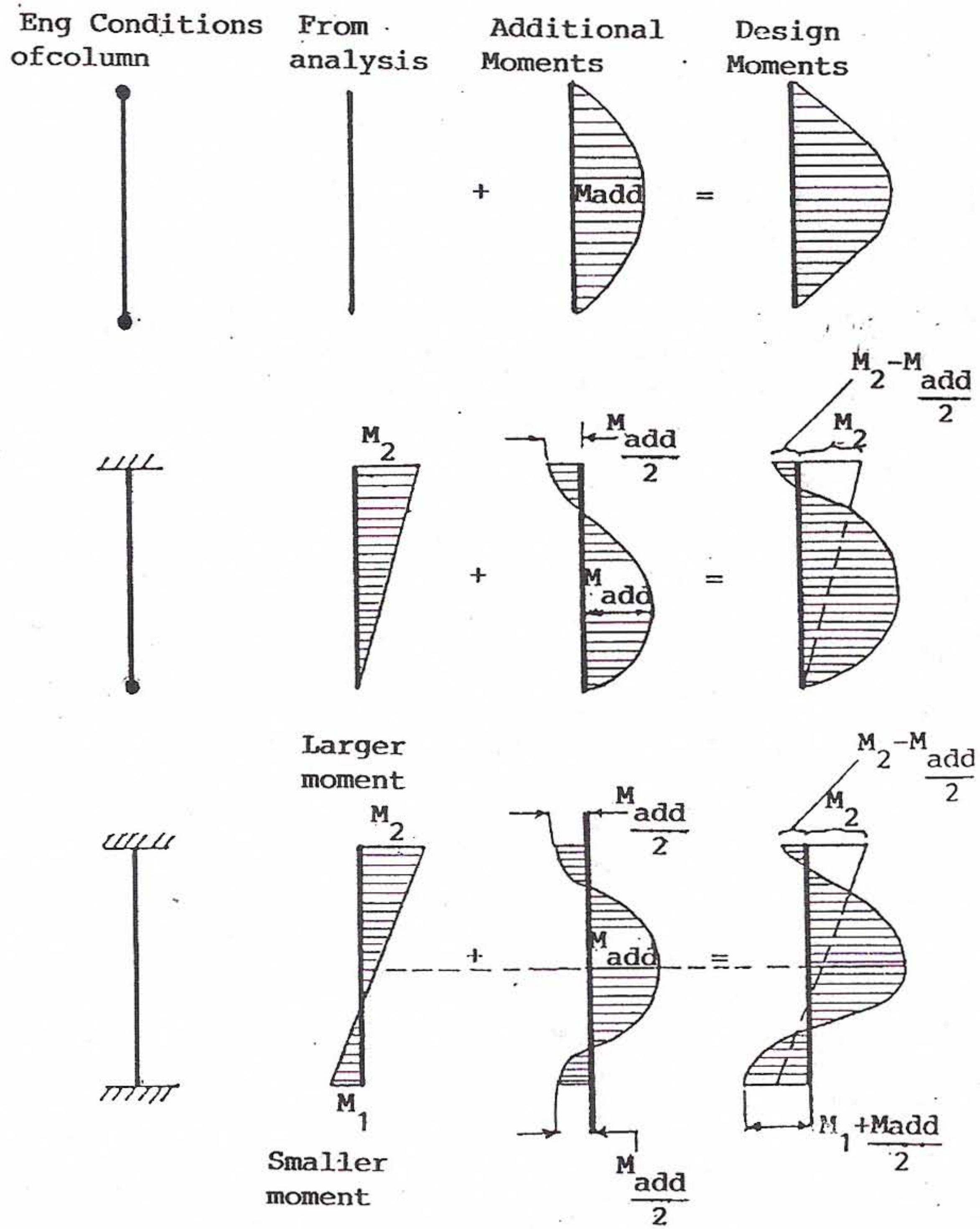


Figure (6-18) Design moments for long braced columns

iii) Design of long braced columns subjected to bending moment about the main axis only:

The column is designed on biaxial moment according to art. (6-4-6) taking the primary momen M_i about the secondary axis equals zero.

Design moment is taken the bigger of M_{add} or $P\ e_{min}$.

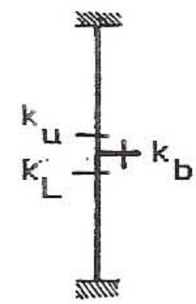
Note : In cases of building consisting of beams end columns on condition that the columns are not subjected to moments due to side sway, it is allowed to simplify the calculations of the moments on the columns as follows:

1-Bending moments at top & bottom (M1 & M2) may be assumed equal to zero for interior columns, which carry beams approximate symmetrical in position and loading.
In case of flat slab system M_1 and M_2 are to be calculated according to article (6-2-7-4), and then the column is to be designed according to article (6-4-5-2-ii).

2-For exterior columns M_1 and M_2 can be assumed according to table(6-11)

Table (6-11) moment on exterior columns

Position of moment	Moment in case of frame with one span	Moment in case of frame with two or more spans
Bottom of upper column	$\frac{K_u M_f}{K_L + K_u + 0.5K_b}$	$\frac{K_u M_f}{K_L + K_u + K_b}$
Top of lower column	$\frac{K_L M_f}{K_L + K_u + 0.5K_b}$	$\frac{K_L M_f}{K_L + K_u + K_b}$



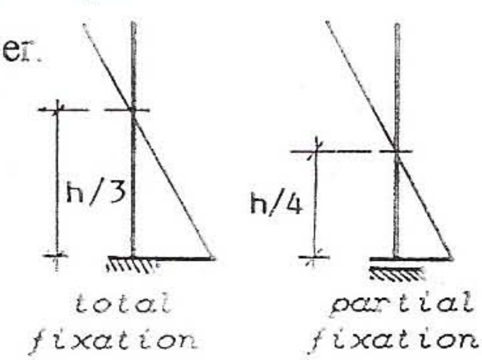
Where M_f= connecting moment at ends of beams which are considered as totally fixed with columns (frame action).

Upper col. stiffness	Lower col. Stiffness	Beam stiffness
$K_u = 4\ EI_u/h_u$	$K_L = 4\ EI_L/h_L$	$K_b = 4\ EI_b/L_b$

h_u, h_L = upper and lower column height respectively.
L_b = beam length
I_u, I_L, I_b = upper col., lower col., and beam moment of inertia respectively.

Equations in table (6-11) could be used for moment at top of lower column to get M and at upper last floor roof ($K_u = 0$).
The above moments are based on the following assumptions.

a) Moment of inertia is constant for each member.
b) Connection points are not subjected to V1 or H1 displacement.
c) All members have the same degree of fixation at far ends.
d) Points of zero moments can be assumed as shown in figure.



6-4-5-3 Design of long unbraced columns

a) Additional moment due to buckling:

For floors which have equal or approximatelyy equal side sway for all columns, the additional moment due to buckling can be obtained from the following equations :

$$M_{add} = P\,\delta_{av} \quad \text{where} \quad \delta_{av} = \sum \delta / n \qquad (6\text{-}39 \;\&\; 6\text{-}40)$$

where : n = number of columns / floor and $\delta = \lambda_b^2\, b/2000$

Note : Values of $\delta > 2\delta_{av}$ are to be neglected in the calculations of δ_{av} . The effect of Madd should be taken into considerations in the design of beams and slabs connected monolithically with columns.

b) Design moments for long unbraced columns subjected to moment in one direction (figure 6-17) :

The design moment acting at the column end is to be taken the greater of:

1) $M_2 + M_{add}$ 2) $P\, e_{min}$

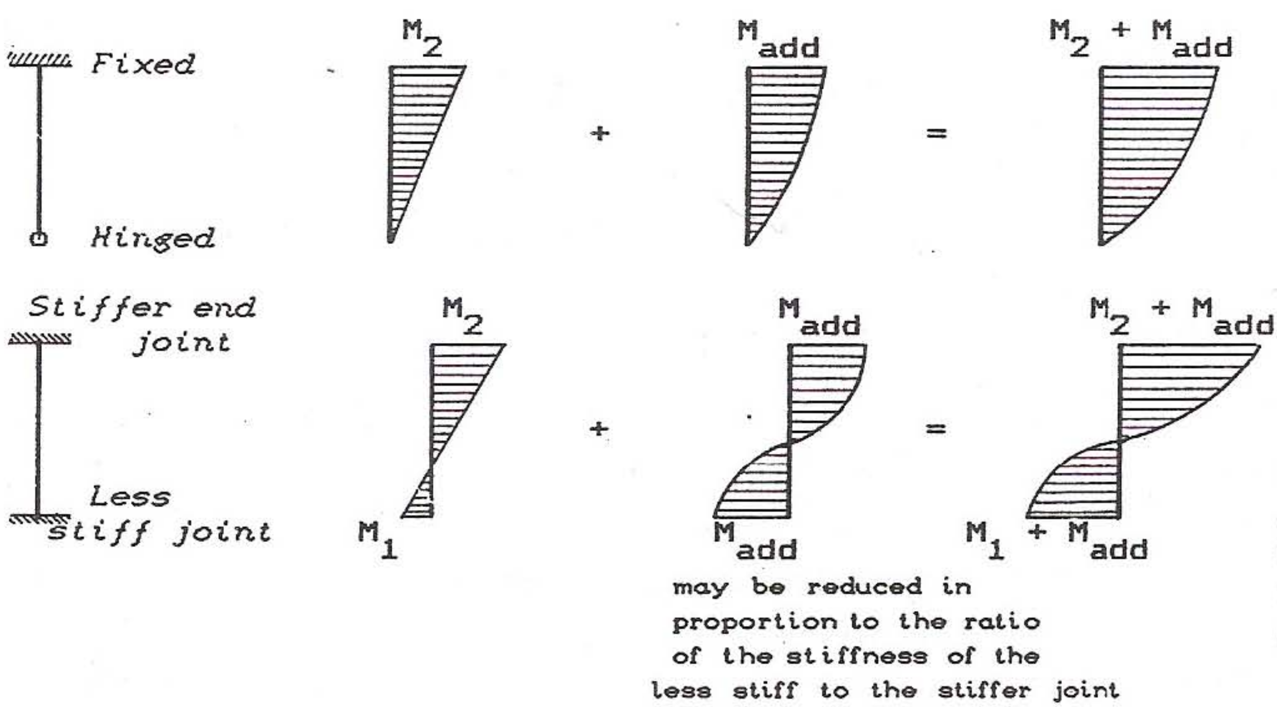


Figure (6-19) Design moment for long unbraced columns

6-4-6 Design of columns subjected to biaxial moments:

i) Columns are designed to resist axial forces and biaxial moments calculated according to article (6-4-4-2) for short columns and according to articles (6-4-5-2-a) & (6-4-5-3) for long columns.

ii) For Rec. sec. With symmetrical rft. subjected to biaxial moments may be assumed acting on one axis using the following approximate method (fig, 6-18).

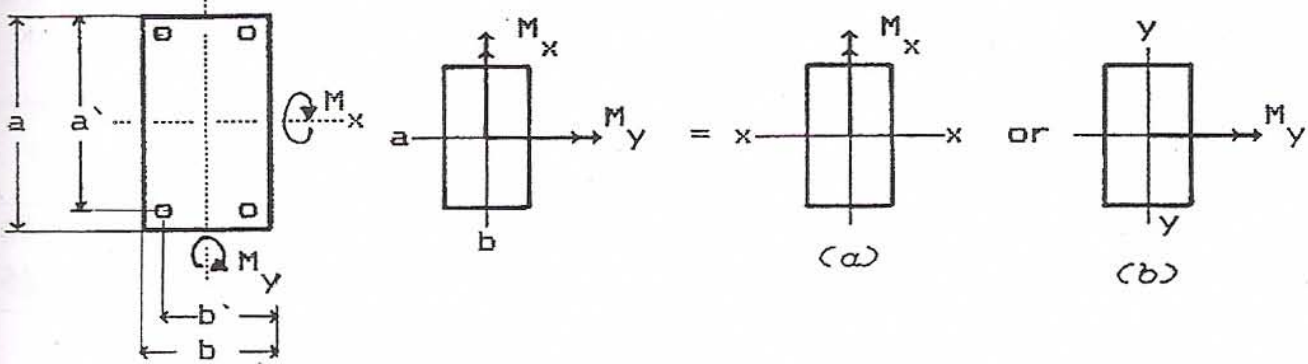


Fig. (6-20-a) Columns under biaxial moment

If $M_x/a` \leqslant (M_y/b`)$ then design moment is taken about y- axis	$M`_y = M_y + \beta\,(b` / a`)\,M_x$	6-41
If $M_x/a` > (M_y / b`)$ then design moment is taken about x- axis	$M`_x = M_x + \beta\,(a` / b`)\,M_y$	4-42

Where values of conefficient β are given in table (6-12):

Table (6-12-a) Values of conefficient β

$P_u / (b\,a\,f_{cu})$	0	0.1	0.2	0.3	0.4	0.5	≥ 0.6
β	0.80	0.80	0.80	0.75	0.70	0.65	0.60

iii) In case of rectangular sections with equal steel reinforcement at each two paralle sides in column sec. (fig.6-21) and the value $P_u/(f_{cu}.b.a)$ must be ≤0 5 Column can be designed by using a simplified method to resist the axial force (Pu), and the following bending moments:

$$M`_x = M_x \,.\, \alpha_b \qquad (6\text{-}43\text{-}a)$$

$$M`_y = M_y \,.\, \alpha_b \qquad (6\text{-}43\text{-}b)$$

Where: α_b can be determined from table (6-21-b)

Table (6-12-b): value of corfficient α_b

$(M_x/a`)/(M_y/b`)$ $R_b=P_u/(f_{cu}b.a)$	∝	3	2	1	0.5	0.33	0
$R_b \leqslant 0.1$	1	1.20	1.25	1.30	1.25	1.20	1
$R_b \leqslant 0.2$	1	1.35	1.50	1.75	1.50	1.35	1
$R_b \leqslant 0.3$	1	1.25	1.35	1.40	1.35	1.25	1
$R_b \leqslant 0.4$	1	0.95	0.95	0.95	0.95	0.95	1
$R_b \leqslant 0.5$	1	0.65	0.70	0.75	0.70	0.65	1

6-4-7 Notes and details:

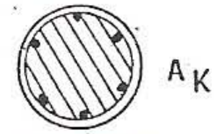
1- Minimum dimensions of rectanglar or circular column = 200 mm

2- Minimum diameter for longitudinal rft, is ϕ = 12 mm

3- All columns should have one longitudinal bar in each conrner.

4- Max. dimension of column side with only corner bars = 300 mm

- Maximum spacing between bars is 250 mm.
- Minimum number of bars for circular columns is 6 bars.



5- Minimum percentage of longitudinal reinforcement:

a	μ_{min} Short column	0.8 % $A_{c\ req}$ and not less than 0.6 % $A_{c\ chosen}$	
b	μ_{min} long column- For rec. section	$\mu_{min} = 0.25 + 0.015\ \lambda_i$ $\mu_{min} = 0.25 + 0.052\ \lambda_b$	6-44 6-45
c	Spiral column μ_{min}	1 % A_g (gross concrete area) or 1.2 A_k (core area-inside spiral)	

6- Maximum percentage of longitudinal rft. is:

Type	Central col.	Edge col.	Corner col.
μ_{max}	4 %	5 %	6 %

7- Minimum diameter of stirrups is the greater of :

1- ϕ 8 mm

2- ¼ max diameter of vertical rft. (¼ ϕ V1 max.)

3- The volume of stirrups > 0.25 % of the concrete volume.

Max. spacing between stirrups	$\not>$	200 mm 15 ϕ (smaller) of V1 bars

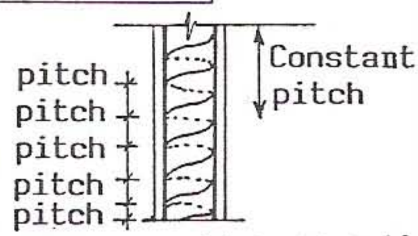
9- For spiral stirrups:

min. diameter = 8 mm

pitch = **80 mm**

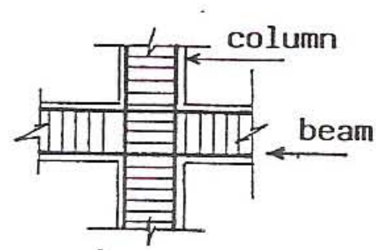
min. pich = 30 mm





It is preferred to keep the pitch constant, and to use 3 turns with half pitch at top and bottom of column with end bar bent inside the column with length 100 mm or 10 ϕ spiral.

10- Both spirals and hoops must continue inside the beams.



11- Minimum splice length for columns bars should not be less than 35 ϕ v1 or 400 mm.

Notes :

1) All forces and moments at the base of the column should be transmitted to the footing by bearing on concrete and by rft (Dowels - mechanical splices). If there is tension on the footing from cases of loading, it must be resisted by steel only and cracking limit state should be satisfied.

2) Bearing stresses for columns and footings should not exceed the values given in article (4-2-4).

3) Dowels and mechanical splices should resist the compression forces greater than the bearing resistance for each of the column and the footing but less than the column rft.

4) Lateral forces, acting on the contact surface, should be transmitted by friction shear (4-2-2-4) or any other method.

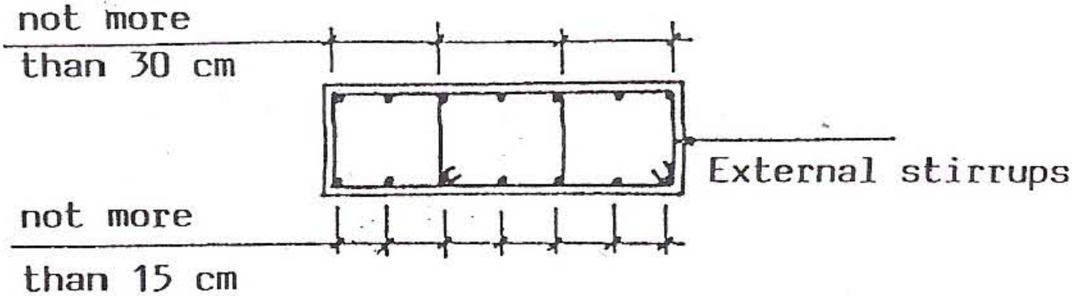
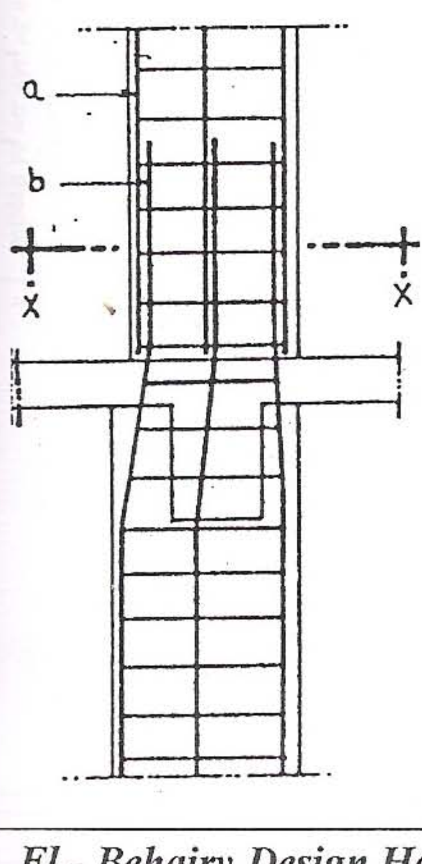


Fig (7-5-a) typical cross - section of a rectangular column



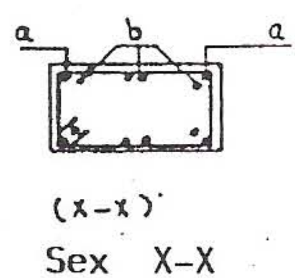


Fig (7-5-b)
Detail of column connection at slabs and beams

DESIGN OF COMPRESSION MEMBERS (h >5b)

$[H_e = K H_o , \lambda_b = H_e/b]$

columns (t < 5b)

SHORT COLUMN

λ_b < 15 braced

λ_b < 10 unbraced

1) TIED COLUMN

$$P_u = 0.35 f_{cu} A_c + 0.67 A_{sc} f_y$$

2) SPIRAL COLUMN

The least of

$$P_u = 0.4 f_{cu} A_c + 0.76 A_{sc} f_y$$

$$P_u = 0.35 f_{cu} A_k + 0.67 A_{sc} f_y + 1.38 V_{sp} f_{yp}$$

LONG COLUMN

15 < λ_b < 30 braced

10 < λ_b < 23 unbraced

$$\delta = \lambda_b^2 \frac{b}{2000}$$

$$M_{add} = p\,\delta$$

BRACED

Moment about Secondary axis (My)	*Moment about Main axis (Mx)*
M_{design} = The bigger of : 1) $M_2 = M_y$ = applied moment 2) $M_1 + M_{add}/2$ 3) $M_i + M_{add}$ 4) $P\,e_{min}$ where $M_i = 0.4M_1 + 0.6M_2 \geq 0.4M_2$	Design as biaxial (Double moment) Mx=applied moment My= The bigger of M_{add} or $P\,e_{min}$

UNBRACED

$M_{add} = P\,\delta_{av}$

where $\delta_{av} = \frac{\Sigma\delta}{n}$

n= No of columns/floor

(Values of $\delta > 2\delta_{av}$ are to be neglected)

M_{design} = The bigger of :

1) $M_{applied} + M_{add}$

2) $P\,e_{min}$

Walls (t > 5b)

SHORT WALL

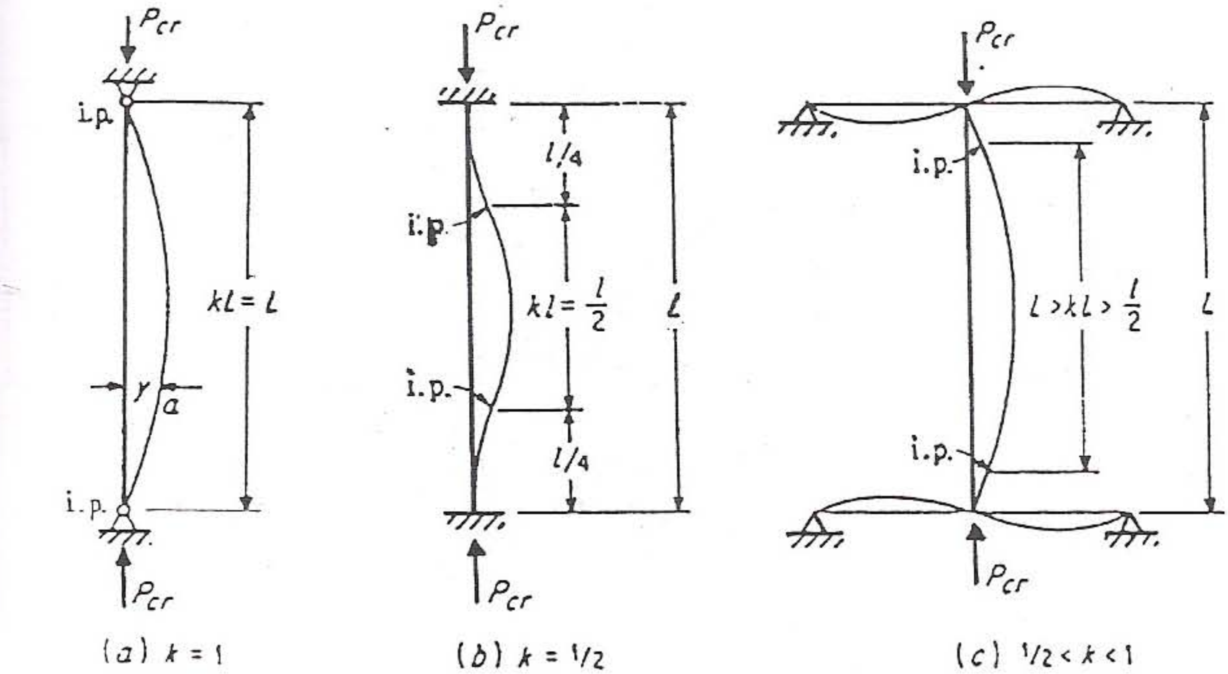
λ_b < 15 braced

λ_b < 10 unbraced

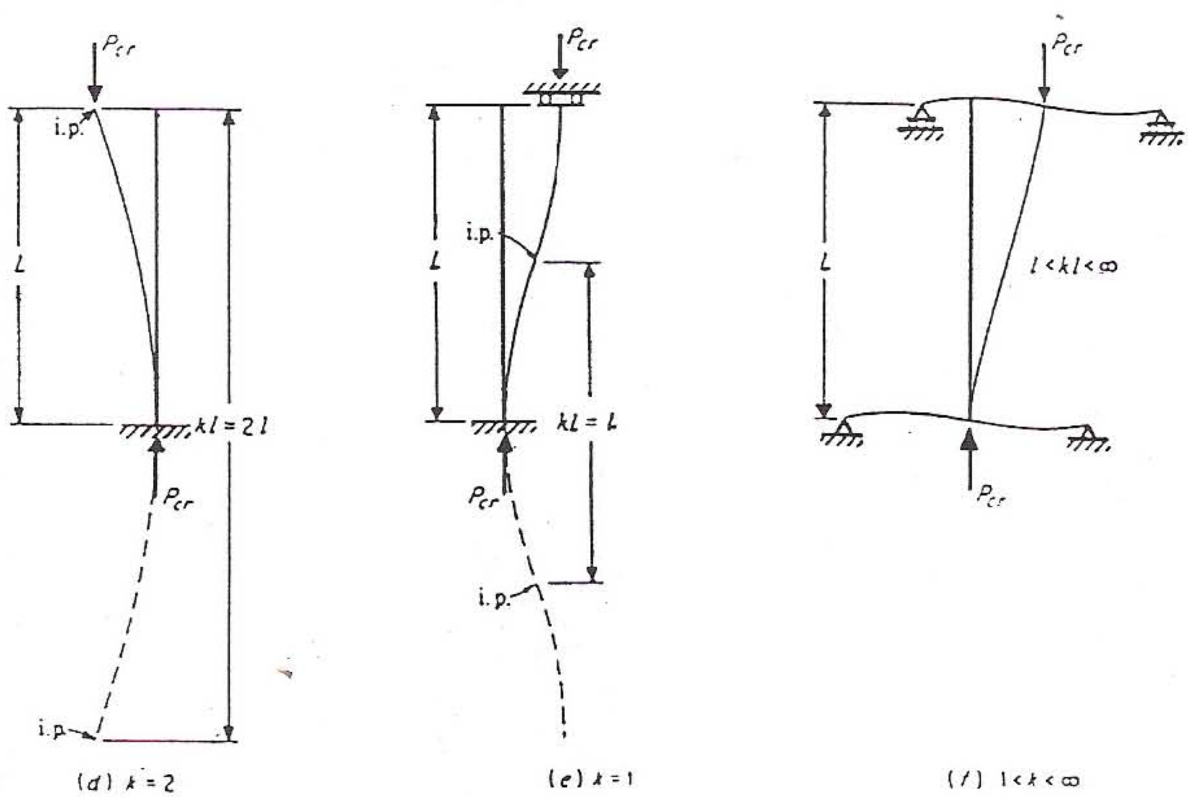
LONG WALL

15 < λ_b < 30 braced

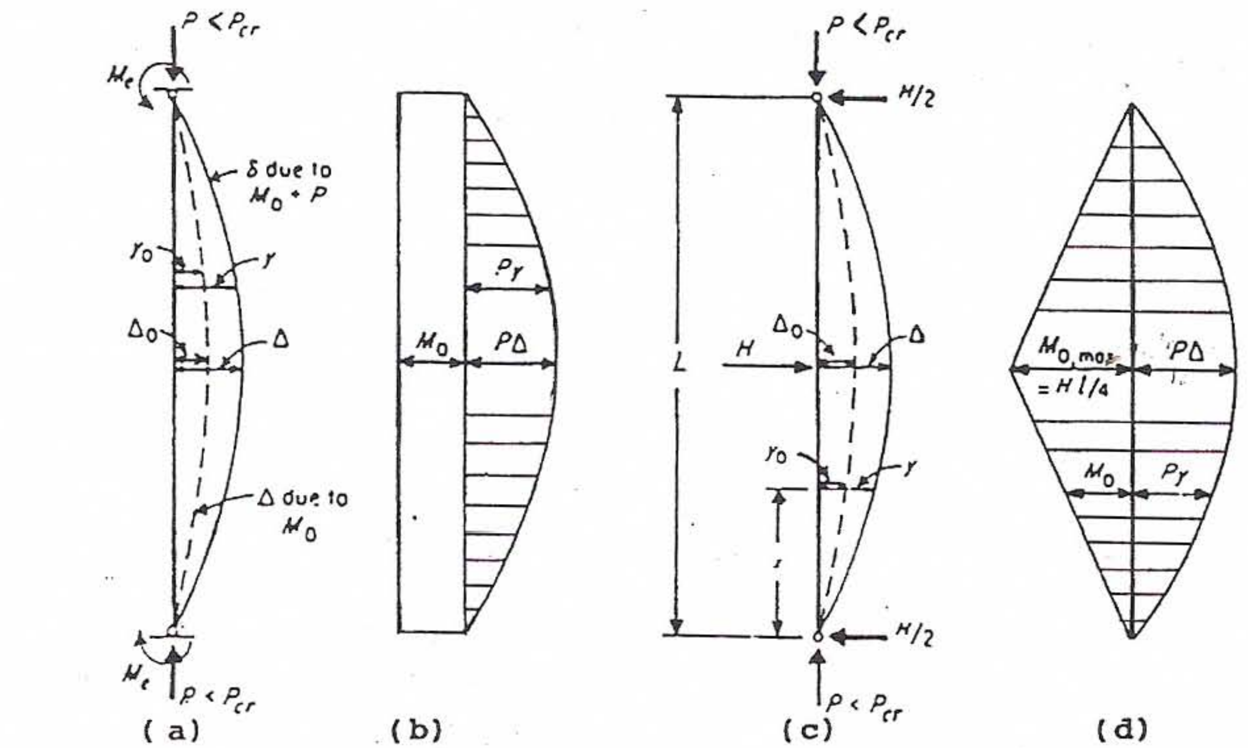
10 < λ_b < 23 unbraced



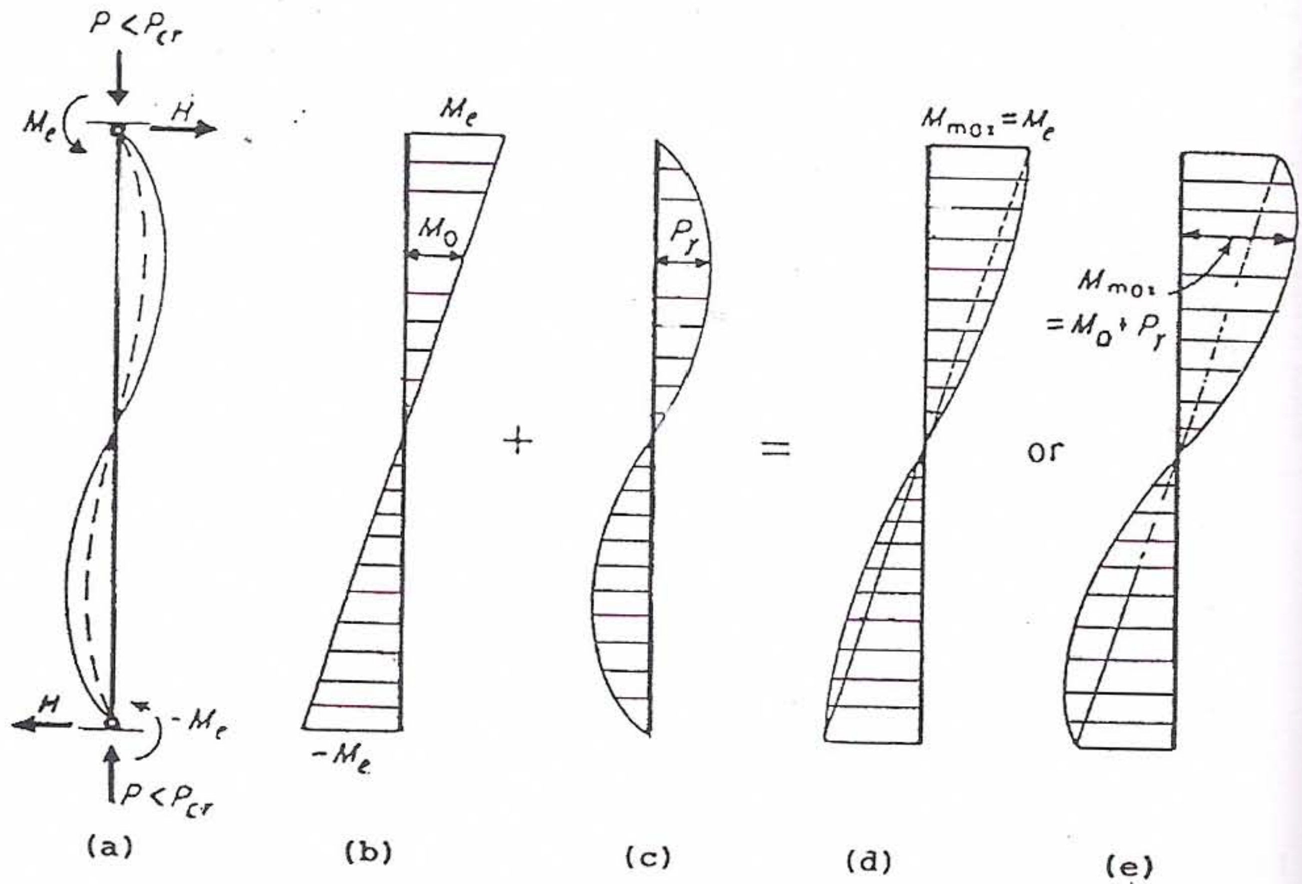
I) BUCKLING & EFFECT. HEIGHT OF AXIALLY LOADED <u>BRACED COLUMN</u>



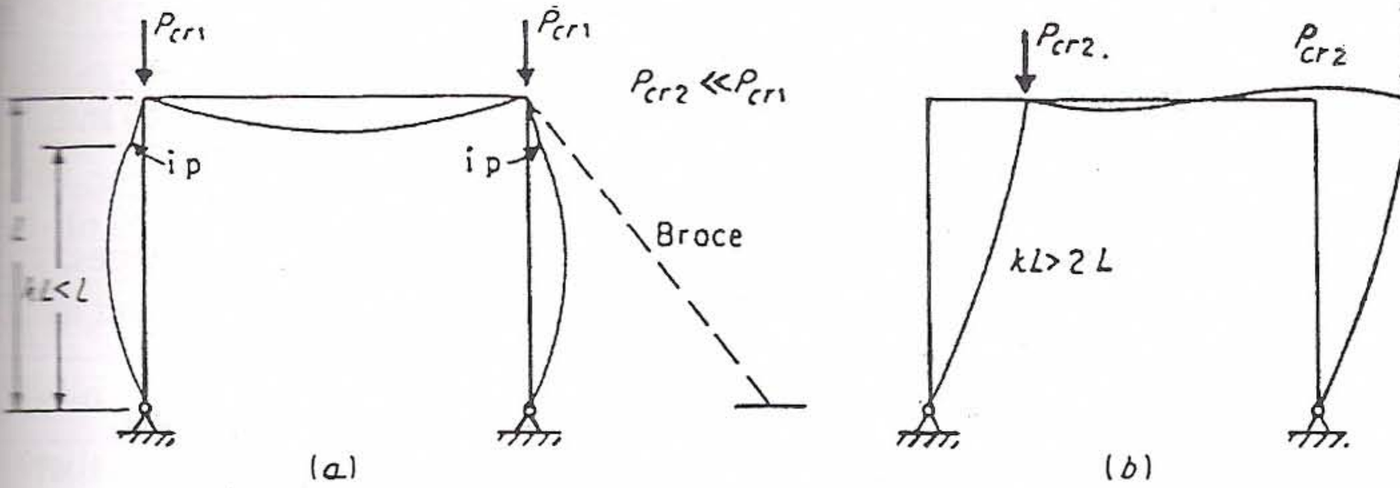
II) BUCKLING & EFF. HEIGHT OF AXIALLY LOADED <u>UNBRACED COLUMN</u>



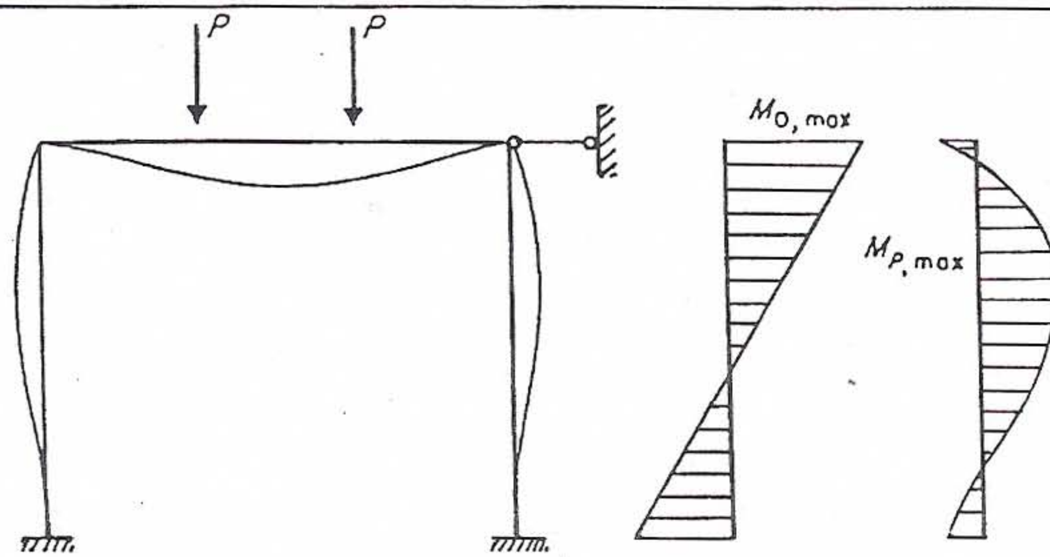
MOMENT IN SLENDER MEMBERS , COMPRESSION PLUS BENDING , SINGLE CURVATURE



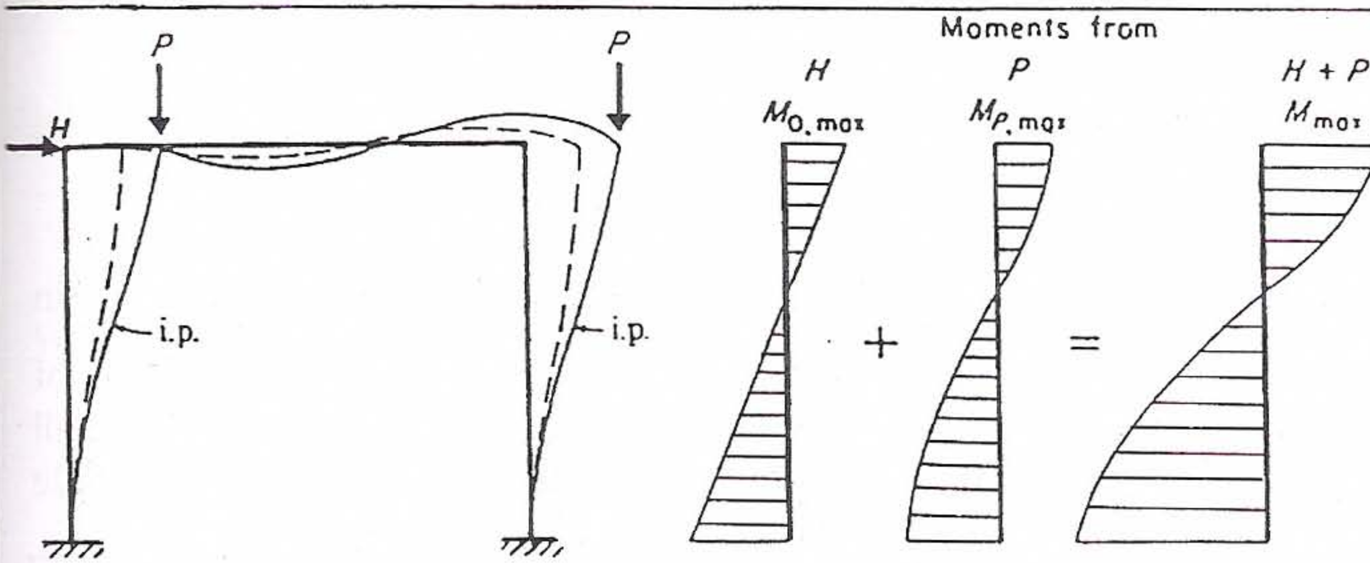
MOMENT IN SLENDER MEMBERS , COMPRESSION PLUS BENDING , DOUBLE CURVATURE



1) TWO HINGED PORTAL FRAME, LATERALLY BRACED



2) TWO FIXED PORTAL FRAME, LATERALLY BRACED



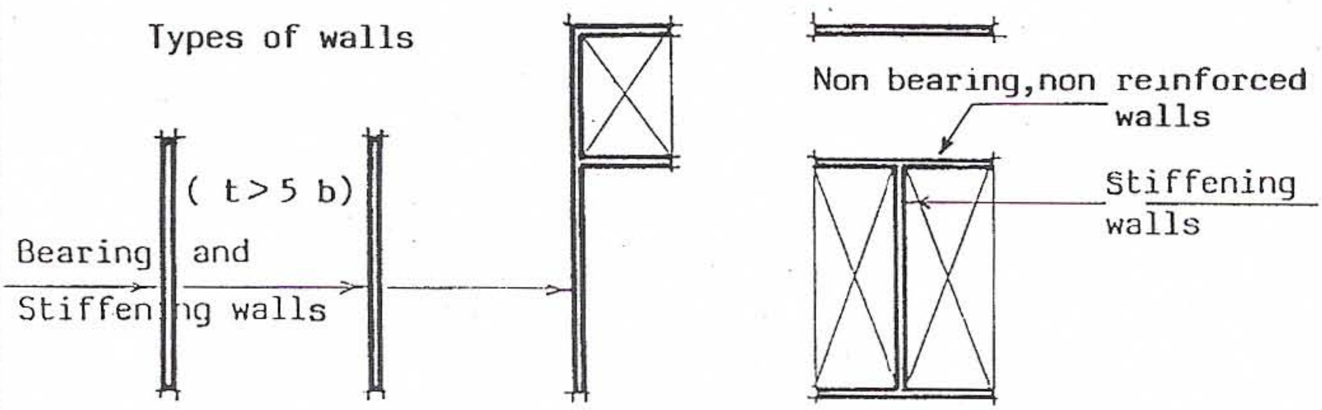
3) FIXED PORTAL FRAME, LATERALLY UNBRACED

6-5 WALLS

6-5-1 Definition

Vertical elements with t > 5b and of minimum thickness 120 mm. Walls can be divided into:

a)Bearing walls : subjected to compression forces and /or horizontal forces.
b)Stiffening walls: to support bearing wall against buckling and can act as wall bearing at the same time.
c)Non bearing walls: subject to own weight and horizontal forces only.
- Wall can be considered braced at any level if the building is braced at that level (art. 6-4-2).



6-5-2 Effective height (H_e)

H_e is to be taken the same values given in article (6-4-5-1).

6-5-3 Slenderness :

If the slenderness ratio $\lambda_b = \frac{H_e}{b}$ exceeds or equals the value given in table (6-13-a) the wall is considered slender (long) and the effect of slenderness must be taken into consideration in statical calculations. In all cases the slenderness ratio should not exceed the values given in table (6-13-b).

Condition	Table (6-13-a)	Table (6-13-b)
	λ_b (short)	λ_b (long)
Braced walls	15	40
Unbraced walls	10	30

Effective height (He = KH) can be determined as follows:

1- In case of more than one horizontal support, K can be taken as follows:

$H/L_{f2} < 0.5$	$k = 1.0$	6-46-a
$0.5 \leq H/L_{f2} \leq 1.0$	$k = 1.5 - H/L_{f2}$	6-46-b
$H/L_{f2} > 1.0$	$k=1/[1+(H/L_{f2})^2]$	6-46-c

2- In case of one bearing wall, K can be taken as following.

$H/L_{f1} < 1.0$	$k = 1.0$	6-47-a
$1.0 \leq H/L_{f1} \leq 2.0$	$k = 1.0 - 0.423\ (H/L_{f1} - 1)$	6-47-b
$H/L_{f1} > 1.0$	$k = 1/\sqrt{[1+0.5\,(H/L_{f1})^2]}$	6-47-c

Where:

H = Clear height of wall

L_{f2}= average horizantal distance between horizontal bearing.

L_{f1}=Horigontal distance between horizontal bearing and the free edge of wall

6-5-2-1-2 Simplified method for design of R.C. walls with rectangular soild section:-

The following simplified method can be used for design of solid concrete wall section with strip reinforcement, if all the following conditions are satisfied:

a) Wall section must be solid rectangular section.

b) Resultant of all ultimate loads including effect of horizontal force that acting on wall section must not be located outside the middle third of rectangular section.

c) Wall Rft. ratio must not be less than the value in art. (6-5-2-2).

d) Wall thickness	$\geq$	least of	0.04 wall heigth
			0.04 wall length
			120 mm

- In this case the ultimate load on section can be calculated according to the following equation.

Where:-

$$P_u = 0.8\ [\ 0.35\ f_{cu}\ A_c(1-(k.h/32t)^2)] \qquad (6\text{-}48)$$

Ac = Area of concrete wall section.

H = Clear wall height between bearings.

K =Factor of wall effective length which prevent the transverse transitional movement above and below the wall, and it is equal:-

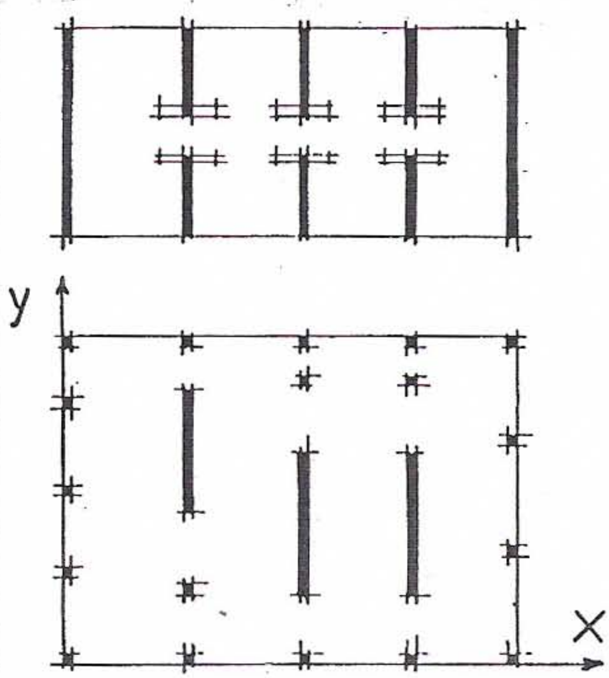
K	0.8	Walls prevented from rotation at ends upper and / or lower ends.
	1	Walls free to rotate at both upper and lower ends.
	2.0	Walls free to move horizontaly and perpendicular to wall plane.

where: t = Wall thickness.

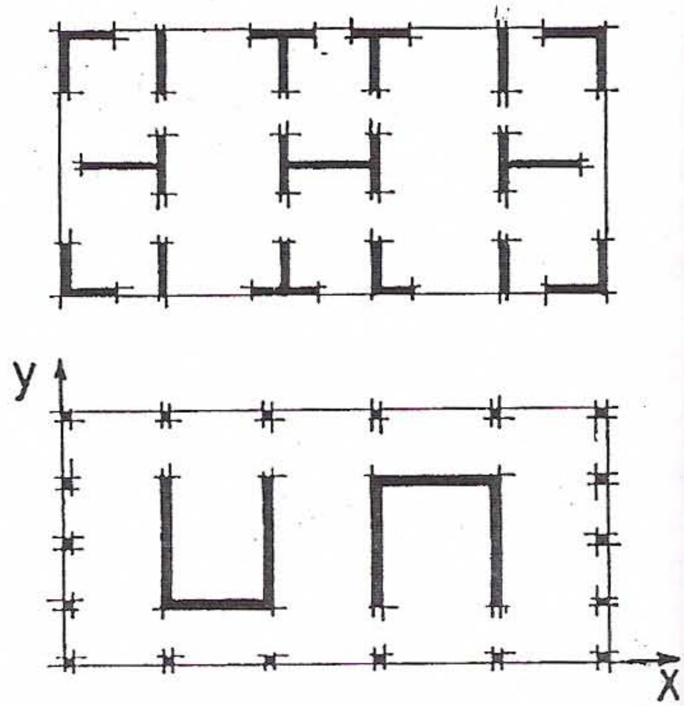
Deviation in verticality: The maximum slope in verticality should not exceed 1:15 and not more than 5mm/m` height with maximum 50mm for the total height of the building.

6-5-4 Braced and unbraced walls

Walls can be considered braced if the building is provided by perpendicular elements able to transmit the lateral forces to the principle structral bracing or to the foundations.



Braced in y- direction
Unbraced in x - direction



Braced in both x & y directions

The building is considered braced if the following condition is satisfied :

4 floors or more	$\alpha = H\sqrt{N/\sum El} \le 0.6$	**(6-46-a)**
less than 4 floors	$\alpha = H\sqrt{N/\sum El} \le 0.2 + 0.1\,n$	**(6-46-b)**

where n = nnumber of floors , N = Σ vertical loads
H = wall height above fixation level at foundations
Σ El = flexural rigidity for all vertical elements sharing in supporting the building

Note : Vertical walls sharing in supporting the building must be casted at the same time with the bearing walls or have enough connection with them. The total equilibrium of buildings with more than 4 floors should not depend in any direction on unbraced walls.

6-5-5 Minimum thickness of walls

Minimum thickness of all types of walls = 120 mm, and should not be less than:

Wall type	minimum thickness
Bearing walls	x / 25
Non - bearing walls	x / 30

Where x is the least of :
- Height between slabs
- H l distance between bracing members

6-5-6 Minimum and maximum percentage of steel

6-5-6-1 Vertical reinforcement

Vertical rft. is to be arranged in two meshes with the min. percentage given in table (6-14) for cracks control.

Table (6-14) min. vl steel ratio $\mu_{min} = (A_s / b\,d)\,\%$

Sec. subjected to	fy = 240 N/mm^2	fy = 400 N/mm^2
tensile stresses	0.8 %	0.45 %
bending moments	0.25 %	0.15 %
comp. stresses	0.4 %	0.4 %

In all cases A_s v l. rft should not be less than 0.5 % A_{creq} & not more than 4 % $A_{c\ actual}$ & of min. diameter ϕ 8 mm.

If Wire Fabric is used, the minimum diameter is 5 mm and the maximum spacing is 250 mm.

6-5-6-2 Horizontal reinforcement

Hl rft. is arranged outside vl rft. to hold it in walls subjted to compression with the following min. percentage :

$f_y = 240$	0.3 % A_c	and $\phi < \frac{1}{4}\phi$ (vl) with min. ϕ 6 mm
$f_y = 400$	0.25 % A_c	for wire mesh fabric : min. ϕ 5 mm

* In case of Vl. rft. > 1% A_c, additional closed stirrups with minimum ϕ 6 mm, or 1/4 max diameter of v1 rft. and with minimum 4 stirrups / square metre are to be arranged to hold v1 rft. on both sides of wall through wall thickness.
* In case of design the wall as a column, distance between branches of stirrups must not be more than 300 mm in both directions.

6-5-7 Horizontal displacement

The horizontal displacement must be calculated if h/L > 12 and must not be more than 1/500 of the wall height.

6-5-8 Concrete cover for steel Refer to article (4-3-2-3-b)

6-5-9 Calculation of forces on lateral supports

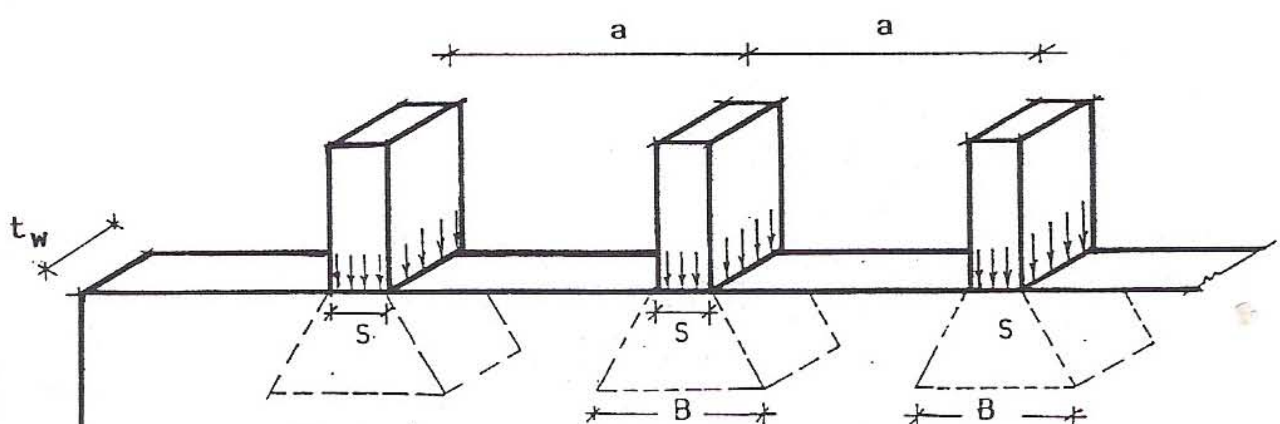
The lateral supports must be able to transmit the following horizontal forces to the foundations :

a) Static action of the total horizontal force at the lateral supports.
b) 1 % of the design ultimate vertical loads at the vertical supports.

6-5-10 Concentrated load on walls

The concentrated load is assumed to be distributed over an effective width equals to the smaller of:

i- Distance between concentrated loads.
ii- Bearing width + 4 times the wall thickness.



$B = s + 4\ t_w \leq a$

Distribution of concentrated loads on walls

6-5-11 UNREINFORCED CONCRETE WALLS

Unreinforced concrete walls are those walls not satisfying the minimum steel percentage according to article (4-5-6). In all cases unreinforced concrete walls should be of min thickness of 120 mm and reinforced by shrinkage and temperature rft. in both v1 & h1 directions with min % of :

0.3 % A_c	**for mild steel**
0.2 % A_c	**for high grade steel and wire mesh fabric**

6-5-11-1 Effective height $H_e = \beta\, h_o$

Where h_o = height between center of slabs or top of footings

β = 1.5 for walls continuous from both sides.

For other cases refer to article (6-4-5-1).

6-5-11-2 Slenderness ratio

For unreinforced concrete walls : $\lambda_{bmax} = \dfrac{H_e}{b}$ not more than 30

where b = the smaller dimension of wall cross-sec.

6-5-11-3 Minimum eccentricity of loads

The minimum eccentricity of the loads to be considered in the design should not be less than (0.05 b) or (20 mm).

6-5-11-4 Eccentricity of loads from slabs and floors

The loads of slabs supported on one side of a wall can be assumed acting on one third of the wall thickness only.

6-5-11-5 Eccentricity of loads in plane of the wall

The eccentricity is to be calculated according to rules of static.

6-5-11-6 Shear strength

Check of shear stresses on unreinforced concrete walls can be neglected if :

i - Design H1. load < 1/4 design V1. load

ii - Average working shear stress < 0.4 N/mm^2 .

6-5-11-7 Opening

For walls with opening, it is recommended to put at the perimeter of the opening 1/2 the steel area omitted at the openings with minimum 2 ϕ 12 or 2 ϕ 16 . For walls with thickness than 150 mm use rft. on both sides.

6-6 Foundations:

Footings area, or piles number and distribution are calculated by using working loads. Stresses on soil or piles loads must not exceed the allowable values, in addition to effects of differential settlement according to the Egyptian Code For Soil Mechanics And Foundations.

6-6-1 Isolated footing:

6-6-1-1 General:

Stresses, on soil can be considered uniformally distributed if footing load is acting in its centroid. Also, on the other hand, stresses on soil can be considered linearly distributed if footing load is eccentric.

6-6-1-2 Design of footings to resist flexure:

1- Critical sections for flexure:

Critical sections for flexure is taken as vertical section passing through footing at:

- Column face, R.C wall face connected to footing fig. (6-24-a).
- At the mid distance between column edge and edge of steel plate that rested on R.C footing below column fig. (6-24-b).
- At the mid distance between middle and edge of masonry wall rested on the footing fig. (6-24-c).

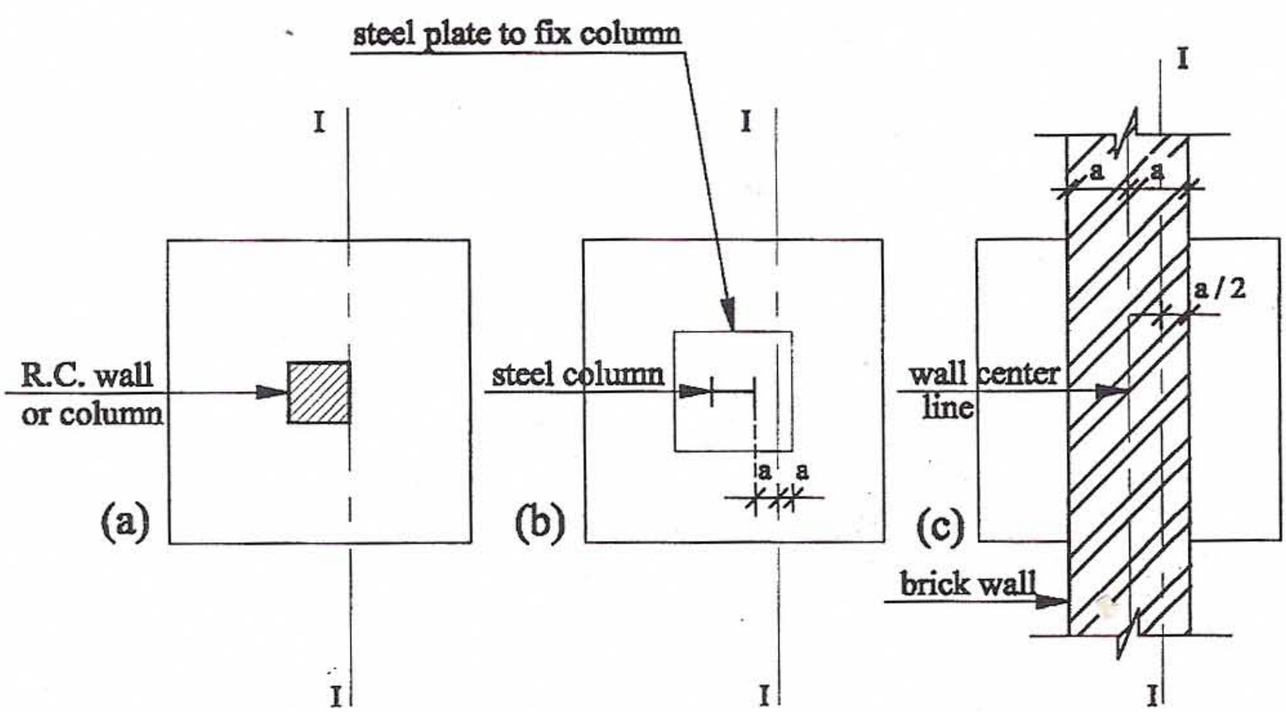


Fig. (6-24) Sec (I-I) Critical section for bending moment

2- Bending moments at critical sections are calculated by taking the moments of all forces acting on the footing at one side from the critical section.

3- For square footings, steel rft. is distributed uniformally overall footing breadth in the two directions, and it can be distributed according to bending moment diagram.

4- For rectangular footings, steel rft. is distributed uniformally or according to bending moment diagram, and it can be distributed as in figure (6-25) according to the following:

- Steel rft. is distributed uniformally in the longitudinal direction.
- In short direction, steel rft. is concentrated in a distance centralized with the column as in fig. (6-25).

The ratio of steel rft. in the concentration zone (A_{sm}) to the total rft. in short direction (A_s):

$$(A_{sm}/A_s) = 2/[(a/b)+1] \quad (6\text{-}49)$$

5- Footing sections are designed according to design conditions of limit states theory (art. 4-2-1), or theory of elasticity art. (5-3-2).

Where: a= Footing long dimension

b=The bigger of footing short direction, or (column length (in b direction) and footing depth).

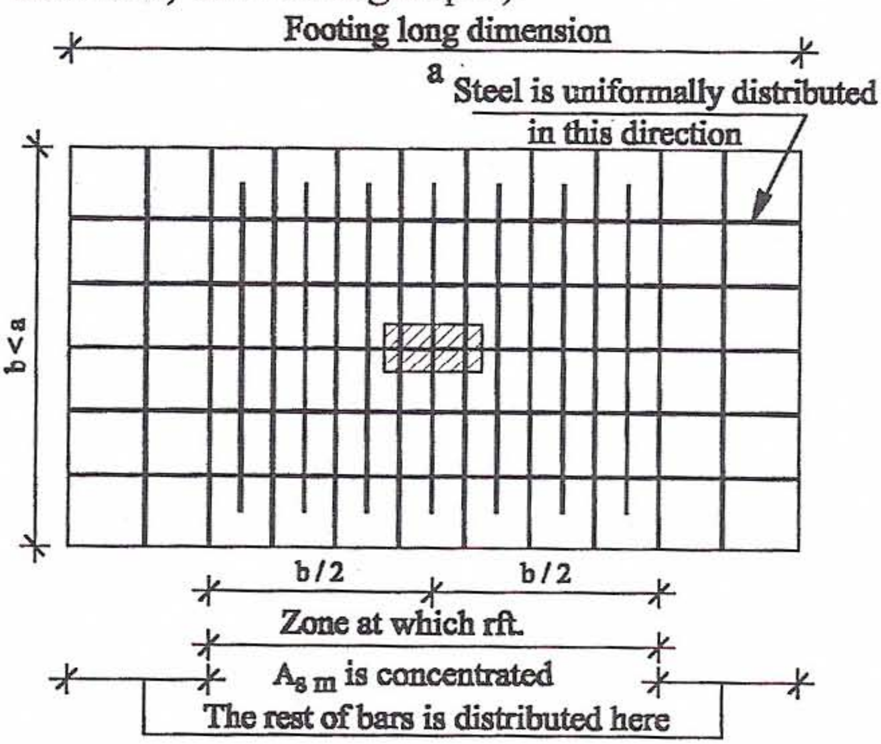
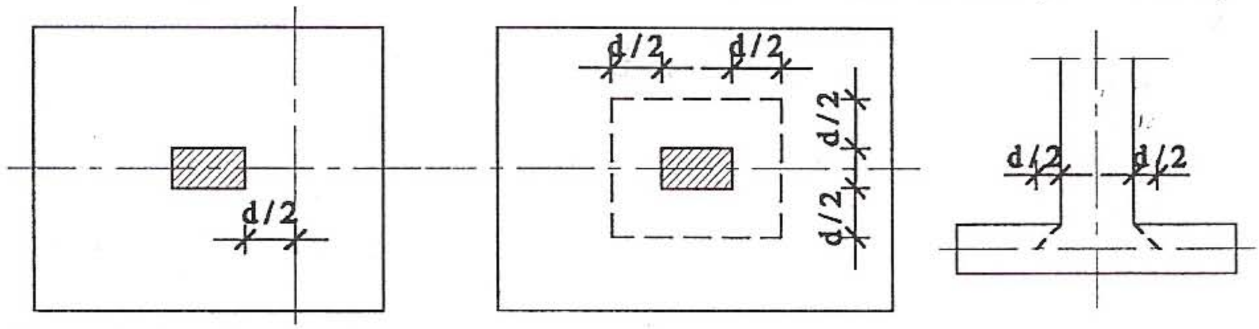


Fig. (6-25) Distribution of steel rft. in rectangular footings

6-6-1-3 Design of footings to resist shear and punching shear forces:

1) Critical sections for shear are taken according to conditions of art. (4-2-2-1-1) and fig. (6-26-a), also critical sections for punching shear are taken according to art. (4-2-2-3) and fig. (6-26-b). In case of using steel plate to fix the column on the concrete footing, the distance d/2 is taken from the position of the critical section determined in art. (6-6-1-2-1).



(a) Critical section for shear (b) Critical section for punching shear

Fig (6-26): Critical sections of shear

2) Shear resistance of footing sections is determined according to art. (4-2-2-2), also punching shear resistance of footing is determined according to art. (4-2-2-3).
3) Shear stresses in pile caps can be calculated according to the following:
- In shear calculations, the total reaction of pile is taken into consideration if pile center is at distance greater than or equal to pile radius outside the critical section fig. (6-27)- case (a).
- Pile reaction is neglected if pile center is at distance greater than or equal to pile radius inside the critical section fig. (6-27) Case (b).
- In other cases which lay between the previons cases, the value of reaction taken into consideration is calculated by linear interpolation between the two previous cases – case (c) in fig. (6-27).

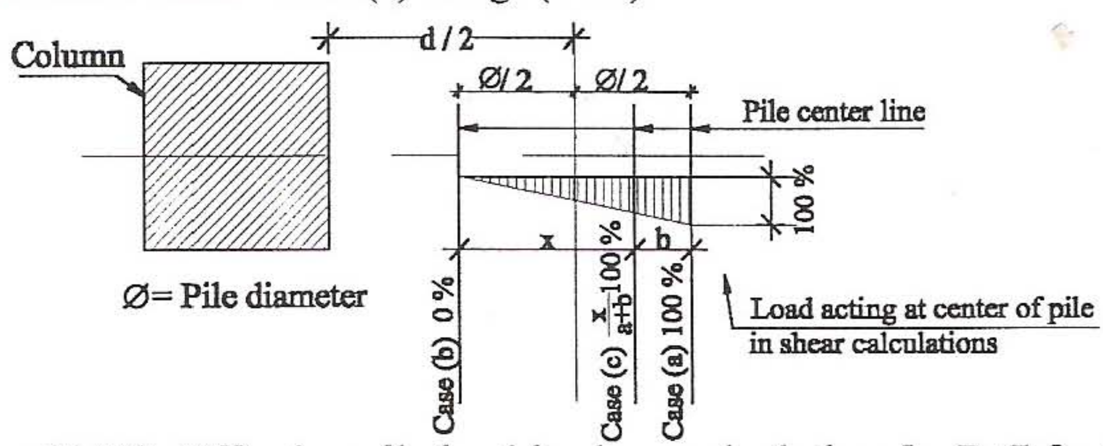


Fig. (6-27): Effective pile load in shear calculation for R.C footing

6-6-1-4 **Design of pile caps using space truss method:**

1) The truss has triangular shapes, its sides connecting between loading center and intersection points between piles centers and main rft. which form the rest of truss tensioned members.
2) If the distance between piles centers is greater than three times pile diameter, the main rft. That is centralized in a distance equal to 1.5 pile diameter (measured from pile center) can be taken in the calculation of resistance of the truss tension member.

5) Min thickness	Isolated footing	Pile cap	$\geq$ column width
	300 mm	400 mm	

6-6-1-6 Development length:

Development length and anchorage length for footing rft. Are determined according to art. (4-2-5).and the critical sections for development length is assumed at the same position of critical sections in art. (6-6-1-2-1).

6-6-2 Combined Footing And Raft Foundation:

1) Combined footing is designed according to relative rigidity between foundation and soil. It is not allowed to analyse combined footing and Mat foundation by the analysis method for slabs (art. 6-2) or flat slab method (art. 6-2-7), and also the beams analysis method (art. 6-3).
2) Combined footing and Mat foundation are considered rigid and stress distribution below them are linear if:

a) Relative rigidity $k_r = E_c.I_B/(E_{soil}.b^3) \geq 0.50$ (6-50-a)

Where: E_c = Concrete modulus of elasticity.

I_B = moment of inertia (for foundation only or foumdation, frames, and shear walls) for one unit of strip length.

E_{soil} = Soil modulus of elasticity.

b = Strip width.

b) Average distance between any two adjacent columns in the two directions < 1.75/β.

Where β is a coefficient:

k:	Winkler coefficient of soil reaction	$\beta = \sqrt[4]{k.b/(4E_c.I}$	6-50-b
E_c.I:	Flexure rigidity of strip section		

Structural analysis is done by satisfing the equilibrium of loads with soil reactions.

3) Combined footing and Mat foundation are considered elastic if the conditions in art. (6-6-2-2) are not satisfied, and the analysis is done by considering it as an elastic slab supported on winkler base or semi-infinite elastic media considering the actual properties of soil. In addition, design shall satisfy stability conditions and compatibility of strains.

6-6-3 Foundations Subjected To Seismic Loads:

6-6-3-1 Footings, Mat. foundation, plie-caps:

1) Rft. bars of columns and R.C walls must be extended inside foundation (footing, Mat, pile-cap) to a distance not less than tension bars development length measured from connection surface between columns (or walls) and foundation, also rft. bars must be extended to the footing bottom rft. with angle.
2) Piles rft.must be extended inside pile-cap to a distance not less than development length of tension bars measured from connection surface between piles and pile cap.
3) If the column (rested on footing or pile-cap) is subjected to tensile forces due to seismic load ,a top rft. must be used to resist bending moments due to these loads.
4) R.C foundation should be used for building located in seismic zones II & III

6-6-3-2 Smells and slabs resting on soil:

1) Smells are designed as a part of the structural elements resisting lateral loads due to earthquake according to conditions in art. (6-7). These elements are designed as a tie connecting foundation. Rft. must be extended overall smell length, and it must extended beyond column C.L. to a distance not less than development length.
2) Art. (6-6-3-2-1) is valid for slabs rested on soil if it is considered as a part of the structural elements resisting lateral loads due to earthquakes.
3) Smell width (b) must be as:-

$L/20 \leq b < 450$ mm, Where: L is the smell clear span, Stiffness limit in art. (6-3-1-8) must be satisfied.

6-6-3-2 Piles:

1- Piles must be designed and reinforced with adequate longitudinal rft. to resist forces and moments due to seismic loads taking into consideration soil properties, also condition of stirrups area and spacing must be satisfied.

3) Pile stirrups must be increased in the following zones:

A- At the joint between pile and the cap to a distance equal to 5 times pile diameter inside the pile and not less than 2.0m measured from the lower surface of pile-cap.

B- At zones which the soil can't peovide side bearing to the pile. and, where soil properties is completely different, volumetric ratio of spiral stirrups must not be less than mentioned in art. (6-7-3-3).

6-7 CONDITIONS FOR RESISTING SEISMIC LOADS:

6-7-1 General:

Strutural systems for earthquake resistance include frames, structural walls, and stringers. One building can contain more than one system.

1) Definition of structural element resisting seismic loads.

Frames are the space structural elements resisting seismic loads its elements and joints resist bending moment, shear, and axial force.

Conditions of art. (6-7-2) are applicable for non-ductile frames,

Conditions of art. (6-7-3) are applicable for ductile frame.

Structural wall is a walls designed to resist shear, bending moment, and axial forces due to seismic loads. It includes reinforced and semi reinforced concrete walls.

2) Seismic loads calculation and determination of seismic zones are according to Egyption code for loads and forces.

Analysis, design, and detailing of structure are done according to chapters.	Structures in zones one & two	Structures in zones three & Four
	3, 4, 6, 7 except articles (6-7-2), (6-7-3)	3, 4, 6, 7 inxluding articles (6-7-2), (6-7-3)

3) Article (1-1-1) must be considered in design of special structures.

6-7-2 Additional Conditions For Non-Ductile Frames:

1- Flat slabs resisting seismic loads:

a) All moments transferred from slab to column are resisted by column strip only.

b) Negative moment (γ_f M_u) given in art. (6-2-7-7) is resisted by effective width of slab (column width + 3 t_s (slab thickness) as shown in fig. (6-28).

c) Rft. of the effective width of slab must be ≥ ½ column strip rft.

d) At least ¼ of column strip top rft. must be extended over the span (fig. 6-29-a).

e) At least ½ of column and field strips bottom rft. must extend over the span (fig. 6-29-a-b) taking into consideration continuity of rft. inside supporting zones with sufficient anchorage length according to art. (4-2-5-3).

f) Bottom continuous rft. (over the span) in column strip must be ≥ 1/3 top rft. of column strip at supporting zones.

g) For non-continuous slab ends, bottom and top rft. at edge support must extend inside supporting zone be a sufficient length according to art. (4-2-5-3).

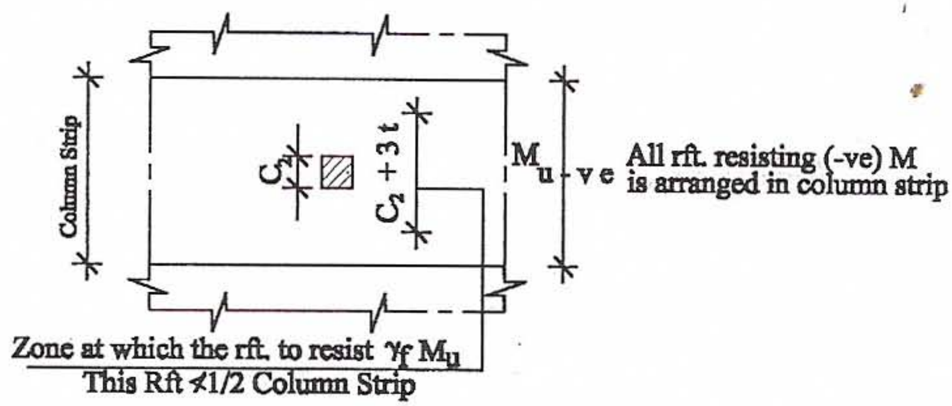
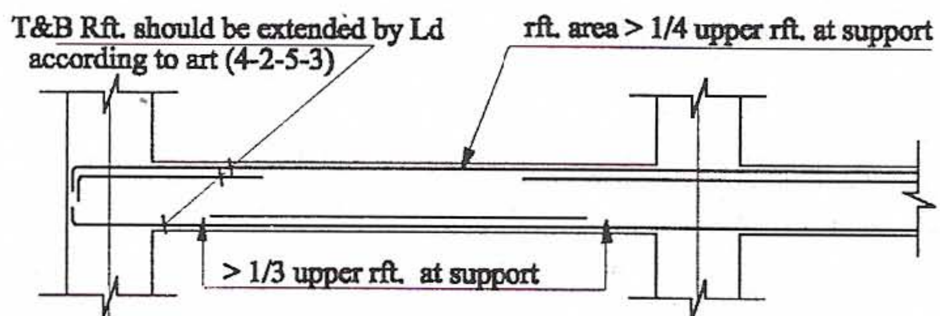
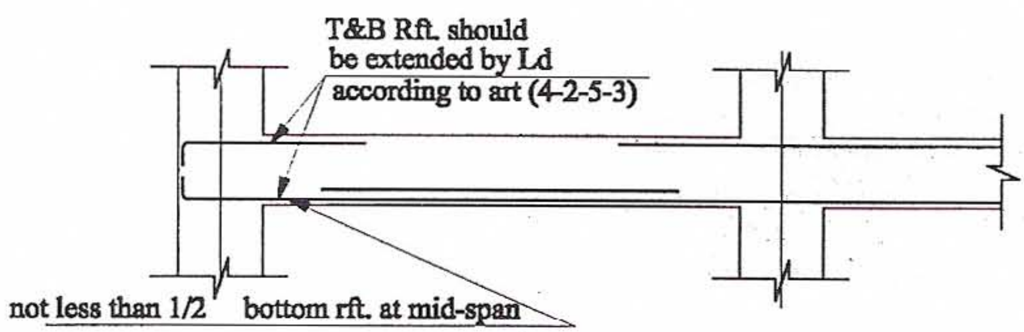


Fig. (6-28): Flat slab effective width



a) Column strip



b) Field strip

Fig. (6-29): Arrangement of rft. in flat slab

2) Beams of R.C frames for resisting seismic loads:

R.C. beams subjected to bending moment or combined bending and axial force $\leq 0.04 f_{cu} A_g$, are designed as following:

a) Beam section at support face is designed to resist ultimate postive moment not less than 1/3 ultimate negative moment resulted in the structural analysis of the structure.

b) Postive or negative moment resistance at any beam section must not be less than 1/5 the max. moment at any face of the two supports.

c) Stirrups in a distance equal to twice beam depth measured from face of support are distributed where the distance between first stirrp and face of support must be less than 50 mm, and the spacing between stirrups is less than the least of the following:-

- ¼ beam depth.
- 8* times smallest long bar diameter in beam section.
- 24* times stirrup diameter.

d	Spacing between stirrups must be less than	0.5 beam depth ≮ 200mm

3) Columns of R.C. frames resisting siemic loads:

Spacing between stirrups must be less than S_o for a distance equal to L_o measured from face of connection between column and beam (or foundation) at both column ends (Fig. 7-6-b).

L_o = the greatest of	1/6 clear height of col.	< S_o	= the least of	8. $\phi_{min.}$ long.
	Col. Big dimension			24. Col. Stirrup diameter
				½ col. Small dimension.
	500 mm			150 mm

The first stirrup is at a distance ($S_o/2$) from face of connection between column and beam. Spacing between stirrups along the rest of column height must be less than $2S_0$ and stirrups must extend inside beam by the same distance S_o.

6-7-3 Additional Conditions For Ductile Frames:

6-7-3-1 Frame beam:

R.C. beams subjected to bending moment with out or with axial compression $\leq 0.04 f_{cu} A_g$, are designed according to art. (6-7-2-2) in addition to the following:

1- Beam width to depth ratio ≯ 0.30.

2- Beam width ≥ 250 mm, ≯[Colmn width + 2 * 0.75 beam depth].

3- Section resistance to positive moment at face of support must not be less than 50 % of its resistance to negative moment at the same face of support.

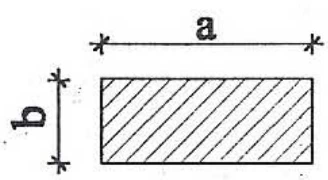
In all cases, section resistance to moment (+ve or – ve) at any section must not be less than ¼ biggest corresponding resistance at support face and at least 2 bars (upper and lower) must be extended along beam length.

4- In case of Lap splices, the following conditions must be satisfied in splice zone:

- Beam transversal rft. in splice zone consists of closed or spiral stirrups.
- Spacing between stirrups in Lap splice zone must be less than the least of the following: ¼ beam effective depth | or | 100 mm
- Its not allowed to make Lap splice at the connection between beam and column, and also at the position where steel rft. may be reach yield stress.
- Distance between beginning of lap splice and face of adjacent support must not be less than twice beam effective depth.

6-7-3-2 Columns of frames:

a- **Column dimension** **a > b**



$b \geq 300$ mm	
$b/a > 0.40$	

b-If $P_u > 0.04\ A_g\ f_{cu}$, ultimate moment strength of frames columns connected with beams must satisfy the following relation:

$$\Sigma M_c \geq 1.2\ \Sigma M_g \quad (6\text{-}51)$$

ΣM_c = Sum of ultimate moment of resistance of columns at joint with beam, calculated at column face by using the value of the axial load that gives min. moment resistance.

ΣM_g = Sum of ultimate moment of resistance of beam at joint with column, calculated at column face. For T-section,.when the slab is subjected to tension due to moment, ultimate resistance of beam is calculated by considering a part of slab width is sharing in the ultimate resistance with condition that slab rft. must be extended with a suitable anchorage length after the critical section. Summation of moment resistance is done so that moment resistance of columns is in the reverse direction of moment resistance of beams.

If this previous condition is not valid, column effect is neglected in calcution of rigidity and ultimate resistance of structure, however conditions of column stirrups in art. (6-7-3-3) must be satisfied.

c-

Column rft. ratio	$> 1\,\%$
	$\leqslant 4\,\%$

d- Lap splices are allowed only at the middle of column height.

e- Lap splices must be designed as tension Lap splices. It is allowed to use weld splices or mechanical splices at any section, with satisfying reqvirements of art. (4-2-5-4-3). considering that more than (¼) bars must not be spliced at the same section, and the distance between splices must not be less than 600 mm, also weld splices must satisfy standard specifications requirements.

<u>6-7-3-3 Joints Frames (connection between frames beams and columns):</u>

a- Forces in long. Rft. of beams are determined at face of column by taking tensile stress in steel rft. equals to 1.25 yield stress.

b- Joint resistance is determined by taking the suitable reduction factor of resistance according to art. (3-2-1-2).

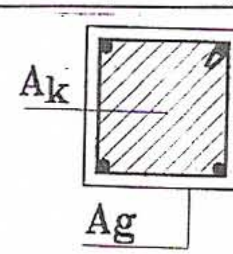
c- Long. Rft. of beam (which stops inside the column) should extend to the far face of column and anchored with suitable development length according to art. (4-2-5-1).

d- If beam rft. is extended through the joint, column dimension in the beam direction must not be less than 20 times largest Rft. diameter.

e- Columns stirrups must extend inside the joint and stirrups area must not be less than:

Greater of	$A_{st} = 0.24\,(s.y_1.f_{cu}/f_{yst})[(A_g/A_k)-1]$	(6-52-a)
	$A_{st} = 0.07\,(s.y_1.f_{cu}/f_{yst})$	(6-52-b)

Where:



A_g = R. Csection area.

A_k = Cross-section area inside stirrups perimeter (core area).

f_{yst} = yeild stress of stirrups.

S = Spasing between stirrups.

Y_1 = Dimension of column core measured from stirrups centerlines.

A_{st} = Total area of stirrups section including crossties through distance (S) and perpendicular to y_1.

f- If beam width is greater than the column width and there is no other intersecting beams, a transversal stirrups must be used through beam depth to anchore the beam long. Rft.

g- Shear force at frame joint must be	$< 0.96\ A_j \sqrt{f_{cu}}$

Where:

A_j= Area of the effective section through the joint in a plane parallel to the plane of the rft. Perpendicular to shear plane in the joint zone.

- Depth of joint zone is the total depth of column.
- Effective width of frame joint must not be less than:

- Beam width + depth of frame joint.
- 2 times the smallest perpendicular dimension measured from beam axis to column edge, and equal to (b+2x) according to fig. (6-30).

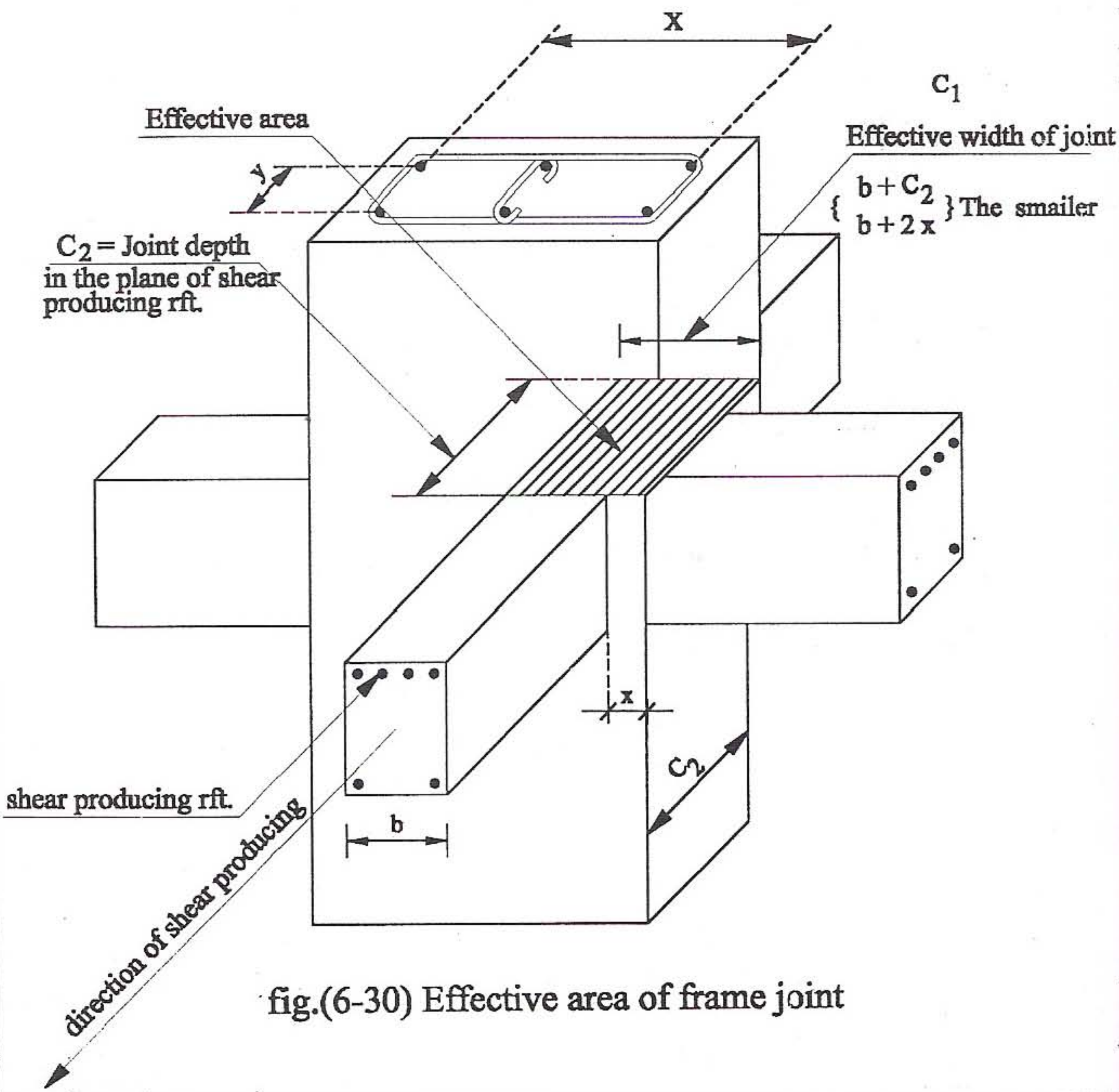


fig.(6-30) Effective area of frame joint

6-8 PRE-CAST CONCRETE:

Pre-cast concrete units are designed according to this article, in addition to all code articles which fit with this article. This article is not sufficient for seismic design.

6-8-1 General:

Pre-cast concrete elements and joints are designed to resist all external loads acting on the element during manufacting, storage, transportation, construction, and using stages, also stresses due to end restraint.

6-8-2 Distribution of design forces between elements:

1-Forces perpendicular to elements plane are distributed according to structural analysis, or empirical test.

2-Forces are transfered between pre-cast slab elements or walls in the same plane according to the following requirements:

a- Continuity of forces transition path in the plane through elements and joints.

b- A continuous path should be provided by using steel rft. to resist the generated tension forces.

c- Splices, ties, and bearing zones are designed to resist all transfering loads including Special Forces due to elastic deformation, creep, shrinkage, or temperature.

6-8-3 Reinforcement of pre-cast elements:

Walls	V^L and H^L Rft. ratio ⩾ 0.25%.
Slabs	Reinforcement Ratio in any direction ⩾ 0.15%.

6-8-4 Structural integrity:

6-8-4-1 Pre-cast concrete structures with two floors, or less should satisfy thr following conditions:

1-Longitudinal, transversal, and vertical ties should be used to ensure connection of the pre-cast elements to structural system resisting lateral loads.

2-For pre-cast slabs behave as rigid horizontal diaphragms, nominal ultimate tensile strength of the connection between these slabs and the vertical elements resisting lateral loads is not less than 4.5 KN/m.

3-A vertical ties must be used in all vertical structural elements, by using splices at horizontal joints according to:

a- Nominal ultimate tensile strength of pre-cast columns must not less than 1.4 A_g (N), where A_g is the required area of the concrete section (mm^2).

b- For pre-cast walls, two ties at least must be used in the wall and the nominal ultimate strength should not be less than 45 KN for each tie. These ties are symmetric about the V^L axis of wall and lay in the external quarter of wall (if possible).

4- Friction resistance due to permant vertical loads must be neglected in the design and detailing of joints.

6-8-4-2 Pre-cast bearing walls structures with height more than two floors, must satisfy the following conditions:

1- Slab structural system should have long. And transversal ties to generate nominal ultimate strength not less than 22 KN/m. these ties should be at bearing zones of internal walls and also between structure elements and external walls, and it should be arranged in a distance not more than 600 mm from floor (or slab) level.

2- Longitudinal ties parallel to slabs spans are arranged at distances not more than 3.0 m, precautions should be taken to ensure forces transition around holes.

3- Transversal ties perpendicular to slabs spans are arranged at distances not more than distance between bearing walls.

4- Ties around external perimeter of each slab are arranged in a distance equal to 1.20 m from slab edge, and it must produce tensile strength not less than 70 KN.

5- Vertical ties are used in all walls, and also it should be continuous through building height. These ties should produce a nominal ultimate tensile strength not less than 40 KN/m of wall length, and each wall should have two ties at least.

6-8-5 Design of joints and bearing zones:

6-8-5-1 Forces can transfere between elements injected joints, or shear connectors, or mechanical splices, or steel rft. splices, or reinforced topping. Mechanical splices are preferable with injected joints or shear connectors in the structure with 3 floors or more.

6-8-5-2 Validity of joints to transfere forces between elements can be determined by analysis or empirical test and when shear is the main acting load, all conditions of art. (4-2-2-4) must be satisfied.

6-8-5-3 In the design of joint with different materials, the relative rigidity of materials, max. strength and ductility must be taken into consideration.

6-8-5-4 If pre-cast slab elements are simply supported, the following conditions must be satisfied:

1-Bearing stresses between contact surfaces must not exceed bearing strength between any of the contact surfaces and supporting element. Concrete bearing strength can be determined according to art. (4-24) or art. (5-6).

2-If the structure analysis or empirical test doesn't prove that there is a defect in the structural behavior of the connection or bearing zones of pre-cast elements, the following conditions must be satisfied:

a- Design dimensions should satisfy that distance between support edge and the end of the pre-cast element ≥ 1/180 of element clear span considering tolerent but not less than: Fig. (6-32).

For slabs	50 mm
For beams	75 mm

b- Bearings are used for non-stiffed ends on a distance not less than 15 mm from face of support.

3-For statically determined pre-cast elements, 1/3 of positive moment rft. must be extended to the middle of supporting length.

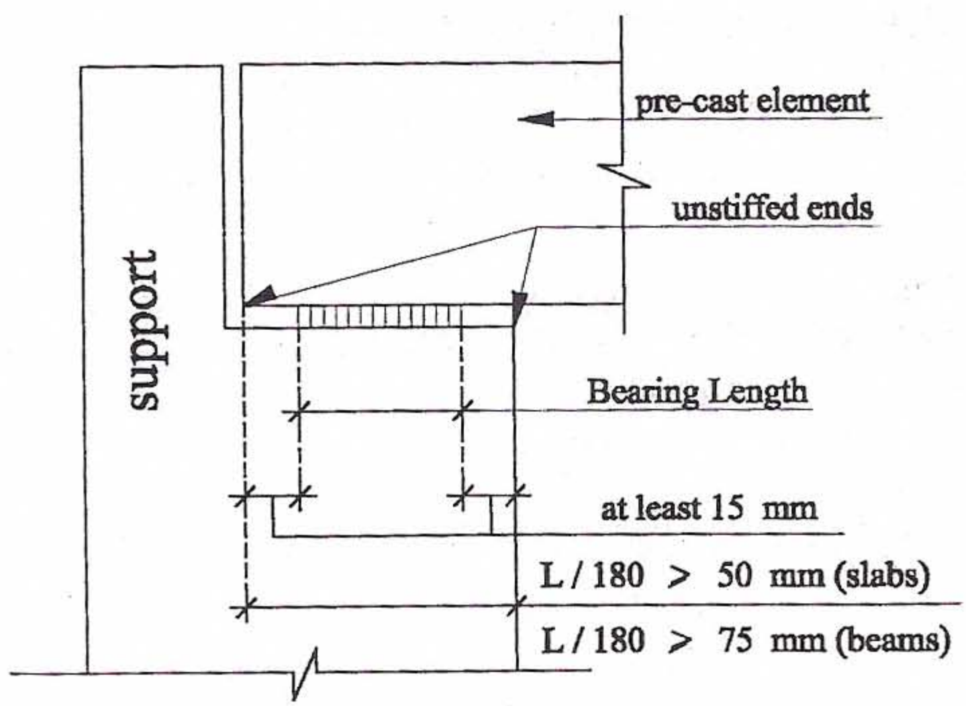


fig.(6-32) Bearing length for pre-cast element

6-8-6 Imbedded parts after casting concrete:

Imbedded parts can be fixed during elasticity stage of concrete as dowels and accessories, which are protruding from concrete surface, but the following must be satisfied:

1- Imbedded parts should not be fixed with concrete rft. or with hooks ends.
2- Imbedded parts should be fixed correctly during elasticity stage of concrete.
3- Concrete should be compacted well around the imbedded parts.

6-8-7 Numbering and differentiation:

1- Each pre-cast element should be numbered to indicate its position and direction in the structure, also its manufacturing date.
2- Signs should be identical to assembling drawings.

6-8-8 Handling:

1- In the design of pre-cast elements, all forces and distortions due to curing, storage, handling, and installation should be taken into consideration.
2- Pre-cast units should be fixed during assembling by using equipments to prevent its movment until finishing the cast of permenant joints.

6-8-9 Evaluation of pre-cast elements strength:

1- Pre-cast elements with cast-insitu concrete, can be tested by loading the pre-cast element only according to the following:
   a- Loading can be done by test loads only if the calculations indicate that the pre-cast element (individually) will not be critical for compression or buckling.
   b- Test load is the load when applied on the pre-cast element individually, gives the same total tensile force in tensile rft. which will be produced when the installed element is loaded by test load according to art. (8-7-7).
2- The pre-cast element is considered acceptable if the conditions in art. (8-7-7) are satisfied.

6-9 Mathematical model and check model for computer modelling of structures:

6-9-1 Conditions which must be satisfied in mathematical model:

It is allowed to use numerical analysis methods by using computers (as finite elements analysis, or any other numerical methed) to determine the internal forces in structures. The used method must satisfy conditions of equilibrium and compatibility of strains in addition to the following conditions that must be satisfied:

6-9-1-1 Geometric conditions:

1- If finite elements analysis is used, rectangularity ratio of grid elements shouled be chosen so that doesn't affect results accuracy.
2- Lines of the grid used in numerical methods is prefered to pass through columns by using grids with different dimension according to solution requirement.
3- The mathematical model should simulate the actual behavior of the structure (geometric shape, supports, loads, and end restraints cases).

6-9-1-2 Structural conditions:

1- Loading path and its transfer from element to another up to foundations must be checked.
2- Different loading cases should be considered to get the max. stresses at any section.
3- Deflection must be checked, taking into account effect of cracking and creep according to art. (4-3).
4- In structures and especially slabs, column rigidity in the two directions and also beams rigidity must be considered.
5- In case of representing slab beams in the two directions as a grid, all internal forces (bending moment, axial force, shear, and torsion) must be considered in the design.
6- Effect of cracks can be taken into consideration in the structural analysis. If it is difficult to take cracks into consideration, moments and the resultant internal forces can be redistributed taking into consideration the expected effects of cracks on the structural behavior of the structure and also on determination of areas and directions of steel rft.

6-9-2 Revision of computer analysis results:

1-Input data (geometric, shape, restraints, loads, materials properties) which used in the analysis must be reviewed, in addition to the general equilibrium of the structure.

2-Results should be compatible with equilibrium requirements between loads, internal forces, compatibility of strains, deformation, expected behavior of the structure, and restraint shapes.

3-The used program in the analysis shoud gives results compatible with the traditional methods with an acceptable accuracy.

6-9-3 Slabs:

In addition to articles (6-9-1) and (6-9-2) the following must be considerd:

1-Slabs elements are represented by using shell elements or plate bending elements.

2-In case of representing slabs by shell or plate elements and representing beams by frames elements, the eccentricity between the element representing the slab and the axis of rigidity of the element representing the beam must be modeled. This eccentricity can be taken into consideration by assuming that the beam element has a rigidity equal to the rigidity of T or L-section with flange width equal to half the flange width in art. (6-3-1-9).

3-Steel rft. can be arranged in perpendicular strips where the distribution of the rft. in each strip is uniform. Bending moment for the strip calculated as follows:

$M_x = (m_x) + (m_{xy})$	6-53-a
$M_y = (m_y) + (m_{xy})$	6-53-b

Where: m_x, m_y are the ultimate bending moments per meter in x,y direction respectively, with candition that m_x or $m_y \leq 1½$ average bending moment in the strip.

- (m_x), (m_y) are the absolute values of the bending moment per unit length in the strip.
- (m_{xy}) is the absolute values of the torsional moment per unit length in the strip.

4-Restraints due to column in the horizontal plan can be distributed on column area and the design is done by using the moments obtained at column faces, but if the column is represented by one point, the design is done by using negative moment value at column perimeter.

5-Generally, main rft. can be arranged in the direction of main tensile stresses with allowable variation about ±15°, or in two perpendicular directions as previous.

6-9-4 Mat foundation:

In addition to art. (6-9-1) and (6-9-2), the following must be considered:

1-Column load in the horizontal plane can be distributed on column area, and the design is done by using the bending moment at columns faces.

2-Soil must be modelled according to code of foundations taking into consideration the allowable values of settlement and stresses on soil.

3-Raft is considered rigid if the conditions in art. (6-6-2-2) is satisfied.

4-In raft analysis, it must be checked that there is no tensile streses between the raft and soil springs.

6-9-5 Beams, columns, and frames:

1- Columns can be analyzed by using a second degree structural analysis (P-Δ effect), but the design moments obtained from this analysis must not be less than the given chapter (6).

2- If frames are analysed as space frames, all forces acting on the section at the same case of loading must be considered.

6-9-6 Deep beams, short cantilevers, and walls:

Numerical methods can be used in the calculation of stresses and strains in deep beams, short cantilevers, and walls, considering that results must not be less than the design values given in this code.

Chapter (10)

Prestressed Concrete

Prestressed Concrete

Types

1- Pre-tensioned
2- Post tensioned
2-1 Bonded prestressing tendons
2-2 Unbonded prestressing tendons
2-2-1 Internal tendons
2-2-2 External tendons
3-1 Full prestressing
3-2 Partial prestressing
4-1 linear prestressing
4-2 Circular prestressing

Loading stages

1- Transfer stage (Jacking)
2- Service load (working) stage
3- Construction stage
4- Cracking load stage
5- Ultimate stage

Design

1- Flexure
2- Axial force
3- Eccentric force
4- Shear force
5- Torsion force

Deformation

1- Elastic deformation
2- Plastic deformation
- deflection
- Camber
- Shrinkage
- Temperature change

Losses

a) Short term losses
- Anchorage slip
- Elastic shortening
- Friction losses

b) Time dependent losses:
- Shrinkage
- Creep
- Steel relaxation

Design of end block
Transfer length
Development length

Buckling

- Thin web and flange
- Unbounded tendons
- External prestressin

10-2 Materials of prestressed concrete:

10-2-1 Concrete:

1) General

Concrete used in prestessed structures should have very high compressive strength to reduce own weight, volumetric deformations (creep & shrinkage), losses and to satisfy limit states requirements.

3) Concrete grade: (characteristic compressive strength)

Number of test results that give a concrete characteistic strength less than 4 N/mm^2 must be less than 1 %.

Table (10-1): Grades of prestressed concrete (f_{cu} N/mm^2)

Concrete grades (N/mm^2)	30	35	40	45	50	55	60
4) Concrete compressive strength at transfer stage must not be less than	Pre-tensioned				Post-tensioned		
	25 N/m^2				22 N/m^2		
5) Safety margin of Mix design (M) (art. 2-5-3)	$M = 2.33\ s - 4.0\ N/mm^2$ (10-1)						
	s = standard deviation N/mm^2.						

10-2-2 Prestressing steel tendons:

Table (10-2): properties of prestressing steel

Steel type	Min. yeild stress / tensile strength	Min elongation at failure %	measurement length
Strands	≮ 85 % *	≮ 3.5 %	610 mm
Wires	≮85 % *	≮ 4 %	250 mm
Smooth bars	≮ 85 % **	≮4 %	20 ∅
Deformed bars	≮ 80 % **		

* Yeild stress is the stress results to an elongation of 1 %.

** Yeild stress is the stress equivalent to residual strain of 0.20 % determined by using elongation under load method.

10-3 Design of prestressed concrete elements:

10-3-1 Basics of design:

1) Prestressed elements are to be designed according to limit states design method. Ultimate strength safety margins (γ_c&γ_s) are taken according to chapter (3), but the symbol γ_s is replaced by γ_{ps} for prestressing steel.
2) For prestressed concrete sections subjected to flexure or eccentric force, the design is to be carried out according to article (4-2-1) taking into consideration the stress-strain relationship of the steel used as shown in Fig. (10-1).
3) Relation between design ultimate tensile strength (f_{pu}) & steel yield strength(f_{py})

Steel type	f_{py} / f_{pu}	
Deformed bars	0.80	(10-2)
Normal relaxation stress relieved strands, wires and smooth bars.	0.85	(10-3)
Low relaxation stress-relieved strands and wires	0.90	(10-4)

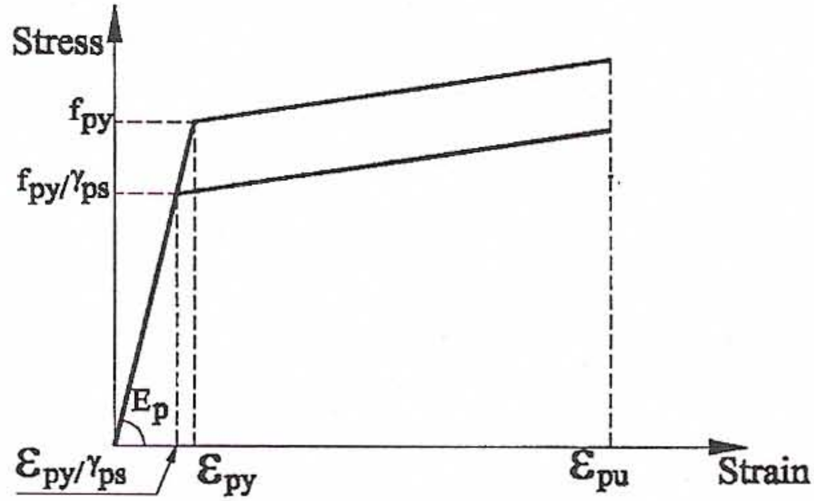


Fig (10-1) Idealized stress and strain curve for prestressing steel.

10-3-2 Working limits requirement:

1) Elastic analysis method (where stiffness of the section are calculated for the full section neglecting cracks) can be used to check stresses at: Transfer stage - construction stage - Working stage - Cracking stage. Design charts and examples for elastic analysis, refer to Design ADIS.

Table (10-3): Allowable working stresses in concrete (N/mm^2)

Concrete grade	f_{cu} *	30	35	40	50	60
Min concrete strength	f_{cui} **	26	30	34	42	51
A- Axial load	$0.25\ f_{cu}$	7.5	8.75	10	12.5	15
B - Flexure:						
1-Transfere stage before time dependent losses (N/mm^2).						
a- Max. compression	$0.45\ f_{cui}$	11.7	13.5	15.3	18.9	23.0
b- Max. tension at ends of simply supported beams	$0.44\sqrt{f_{cui}}$	2.24	2.4	2.56	2.84	3.14
c- Max tension except that in (b)	$0.22\sqrt{f_{cui}}$	1.12	1.20	1.28	1.42	1.57
2-Working stage after all losses.						
a- Max. compression due to prestressing and peremenant loads	$0.35\ f_{cu}$	10.5	12.25	14	37.5	21
b- Max. compression due to prestressing and all loads	$0.40\ f_{cu}$	12	14	16	20	24
c- Max. tension in pre-compressed tensile zone	$0.44\sqrt{f_{cu}}$	2.41	2.60	2.80	3.11	3.41

* f_{cui}= Concrete characteristic compressive strength at transfer stage.

** f_{cu} = Concrete characteristic compressive strength at working stage.

Table (10-4): Allowable stresses for prestressing steel tendons (N/mm^2)

a- Due to primary tension force*.	$0.9 f_{py} \leq 0.75 f_{pu}$
b- During tensioning of bars (transfer stage).	$0.7 f_{pu}$
c- After transfer of stresses to concrete (working stage)	$0.8 f_{py} \leq 0.7 f_{pu}$
d- At end anchorage and positions of splices in post-tensioned concrete.	

* Must not be more than the design value suggested by the manufacturer of the end anchorages.

10-3-2-4 Limit state of deflection and camber (SLS):

1) Immediate (short-term) deflection and camber are to be calculated for fully prestressed beams by using theory of elasticity considering the moment of inertia for the gross section (Ig). Calculations are carried out for different cases of loading including prestressing force (without creep and shrinkage).
2) Long term deflection is calculated under the effect of all permanent loads including shrinkage, creep and steel relaxation. (Stresses in concrete and pestressing steel are calculated after occurrence of all losses).
3) Camber values must not exceed the limits, which negatively affect the form and the function of the structure.
4) Deflection should not exceed the following limits according to art.(4-3-1)

Max.allowed deflection (Δ)

Δ (dead load + live load + temp + shrinkage + creep)	≯	L / 250	One way slabs & beams	4-60-a
		L\` / 450	Cantievers	4-60-b
Δ (dead load + finishes + temp + shrinkage + creep)	≯	L / 350 or 20mm	One way slabs & beams	4-60-c
		L\` / 600	Cantilevers	4-60-d

10-3-3 Design for Ultimate Strength Limit State:

10-3-3-1 Sections subjected to flexure:

1) Ultimate limit bending moment for a prestressed concrete section can be determined by using limit states method taking into consideration the stress distribution shown in figure (10-2).

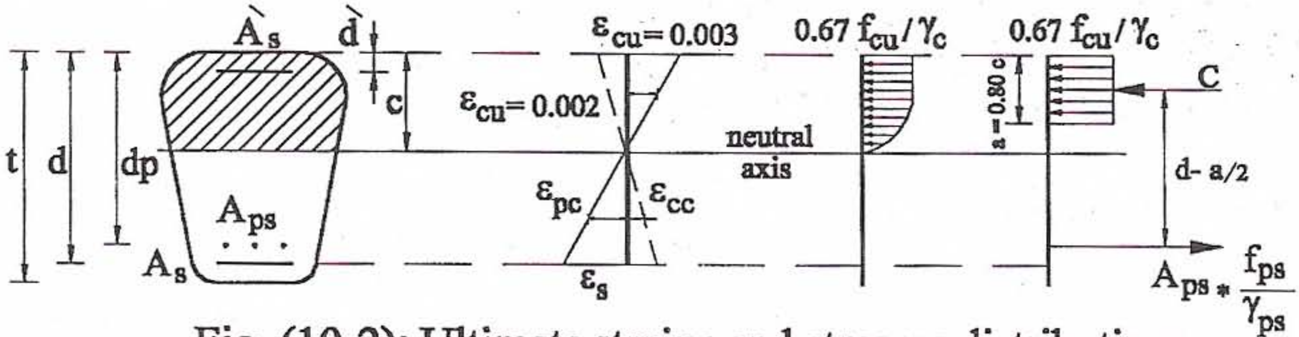


Fig. (10-2): Ultimate strains and stresses distributions

2) Total strain in prestressing steel	$\varepsilon_{ps} = \varepsilon_{pe} + \varepsilon_{ce} + \varepsilon_{pc}$	(10-5)

Where: ε_{pe} = strain in prestressing steel due to the prestressing force taking into consideration the effect of all losses.

ε_{ce} = strain in concrete at the level of prestressing steel due to prestressing taking into consideration the effect of all losses.

ε_{pc} = strain in prestressing steel due to compatibility of strains at the ultimate limit of bending moments.

3) Stress in prestressing steel (f_{ps}) at ultimate limit bending moment of the section is calculated according to the total strain obtained from eqn (10-5) and the idealized curve of prestressing steel (fig. 10-1).

4) **Sections with prestressing steel at tension side only:**

Ultimate limit moment for rectangular sections with

Prestressing steel at tension side only	$M_u = A_{ps}\,(f_{ps}/\gamma_{ps})\,(dp-a/2)$	10-6-a
With additional ordinary steel rft.	$M_u = A_{ps}\,(f_{ps}/\gamma_{ps})\,(d_p - a/2) + A_s\,(f_y/\gamma_s)\,(d - a/2)$	10-6-b

Where: d_p is the distance between the fibers subjected to max. compressive stresses and the centroid of prestressing steel.

b = section width & a = see fig. (10-2).

Effect of any steel rft. in the section can be considered by using equilibrium method and compatibility of strains.

5) **Approximate equations (10-7) ,(10-8) for calculation of f_{ps}**

May be used if the stress in prestressing steel after all losses $f_{pe} \geq 0.5\, f_{pu}$.

5-1- Bonded prestressing tendons:

a) Sections with tension and compression rft.	$f_{ps} = f_{pu} [1 - \eta_p (\mu_p f_{pu} / f_{cu} + d/d_p (\omega - \omega`))]$	10-7
b) To take compression rft. into consideration eqn. (10-8) should be satisfied	$[(\mu_p . f_{pu} / f_{cu} + d / d_p (\omega - \omega`))] \geqslant 0.17$ $d` \leqslant 0.15\, d_p$	10-8

5-2- Unbonded prestressing tendons:

Case a) Span / depth $\leqslant 35$	$f_{ps} = f_{pe} + 70 + (f_{cu} / 125\, \mu_p)$ N/mm^2	(10-9)
	$f_{ps} <$ the least of:	f_{py} $(f_{pe} + 420)$ N/mm^2
Case b) Span / depth > 35	$f_{ps} = f_{pe} + 70 + (f_{cu} / 375\, \mu_p)$ N/mm^2	(10-10)
	$f_{ps} <$ the least of:	f_{py} $(f_{pe} + 200)$ N/mm^2

η_p = is a coefficient depends on type of steel	0.68	0.50	0.35
(f_{py} / f_{pu}) not less than	0.80	0.85	0.90

$\omega_p = \mu_p f_{ps} / f_{cu}$,	$\mu_p = A_{ps} / (b.d_p)$
$\omega = \mu\ f_y / f_{cu}$,	$\mu = A_s / (b.d)$
$\omega` = \mu`\ f_y / f_{cu}$,	$\mu` = A`_s / (b.d)$

6) Max. and Min. limits for prestressing steel (Asp) & normal steel (As):

a- Max. steel ratio A_{ps} (max.) & A_s (max.)	$\omega p \leqslant 0.28$	10-11-a
R – sec.	$[\omega_p + (d/d_p)(\omega - \omega`)] \leqslant 0.28$	10-11-b
T – sec.	$[\omega_{pw} + (d/d_p)(\omega_w - \omega`_w)] \leqslant 0.28$	10-11-c

Otherwise use compatibility method to calculate strength of the R.C. section

Where: μ_p= percentage of prestressing steel in the x-section,

ω_{pw}, ω_w, $\omega`_w$ of T-sections are similar to wp,w, and w` of R-sections except that b shall be the web width, and rft. area shall be the required to develop compressive strength of web only.

b- Min. steel ratio $A_{ps\,(min)}$ & $A_{s\,(min.)}$

Min values of (A_{sp}) & (A_s) shall be adequate to develop $M_u > 1.2$ Mcracking (art. 4-3-1-4). This condition is not valid for the following cases:

a) Unbonded post-tensioned slabs.

b) Elements subjected to flexure with shear and flexure strength equal to twice the required values from calculations.

2- Min. bonded reinforcement in elements with unbonded prestressing steel

a) members with unbonded prestressing tendons		$A_{s\,min} = 0.004\ A$	10-12-a
b- Flat slabs & two way slabs	In +ve moment areas	$A_{s\,min} = N_c / (0.5\ f_y)$	10-12-b
	In -ve moment areas	$A_{s\,min} = 0.00075\ t_s\ L$	10-12-c

Where t_s = slab thickness

A_{ct} = area of that part of section between flexural tension face and neutral axis of stress.

N_c= tensile force in concrete due to unfactored dead load plus live load.

L = length of span in direction parallel to that of the rft. being determined

The following condition should be followed:

1)- Rft. shall be uniformly distributed over pre-compressed tensile zone as close as practicable to extreme tension fiber.

2)- Rft. should be distributed within a slab width equals to ($c+3t_s$) where c is column width. The bars should not be less than 4 bars with spacing > 300 mm. f_y shall not exceed 400 N / mm^2.

10-3-3-2 Development length and transfer length of prestressing steel.

For 3 or 7 wire prestressing strands: Development legnth: $L_d = Lt + La = \phi/7\ (f_{ps} - 2/3\ f_{pe})$ mm.	(10-13)
Where: L_t (transfer length) = $(f_{pe} / 3)\ \phi/7$ mm.	(10-14-a)
L_a the length after the critical section = $(f_{ps} - f_{pe})\ \phi/7$ mm.	(10-14-b)

Where ø is tendon diameter in mm. , and f_{pe}, f_{ps} in N/mm^2.

10-3-3-3 Ultimate strength design shear:

1) For prestressed beams, shear stress can be calculated at

a- t/2 from the inner face of support or

b- at the first change in web width (the most critical of them).

2) Vertical component of prestressing force of inclined tendons should be considered.

$Q_{du} = Q_u - Q_{pv}$	10-15

Where Q_{du} is the design ultimate shear force,

Q_u is the ultimate shear force due to dead and live loads,

Q_{pv} is the vertical component of prestressing force

3) Nominal concrete shear strength:

a) The effect of openings should be considered in the calculation of ultimate shear strength q_{cu}.

b) $\sqrt{f_{cu}} \ngtr 7.00$ N/mm^2.

q_{cu} (Nominal concrete shear strength without rft.) N/mm^2.

f_{cu} N/mm^2	30	35	40	45	50	55	60
$q_{cu\,min} = 0.125\sqrt{f_{cu}}$	0.70	0.74	0.79	0.84	0.89	0.93	0.97
$q_{cu\,max} = 0.35\sqrt{f_{cu}}$	1.92	2.07	2.21	2.35	2.47	2.60	2.71

Where: M_u is the ultimate moment at the critical section in shear.

c) In elements in which effective prestressing force is greater than 40 % of tensile strength of flexure rft. q_{cu} can be calculated approximatelly eq$_n$. (10-16) (If accurate calculation will not be carried out as in art. (d-1 & d-2)).

d) Accurate nominal concrete shear strength (q_{cu}) is the least of q_{cw} and q_{ci} as follow:

Approximate calculation of q_{cu}.	$q_{cu} = (0.045\sqrt{f_{cu}} + 5.0\,Q_u.\,d_p/M_u)$ N/mm^2 $q_u\,d_p / M_u > 1.0$	10-16
d-1) Shear strength (q_{ci})	$q_{ci} = 0.045\sqrt{f_{cu}} + q_d + q_i\,M_{cr}/M_{max}$ N/mm^2	10-17
	$q_{ci} \geqslant 0.125\sqrt{f_{cu}}$	
	$M_{cr} = (I/y_t)\,(0.60\sqrt{f_{cu}} + f_{pce} - f_{cd})$ N/mm^2	10-18
d-2) Shear strength(q_{cw})	$q_{cw} = 0.27\sqrt{f_{cu}} + 0.30\,f_{pcc} + q_{pv}$ N/mm^2	10-19

Where: $M_{max.}$ = Ultimate moment due to external loads.

q_i = Stresses due to ultimate shear forces produced from external loads corresponding to $M_{max.}$.

q_d = Shear stresses due to unfactored dead load.

f_{cd} =Stress due to unfactored dead loads at the section edge where external loads produce tensile stress.

f_{pce} =concrete compressive stress due to effective prestressing force only (after all losses) at the pre-compressed tension zone.

Y_t = distance between max tension fiber and neutral axis of the total section neglecting the effect of prestressing steel and ordinary steel.

I = gross moment of inertia neglecting the effect of prestressing steel and ordinary steel.

q_{pv} = Shear stress due to the prestressing loads.

f_{ppc}= Concrete compressive stress (after all losses) at section centroid or at the connection between beam web and slab when the centroid lays inside the slab.

As alternative solution, q_{cw} can be taken equal to the shear stress due to dead and live loads that produce main tensile stresses equal to 0.30 $\sqrt{f_{cu}}$ at element neutral axis or at the connection between flange and web when element axis lies inside the flange.

Effective depth (d_p)	Greater of	Distance between max. compression fiber and centroid of prestressing steel
		0.8 t

d-3) For pre-tensioned members, in case the distance t/2 from face of support is less than transfer length L_t (eqn. 10-14), the corresponding prestressing force shall be taken in calculation of (q_{cw}). This force is calculated by assuming that the prestressing force increases linearly from zero (at tendon end) to max. value (at distance L_t).

d-4) For pre-tensioned members if some tendons are anchored or stopped before end of the element, the reduced prestressing force shall be used in calculation of shear stress according to eqns (10-17), (10-19), and the change in prestressing force can be assumed linearly from zero (at tendon end) to max. value (at distance L_t).

d-5) When calculating shear strength in ribs that have grouted ducts with diameter $\varnothing > b_w/8$ (where b_w is rib width), the effective rib width shall be taken equal to (b_w-0.5 $\Sigma\varnothing$) where: $\Sigma\varnothing$ is the sum of ducts diameters at the plane which contains the max. number of tendons.

4) Nominal ultimate shear strength of web rft.:

If the ultimate shear stresses (q_u) are greater than nominal concrete shear strength (q_{cu}), a web rft. shall be used according to art. (4-2-2-1-4), to resist the following stress:

$$q_{su} = q_u - 0.50\, q_{cu} \quad (10\text{-}20)$$

10-3-3-3 Ultimate Torsion Strength:

1) Critical sections for torsion are determined according to art. (4-2-3-1).
2) Ultimate shear stresses due to torsional moment (q_{tu}) are determined according to art. (4-2-3-2).
3) Ultimate shear stresses $q_{tu(max)}$. for section with web and longitudinal rft. subjected to shear force and torsional moment should not be more than the values in art. (4-2-3-4).
4) Effect of torsional moment can be neglected if :

$$q_{tu} < 0.06 \quad \sqrt{f_{cu}/\gamma_c}\sqrt{1+f_{pcc}/(0.3\sqrt{f_{cu}})}\ N/mm^2 \quad (10-21)$$

5) Reinforcement for combined shear and torsion:

If the calculated stresses q_{tu} (art. 10-3-3-4-2) are greater than the value obtained from eqn. (10-21) and less than $q_{tu\ max}$ (art. 4-2-3-4), a torsional rft. shall be used (closed stirrups and longitudinal rft.) in addition to any other rft. according to table (4-5).

Area of torsion rft.	
a-Area of one branch of closed stirrups: $A_{str} = M_{tu}.S/[2A_0(f_{yst}/\gamma_s) \cot \theta]$	10-22
Case of rectangular section: $A_{str} = M_{tu}.S/[1.7(x.y_1)(f_{yst}/\gamma_s) \cot \theta]$	10-23
b- Area of additional longitudinal rft: $A_{sL} = (A_{str}.P_h/s)(f_{yst}/f_y) \cot^2\theta]$	10-24
-Min. longitudinal rft.: $A_{sLmin}=(0.46\sqrt{f_{cu}/\gamma_c}\ A_{cp}/f_y)-(A_{str}/s)P_h(f_{yst}/f_y)$ Where: $(A_{str}/s) \geqslant 1/6\ (b/f_{yst})$	10-25

Where: $\theta = 45°$ If the effective prestressing force is less than 40% tensile strength of flexure rft.

$\theta = 37.5°$ If the effective prestressing force is greater than 40% tensile strength of flexure rft.

f_{cu}, f_y, f_{yst} are in N/mm^2.

All conditions in art. (4-2-3-5) shoud be fulfilled.

6) For statically indetermined structures where the torsional moment results from compatibility of strains (compatibility torsion), the ultimate torsional moment can be reduced to the following value:

$$M_{tu} = 0.316\ (A^2_{cp}/p_{cp})\ \sqrt{f_{cu}/\gamma_2}\sqrt{1+f_{pcc}/(0.3\sqrt{f_{cu}})} \qquad (10-26)$$

Where: A_{cp} and P_{cp} are defined in art. (4-2-3-6)

10-3-3-5 Tendon anchorage zones:

a- For post–tensioned members, dimensions of end blocks are determined such that it achieves the ultimate limit design for concrete bearing strength (art.4-2-4).

b- Reinforcement shall be provided where required in tendon anchorage zones to resist splitting and spalling forces induced by tendon anchorages and it can be calculated according to theory of elasticity. Strut and tie method is a good design tool for design of these anchorage zones.

c- Forces and reinforcement of tendon anchorage zones shall be calculated to resist maximum jacking force and the requirements of the tendon fabricator.

10-3-3-6 Members subjected to combined flexure and axial loads:

Members subjected to eccentric loads are designed by using limit states method (according to art. 4-2-1). Design shall satisfy stability conditions and compatibility of strains, taking into consideration stresses of prestressing steel according to art. (10-3-3-1).

10-3-4 LOSSES OF PRESTRESSING:

10-3-4-2 Short term losses:

1) Losses due to anchorage slippage

In case of post-tensioning systems only friction wedges used to hold the wires will slip a little distance before the wires can be firmly gripped.

The amount of slippage varies between 2-3 mms. It depends upon the type of wedge and the stress of the wires ...etc. In most cases these values are given by the tendon fabricators.

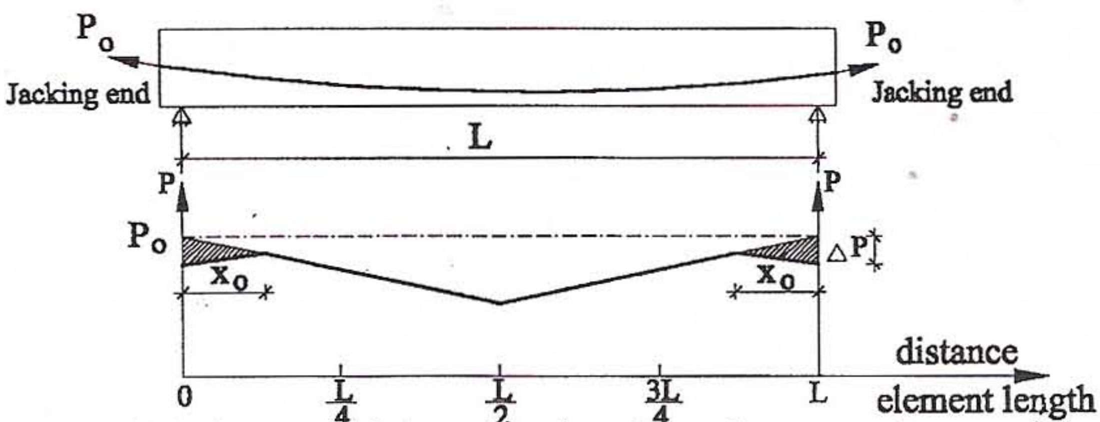


Fig (10-3) Losses in prestressing force due to anchorage slip.

2) Losses due to elastic shortening

a) Pre - tensioning:

As the prestress is transformed to the concrete, the member shortens and the prestressed steel shortens with it, hence there is a loss of prestress.

$$\text{Strain in concrete } \varepsilon_c = \frac{fc}{Ec} = \frac{P_o}{Ac\,Ec} = \varepsilon_s \text{ strain of steel}$$

Losses of prestress due to elastic shortening	$\Delta f_{pe} = (E_p / E_{ci}) f_{pci} = n f_{pci}$	10-27

Where:

E_p = young's (elastic) modulus for prestressing steel.

E_{ci} = young's (elastic) modulus for concrete at prestressing age.

F_{pci}= primary stress in concrete around pre-stressing steel before tosses depends on time.

b) Post tensioning

In case of applying the tension force for the prestressing steel one time, losses will be zero. sequence of prestressing effect is taking into consideration as follow:

$$\Delta f_{pe} = \frac{1}{2} (E_p / \varepsilon_{ci}) f_{pci} = \frac{1}{2} n f_{ci} \qquad (10\text{-}28)$$

3) Losses due to friction:

In post-tensioning only to consider losses due to wobble effect and curvature effect.

Its value depends on:

The coefficient of friction between the duct-material and the strands, the workmanship, the total change in angles (degree of curvature), the stress in the steel due to curvature.

a) Due to jack internal friction:

Jacks internal friction must be considered according to the given data of the tendon fabricator.

b) Losses due to wobble:	$P_x = P_o . e^{-kx}$	10-29
c) Losses due to curvature:	$p_x = p_o . e^{(-\mu x / r_{ps})}$	10-30
If $(\mu x / r_{ps}) \leq 0.20$ eqn (10-30) can be simplified to	$P_x = P_o(1-\mu x/r_{ps})$	10-31
If $(k_x + u_x / r_{ps}) \leq 0.20$ all friction losses can be calculated using:	$P_x = P_o [1- (kx + \mu x/r_{ps}]$	10-32

Where:

p_o = prestressing force at jack (N).

x = distance from tension side (m) (Fig 4-10).

P_x = prestressing force at x distance (N).

k = factor depend on strands type, method of construction… Etc.

k = 0.0033/m\` (normal case), k = 0.0017 /m\` rigid strands fixed in forms

r_{ps} = radius of curvature of cables .

μ = 0.55 for steel moving on concrete.

= 0.30 for steel moving on steel.

= 0.25 for steel moving on lead.

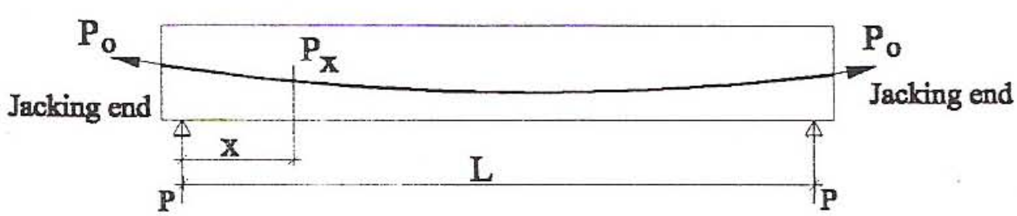


Fig (10-4) Losses of prestressing due to friction.

10-3-4-3 Time dependent losses:

1) losses due to shrinkage of concrete

$$\Delta f_{psh} = \varepsilon_{sh} \cdot E_p \quad (10\text{-}33)$$

ε_{sh} = shortening strain in concrete due to shrinkage.

- Strain in concrete due to shrinkage is determined from table (2-9-a), and if there is no data available, the value can be taken from table (10-5).

In case of construction stage for the prestressing members, 0.5 ε_{sh} is considered during first month, 0.75 ε_{sh} is considered during the first six months after construction.

Table (10-5) strain due to shrinkage ε_{sh}

pre-stressing type	ε_{sh}
pre-tension (3-5 days after casting)	300×10^{-6}
post-tension (7-14 days after casting)	200×10^{-6}

2) losses due to creep of concrete:

Table (6-10) Strain due to Creep ε_{cr}

Prestressing Type	ε_{cr} / (N/mm^2) of working stress	
	f_{ci}(concrete strength before prestressing process) N/mm^2	
	$f_{ci} \leq 40$	$f_{ci} > 40$
Pretensioned	$(40/f_{ci})x48x10^{-6}$	$48x10^{-6}$
Post-tensioned	$(40/f_{ci})x36x10^{-6}$	$36x10^{-6}$

Case of increasing working stresses over 1/3 (f_{cu}):

$$\varepsilon_{cr}^{*} = \varepsilon_{cr} \cdot \propto \quad (10\text{-}34)$$

Where $\propto$ = coefficient from figure (10-5).

- In case of construction stage for the prestressing element:-
  0.5 ε_{cr} could occurr during the fist month.
  0.75 ε_{cr} could occurr during the six months.

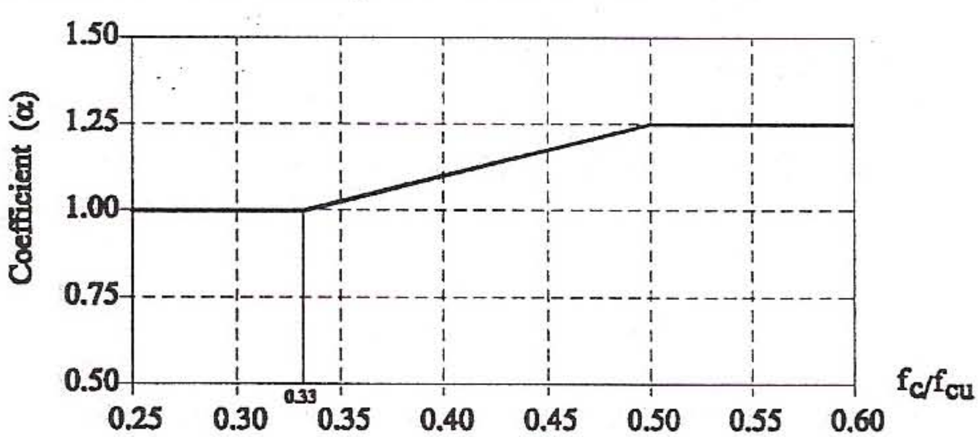


Fig (10-5) change of coefficient α with concrete stress

For bonded prestressing	Creep losses $\Delta f_{pcr} = \phi . E_p / E_c . f_{cs}$	10-35
	$\phi = \varepsilon_{cr} / \varepsilon_{el}$	10-36

Where: ϕ =2 (Pre-tensioned), ϕ =1.6 (Post-tensioned).

ε_{el} = Elastic strain, $f_{cs} = f_{cs}^* - f_{csd}^*$ (10-37)

f_{cs}^* =concrete stresses at prestressing steel level due to prestressing force at transfer stage.

f_{csd}^* =concrete stresses at prestressing steel level due to semi-permanent loads at transfer stage.

3)Losses due to prestressing steel relaxation

Over tensioning the cables by (5-10) % can reduce relaxation in steel and the Jacks should be held for 2-3 minutes.

c) Losses due to steel relaxation

$$\Delta f_{pr} = f_{pi}(\log t)(\frac{f_{pi}}{f_{py}} - 0.55)/k_1 \quad (10\text{-}38)$$

Where: f_{py} = Yield strength of prestressing tendons.

t = time from the beginning of prestressing in hours (max 1000 hour).

f_{pi}= initial generated stress in the adjacent concrete to the prestressing after short team losses and before time dependent losses.

k_1 = Factor depend on prestressing steel type:

k_1 = 10 normal relaxation stress – relieved steel.

k_1 = 45 low relaxation stress – relieved steel.

10-4 Analysis of prestressed structures:

Analysis and design of prestressed structures (statially determined or statically indetermined) should satisfy the requirem of the ultimate strength limit state and the working limit states.

10-4-1 Statically indetermined structures:

1) Performance at service load conditions shall be determined by elastic analysis, considering reactions, moment, shear, and axial forces produced by prestressing, creep, shrinkage, temperature change, axial deformation, restraint of attached structural elements, and foundation settlement.
2) Used moments to compute required section strength shall be the sum of the moments due to reactions induced by prestressing (with a load factor of 1.0) and the moments due to factored loads as in art. (3-2-1).
3) Empirical methods should not be used for calculation of internal forces.

10-4-2 Moments redistribution:

It is allowed to redistribute moments calculated by elastic theory due to any suitable cases of loading on the spans under the following conditions:

- Equilibrium between internal and external forces for any loading case.
- Allowable reduction in positive or negative moments according to elastic theory (should cover all loading cases) should be less than 10%

Ductility condition in sections at which moments are redistributed.

10-4-3 Prestressed slabs:

1- Bending moments and shearing forces can be determined by using frames equivalent method (6-2-7-4).
2- Internal stresses can be calculated by any advanced method.
3- Ultimate moment capacity of prestressed slabs at every section shall not be less than the required strength according to art. (10-3-3).
4- Shear strength of prestressed slabs at columns should not be less than values given in art. (6-2-7-7).
5- All serviceability limit states should be satisfied.
6- **Details of slab reinforcement:**

*For normal live loads and uniformly distributed loads, spacing of prestressing tendons or groups of tendons in one direction shall not exceed neither 6 times the slab thickness, nor 1.5 m, nor duct bigger dimension (for non-circular ducts).

*Spacing of tendons also should provide a min. average prestress (after all losses) of 0.90 N/mm^2 on the slab section tributary to the tendon or tendon group.

-Two tendons at least should be provided in each direction through the critical shear section over columns.

10-5 Structural details:

10-5-3 Tendons concrete cover:

Concrete cover is determined according to durability requirements, fire resistance, and design requirements.

10-5-3-1) Bonded tendons.

a) Concrete cover required for corrosion protection:

Table (10-7): Min. thickness of concrete cover (mm)

Min. f_{cu} (N/mm^2)		≤35	40	45	50
Degree of exposure*	Category one	25	25	25	25
	Two	--	40	30	30
	Three	--	50	40	40
	Four	--	--	60	50
Max. (free water/cement) ratio		0.50	0.45	0.40	0.35
Min. cement content (kg/m^3)		350	400	425	450

* Degree of exposure is according to (4-11)

b) Concrete cover required for fire resistance:

Conditions in table (2-13) should be satisfied and the concrete cover should not be less than values in table (10-8).

Table (10-8): Concrete cover according to the required periods for fire resistance*

The required period for fire resistance (hour)	Concrete cover (mm).					
	Beams		Slabs		Ribs	
	Simple	Continuous	Simple	Continuous	Simple	Continuous
0.50	25	25	25	25	25	25
1.00	25	25	25	25	35	30
1.50	35	30	30	25	45	35
2.00	60	35	40	35	55	45
3.00	70	60	55	45	65	55
4.00	80	70	65	55	75	65

* If thickness of concrete cover is greater than 45 mm, precautions shall be considered to prevent splitting of concrete cover, and this can be achieved by decreasing shrinkage of concrete mix and crack control.

<u>**10-5-3-2) Concrete cover and allowable distancec between tendons:**</u>

Concrete cover	the greatest of	50 mm
		Values in table (10-7) and (10-8) + stirrup diameter
		Values in figure (10-7) and (10-8)

a ≥	min. concrete cover + stirrup diameter	2.5 ϕ	--
B ≥	Max. aggregate size + 5mm.	2 ϕ	20 mm
C ≥	Max. aggregate size	2 ϕ	10 mm

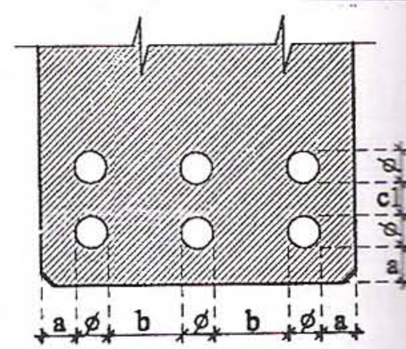


Fig. (10-6): Pre-tension system (wires or tendons).

a ≥	C (10-5-3-1) + stirrup diameter
	ϕ if ϕ ≤ 80 mm
	0.75 ϕ if ϕ > 120 mm
	50 mm
b1≥	ϕ or 40 mm
c1≥	ϕ or 50 mm

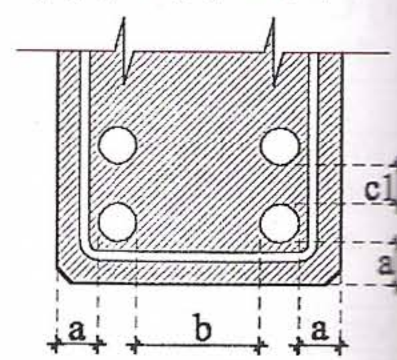


Fig. (10-7): Post-tension system (single tendons)

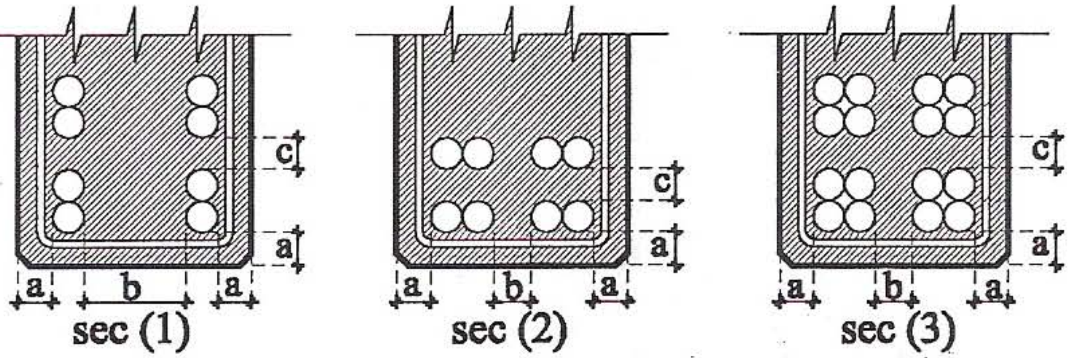


Fig (10-8) Min. allowable distance between tendons ducts

		Sec. (1)	Sec. (2)	Sec. (3)
a	⩾	Fig. (10-9)	⩾1.5 ϕ	⩾1.5 ϕ
b	⩾	1.5 ϕ	⩾1.5 ϕ	⩾1.5 ϕ
c	⩾	1.2 ϕ	⩾1.2 ϕ	⩾1.2 ϕ
ϕ	⩽	50 mm	⩽ 50 mm	⩽ 50 mm

10-5-3-3 External tendons:

In case of protection of external tendons by using concrete cover, the used concrete strength shall not be less than 40 N/mm^2, the thickness of concrete cover shall not be less than the previous values, and shall be anchored with prestressed element by using steel rft. In addition, cracks control requirement as in chapter (4) shall be met.

10-5-4 Distance between prestressing tendons:

I - pre-tensioning:

-Distance between cables is determined according to fig. (10-6). If the cables is arranged in two groups or more separated from each other, the possibility of longitudinal splitting in structural element shall be taken into consideration and it can be avoided by adding stirrups and longitudinal rft.

-In pre-tensioned elements with bonded tendon, the distance between wires or strands at ends shall satisfy requirements of art. (10-3-3-2), (10-3-3-5).

II- Post-tensioning:

According to fig. (10-7) and fig. (10-8)

10-5-5 Curved tendons:

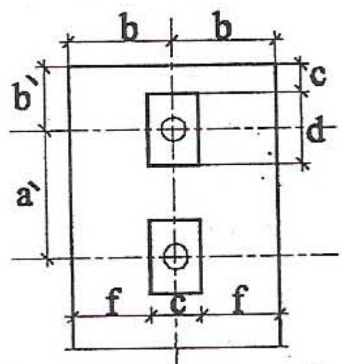
In case of using curved tendons in post-tensioning, positions of tendons ducts are determined by 3-D coordinates, and also sequence of tendons tensioning are determined to avoid the following:

a- Crash of side concrete cover perpendicular to ducts curvature plane.
b- Crash of cover in the same plane of ducts curvature.
c- Failure of concrete between ducts in curvature plane and the perpendicular plane.

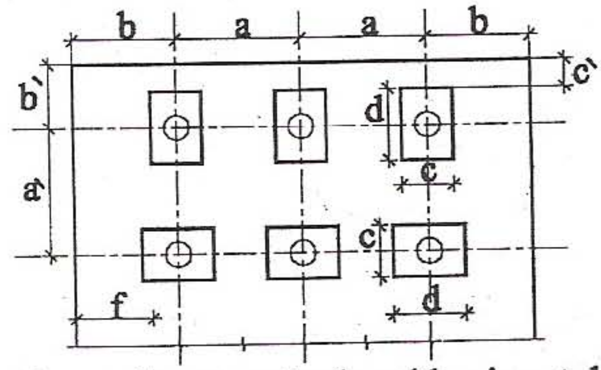
Thickness of concrete cover and distance between tendons shall not be less than values in art. (10-5-3) and (10-5-4).

10-5-6 Tendon anchorage zone:

Fig (10-9) shows the conditions for distances between tendon anchorage.



I) anchore one vertical line



II) anchore vertical and horizontal

Fig (10-9) distance between anchore plates

Where:

C, D= are smaller and bigger dimensions for anchor plate as from fabricator.

a_* = min. allowable distance between anchor plate axes as from fabricator.
$\geq$ (C or D) + 30 mm

b_* = min. allowable distance between anchor plate axis and concrete edge as from fabricator.

a = horizontal actual distance between anchor plate axes.

b = horizontal actual distance between anchor plate axis and concrete edge.

a` = vertical actual distance between anchor plate axes.

b` = vertical actual distance between anchor plate axis and concrete edge.

f = distance between anchor plate edge and concrete edge as in next table:.

Prestressing force (1000KN)	0.5	0.5-1.0	1.5-3.0	3.0-4.0	>4.0
Distance f (mm)	30	50	70	80	100

The next conditions should satisfied:

$a \geq a_*$, $a` \geq a_*$	$b \geq 1.50\, b_*$
$b \geq b_*$, $b` \geq b_*$	$a\, b` \geq 1.60\, b_*^2$
$a`\, b \geq 1.60\, b_*^2$	$a`\, b \geq$ Initial tension force / f_{cu}
$a\, a` \geq 1.50$ Initial tension force / f_{cu}	

PART (III)

DESIGN AIDS

A) ***TABLES AND CHARTS FOR (USD) DESIGN OF SECTIONS***

B) ***PRESTRESSED CONCRETE STRUCTURES***

- *DESIGN OF SECTIONS WITH TENSION STEEL ONLY*
- *DESIGN OF SECTIONS WITH COMPRESSION STEEL*
- *DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMPRESSION FORCE* ***(INTERACTION DIAGRAMS)***
- *DESIGN OF PRESTRESSED CONCRETE SECTIONS*

Ultimate Limit Design Charts & Tables for Simple Bending

CHART (A & B) Page (398&399)

$$R_u = \frac{M_u}{b.d^2}$$

$$A_s = \mu . b . d$$

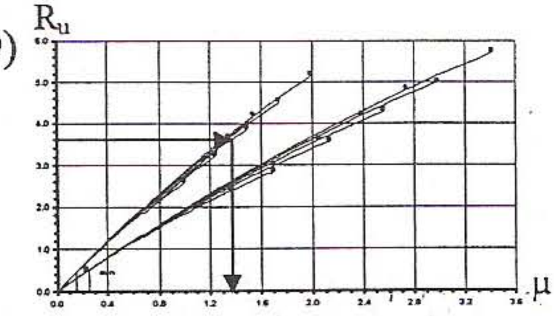


CHART (C) Page (401)

$$d = C_1 \sqrt{\frac{M_u}{f_{cu}.b}} \qquad A_s = \frac{M_u}{f_y.d.j}$$

Assume: C1 = 3⇒4 (for Beams)

= 4⇒5 (for Slabs)

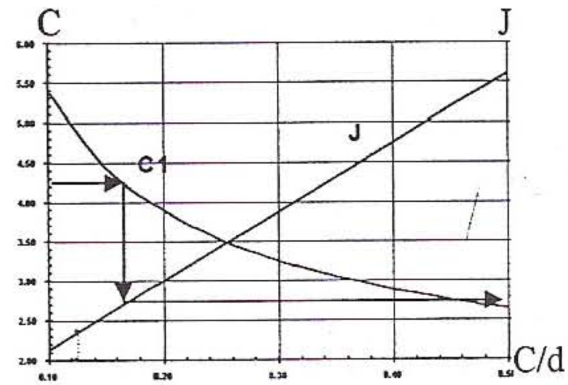


CHART (D) Page (400)

$$d = C_1 \sqrt{\frac{M_u}{f_{cu}.b}} \quad \text{or} \quad R_1 = \frac{M_u}{f_{cu}.b.d^2} = \frac{1}{C_1^2}$$

$$A_s = \omega \frac{f_{cu}.b.d}{f_y}$$

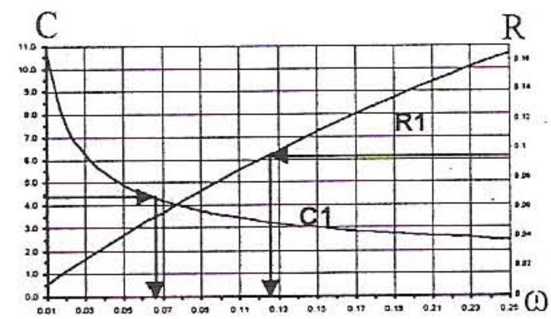
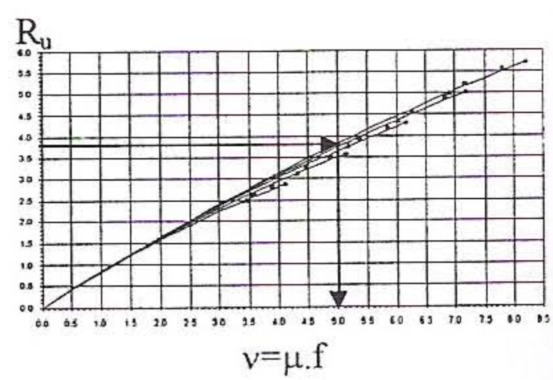


CHART (E) Page (402)

$$R_u = \frac{M_u}{b.d^2}$$

$$\nu = \mu.f_y \Rightarrow\Rightarrow \mu = \frac{\nu}{f_y}$$

$$A_s = \mu.b.d$$



Ultimate limit design tables (R & μ): **Pages (403 & 404)**
Ultimate limit design tables (Sec. with compression Rft.): **Pages (405 - 406)**

DESIGN AIDS

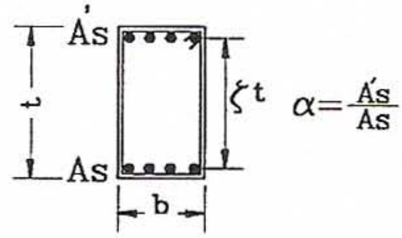
INTERACTION DIAGRAMS FOR ECCENTRIC FORCES

RECTANGULAR SECTIONS

i–top & Bott. R.F.T. Only

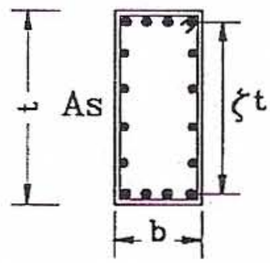
N/mm²

f_y = 280 / 360 / 400 → α = 1.00 / 0.80 / 0.60 → ζ = 0.90 / 0.80



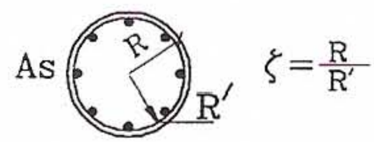
ii–R.F.T ALong The Perimeter

f_y = 280 / 360 / 400 → ζ = 0.90 / 0.80



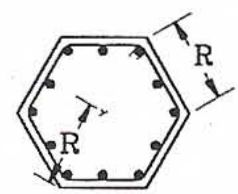
CIRCULAR SECTIONS

f_y = 280 / 360 / 400 → ζ = 0.90 / 0.80



HEXAGONAL SECTIONS

f_y = 280 / 360 / 400 → ζ = 0.90 / 0.80

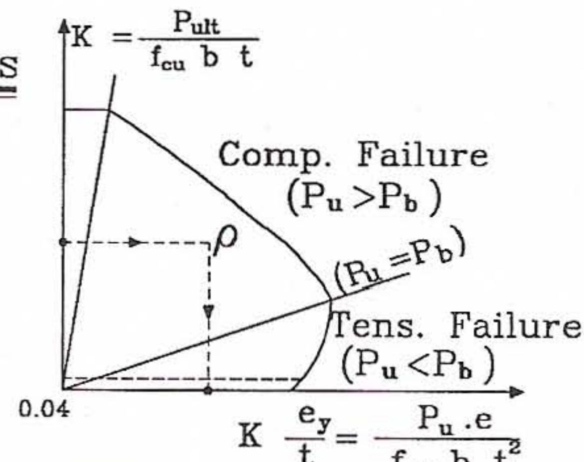


How To Use Interaction Diagrams

$$K=\frac{P_u}{f_{cu}.b.t} \qquad \mu=\rho\, f_{cu}10^{-4}$$

$$K\frac{e}{t}=\frac{P_u.e}{f_{cu}.b.t^2} \qquad As=\mu.b.t$$

$$As'=\alpha\, As$$



III. ULTIMATE LIMIT STATE (DESIGN FOR FLEXURE)

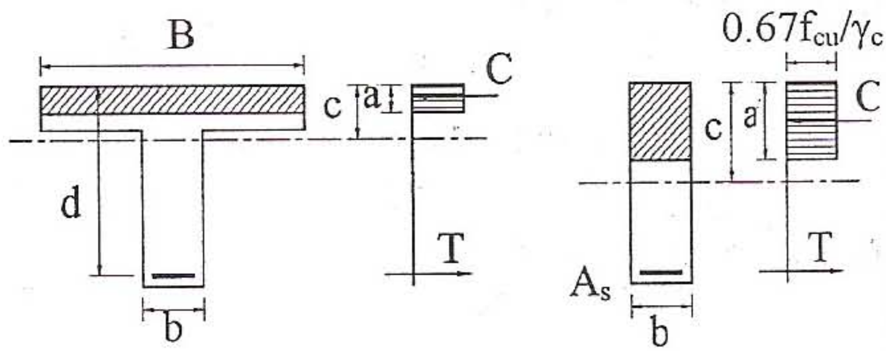
2.1-GENERAL CONSIDERATIONS:

The ultimate limit flexure strength of the cross-section ($M_{ultimate\ limit}$) must be equal to or greater than the required strength (M_u). Where M_u = Load factor . $M_{working}$.

2.2-SECTIONS SUBJECT TO SIMPLE BENDING WITH TENSION RFT. ONLY:

a) Rectangular Sections (beams and Slabs) and T-section with ($a \le t_s$):

For the design of rectangular sections (beams and slabs) and T-section with ($a \le t_s$) with tension reinforcement only, the conditions of equilibrium are:



The force equilibrium:

$$T = C$$

where, b^* = (b) web for R – sec.
= (B) flange for T – sec.
= unit width for slabs

$$A_s \frac{f_y}{\gamma_s} = 0.67 \frac{f_{cu}}{\gamma_c} a.b^*$$

$$\boxed{a = A_s \frac{f_y}{\gamma_s} \frac{1}{0.67 \frac{f_{cu}}{\gamma_c} b^*}} \quad \text{.......... code eqn.(4-2)}$$

$$\frac{a}{2d} = \frac{As}{b^*d} \frac{f_y}{\gamma_s} \frac{\gamma_c}{1.34 f_{cu}} = \frac{\mu}{1.34} \frac{f_y}{\gamma_s} \frac{\gamma_c}{f_{cu}} \qquad \text{where, } \mu = \frac{A_s}{b.d}$$

The moment equilibrium:

$$\boxed{M_u = (T \text{ or } C).(d - \frac{a}{2}) = A_s \frac{f_y}{\gamma_s} (d - \frac{a}{2})} \quad \text{.......... code eqn.(4-1)}$$

$$M_u = A_s \frac{f_y}{\gamma_s} d(1 - \frac{a}{2d}) = A_s.d \frac{f_y}{\gamma_s} (1 - \frac{\mu}{1.34} \frac{f_y}{\gamma_s} \frac{\gamma_c}{f_{cu}})$$

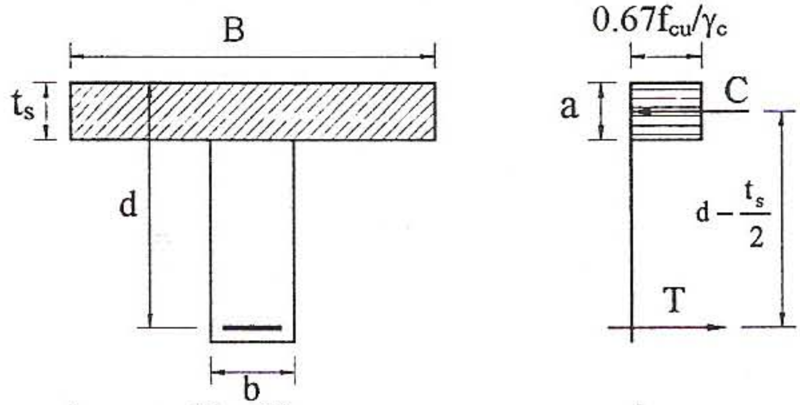
$$\frac{M_u}{bd^2} = \frac{A_s}{bd}\frac{f_y}{\gamma_s}(1 - \frac{\mu}{1.34}\frac{f_y}{\gamma_s}\frac{\gamma_c}{f_{cu}}) = \mu\frac{f_y}{\gamma_s}(1 - \frac{\mu}{1.34}\frac{f_y}{\gamma_s}\frac{\gamma_c}{f_{cu}})$$

For simple bending : $\gamma_c = 1.5$ & $\gamma_s = 1.15$

$$R_u = \frac{M_u}{bd^2} = \mu\frac{f_y}{1.15}(1 - 0.973\mu\frac{f_y}{f_{cu}}) \quad \text{.......... } (4-1-a)$$

b) Design T-section with ($a > t_s$):

The part of the web in comp. can be neglected, then:



$$M_u = 0.67\frac{f_{cu}}{\gamma_c}B.t_s(d - \frac{t_s}{2})$$

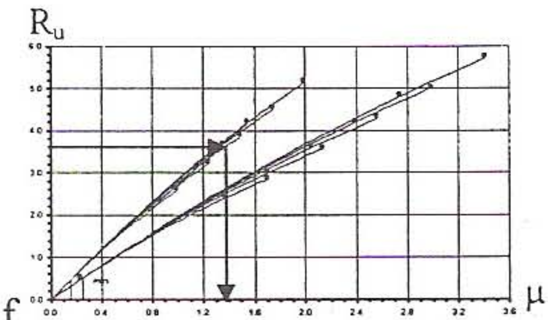
$$M_u = A_s\frac{f_y}{\gamma_s}(d - \frac{t_s}{2}) \quad \text{......... code eqn.} (4-8)$$

2.3-DESIGN AIDS FOR FLEXURE IN THE FORM OF CHARTS AND TABLES:

i) Design Charts (A & B):

Equation No. (4 - 1 - a)

$$R_u = \frac{M_u}{bd^2} = \mu\frac{f_y}{1.15}(1 - 0.973\mu\frac{f_y}{f_{cu}})$$



gives $\Rightarrow R_u = \frac{M_u}{b.d^2}$ as a function of μ, f_y & f_{cu}

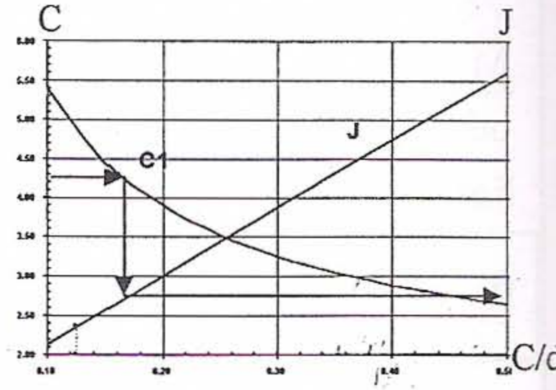
For given values f_{cu} & f_y curves can be developed for μ versus R_u

The same relation can also be expressed in tables as shown on pages number (403 & 404)

ii) Design Chart (C):

represents eqn. (4-1) in the following alternative form:

$$d = C_1\sqrt{\frac{M_u}{f_{cu}\; b}} \quad \& \quad A_s = \frac{M_u}{f_y . j . d}$$



$$j = \frac{M_u}{A_s . f_y . d} = \frac{A_s \frac{f_y}{\gamma_s}(d - \frac{a}{2})}{A_s . f_y . d} = \frac{1}{\gamma_s}(1 - \frac{a}{2d})$$

i.e. $$\boxed{j = \frac{1}{\gamma_s}(1 - 0.4\frac{c}{d})} \quad .... (4-1-b)$$

and $$C_1 = d\sqrt{\frac{f_{cu} . b}{M_u}} = d\sqrt{\frac{f_{cu} . b}{0.67\frac{f_{cu}}{\gamma_c}ab(d - \frac{a}{2})}} = 1/\sqrt{\frac{0.67}{\gamma_c}\frac{a}{d}(1 - \frac{a}{2d})}$$

i.e. $$\boxed{C_1 = 1/\sqrt{0.3573\frac{c}{d}(1 - 0.4\frac{c}{d})}} \quad .... (4-1-c)$$

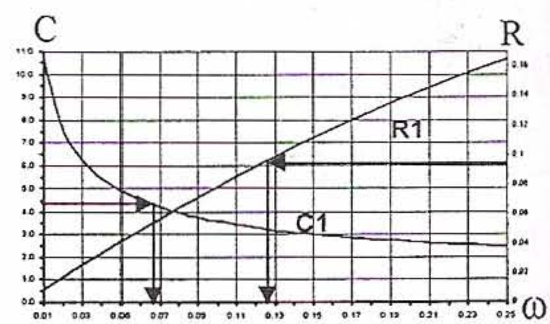
Where C_1 & j are function of c/d (a=0.80c)

For different values of c/d calculate C1 & j values and thus design chart (C) can be easily drawn.

Assume: C1 equal to [3 $\Rightarrow$ 4 (for beams)] & [4 $\Rightarrow$ 5 (for slabs)]

iii) Design Chart (D)

One curve for all concrete grades and types of steel can be developed as follows:



Assume $\omega = \mu\frac{f_y}{f_{cu}}$

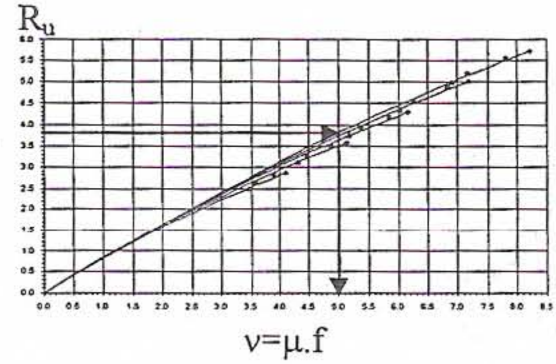
and $$\boxed{R_1 = \frac{M_u}{f_{cu}\; b d^2} = \frac{\omega}{\gamma_s}(1 - 0.973\omega)} \quad .... (4-1-d)$$

Design chart (D) gives two curves for

R_1 and $C_1 = \frac{1}{\sqrt{R_1}}$, versus ω , where $\boxed{d = C_1\sqrt{\frac{M_u}{f_{cu}\; b}} \quad \& \quad A_s = \omega\frac{f_{cu}}{f_y}b\,d}$

iv) Design Chart (E):

Equation No. (4-1-a) gives $\boxed{R_u = \frac{M_u}{b.d^2}}$ as a function of $\mu, f_y \& f_{cu}$ For given values of f_{cu} $\& f_y$ a curve can be developed for $(\nu = \mu.f_y)$ versus $R_u = \frac{M_u}{b.d^2}$

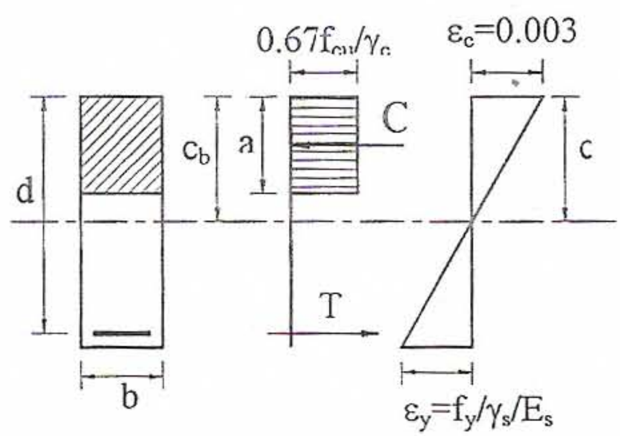


Alternatively a computer program using the pervious equations for the design of flexure members can be developed.

2.4- Modes of Failure:

Failure of reinforced concrete sections has two modes, brittle or ductile. Brittle failure occurs when the concrete strain on compression side reaches the max. compressive strain before the reinforcement reaches it's yield strain.

Balanced section is the section at which the concrete on compression side reaches its ultimate strain ($\varepsilon_c = 0.003$) at the same time when the steel in tension side reaches its yield strain ($\varepsilon_y = f_y/\gamma_s/E_s$). This condition gives the position of the neutral axis.



To avoid brittle failure the code specify that : $(\frac{c}{d})_{max.} = 0.67(\frac{c}{d})_{balanced}$

which gives RC section with ductile failure

2.5- Max. ultimate moment using tension steel only and max.% of steel:

$$\mu^*_{max.} = \frac{A_s}{b.d} = 0.67\frac{f_{cu}}{\gamma_c}\frac{\gamma_s}{f_y}\frac{a_{max.}}{d} = 0.536\frac{f_{cu}}{\gamma_c}\frac{\gamma_s}{f_y}\frac{c_{max.}}{d}$$

$$R_{max.} = \frac{M_{u\ max.}}{(f_{cu}/\gamma_c)bd^2} = 0.536\frac{c_{max.}}{d}(1-0.4\frac{c_{max.}}{d})$$

Table (4-1)

The maximum values for [c/d & μ & R & ω & R_1 & C_1]

Steel type N/mm^2	c/d Balance	C/d Max.	$R_{max.}$	$\mu_{max.}$	$\omega_{max.}$	$R_{1\,max.}$	$C_{1\,min.}$
240/350	0.742	0.50	0.214	$8.56 \times 10^{-4} f_{cu}$	0.204	0.142	2.651
280/450	0.711	0.48	0.206	$7.00 \times 10^{-4} f_{cu}$	0.196	0.138	2.693
360/520	0.657	0.44	0.194	$5.00 \times 10^{-4} f_{cu}$	0.180	0.129	2.780
400/600	0.633	0.42	0.187	$4.31 \times 10^{-4} f_{cu}$	0.172	0.124	2.830
450/520	0.605	0.40	0.180	$3.65 \times 10^{-4} f_{cu}$	0.164	0.120	2.890

Table (4-2) (Moment redistribution ± 10%)

The maximum values for [c/d & μ & R & ω & R_1 & C_1]

Steel type N/mm^2	c/d Balance	c/d max.	$R_{max.}$	$\mu_{max.}$	$\omega_{max.}$	$R_{1\,max.}$	$C_{1\,min.}$
240/350	0.597	0.40	0.180	$6.85 \times 10^{-4} f_{cu}$	0.164	0.120	2.887
280/450	0.567	0.38	0.173	$5.58 \times 10^{-4} f_{cu}$	0.156	0.115	2.940
360/520	0.507	0.34	0.157	$3.88 \times 10^{-4} f_{cu}$	0.139	0.105	3.090
400/600	0.477	0.32	0.150	$3.29 \times 10^{-4} f_{cu}$	0.132	0.100	3.160
450/520	0.447	0.30	0.142	$2.74 \times 10^{-4} f_{cu}$	0.124	0.095	3.250

Table (4-1-a)

The maximum values for [$\mu_{max.}$ (%)] for the different f_{cu} values

Steel type N/mm^2	20	25	30	35	40
240/350	1.712	2.14	2.568	2.996	3.424
280/450	1.40	1.75	2.10	2.45	2.80
360/520	1.00	1.25	1.50	1.75	2.00
400/600	0.862	1.0775	1.293	1.5085	1.724
450/520	0.73	0.9125	1.095	1.2775	1.46

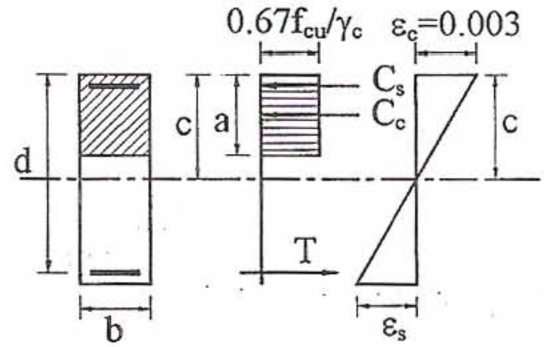
Table (4-2-a)

The maximum values for [$\mu_{max.}$ (%)] for the different f_{cu} values

Steel type N/mm^2	20	25	30	35	40
240/350	1.37	1.7125	2.055	2.3975	2.74
280/450	1.116	1.395	1.674	1.953	2.232
360/520	0.776	0.97	1.164	1.358	1.552
400/600	0.658	0.8225	0.987	1.1515	1.316
450/520	0.548	0.685	0.822	0.959	1.096

2.6- DESIGN OF SECTIONS SUBJECTED TO SIMPLE BENDING WITH TENSION AND COMPRESSION RFT.:

According to clause (4-2-1-2) of the code when $\mu_{max.}$ and $R_{max.}$ are reached the max. ultimate moment of the section can be increased by using steel rft. In the compression zone. For a given moment M_u^* greater than the maximum moment of singly reinforced section $M_{umax.}$ then add steel area $A_s`$ at both compression and tension side, thus:



$$\Delta M = A_s` \frac{f_y}{\gamma_s}(d-d`) \quad \& \quad M_u^* = M_{u\,max.} + \Delta M$$

$$M_u^* = R_{max.} \frac{f_{cu}}{\gamma_c} . b . d^2 + A_s` \frac{f_y}{\gamma_s}(d-d`) \quad \text{......... code eqn.(4-6)}$$

$$A_s \frac{f_y}{\gamma_s} = 0.67 a_{max.} . b \frac{f_{cu}}{\gamma_c} + A_s` \frac{f_y}{\gamma_s} \quad \text{......... code eqn.(4-7)}$$

$$A_s = \mu_{max.} . b . d + A_s`$$

NOTES FOR SECTIONS SUBJECTED TO SIMPLE BENDING:

The min. rft. must be checked as As min. = The least of $[\frac{1.1}{f_y}.b.d]$ or $[1.3\ A_{s\,req.}]$

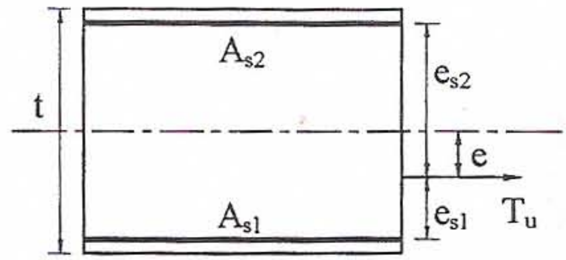
But not less than 0.25% b.d for st 240/350 & 0.15% b.d for st 360/520 and in T-section, the value of b is for the web.

The ratio (a/d) must not be less than (0.10).

$\mu_{min.}$		f_y			
	The least of 1.30 $As_{req.}$ OR →	**240**	**280**	**360**	**400**
		0.46%	0.40%	0.31%	0.275%
	But not less than	0.25%	0.25%	0.15%	0.15%

Section subjected to eccentric tensile force:

Axial tension or eccentric tension acting between steel rft. (force is carried by steel only) [e < (d-d`)/2]



i – Calculate $e = \dfrac{M_u}{T_u}$, $(e \le t/2 - cover)$

ii – Caculate $e_{s1} = t/2 - e - cover$ & $e_{s2} = t/2 + e - cover$

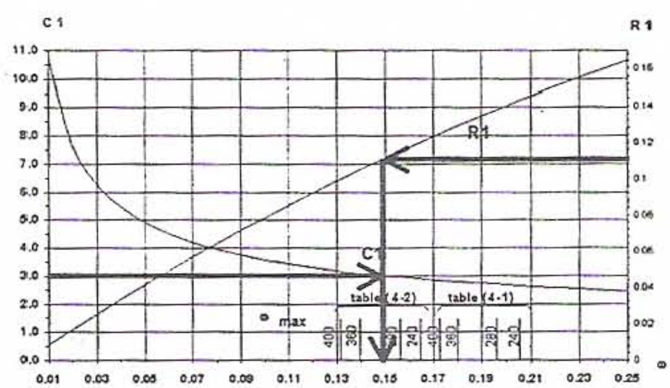
iii – Calculate $\boxed{A_{s1} = \dfrac{T_u\, e_{s2}}{e_{s1} + e_{s2}} / \dfrac{f_y}{\gamma_s} \quad \& \quad As2 = \dfrac{T_u\, e_{s1}}{e_{s1} + e_{s2}} / \dfrac{f_y}{\gamma_s}}$

Solved Examples for Sections Subjected to Flexure

Example 1:

Given: M_u= 300 kN.m, b = 300 mm, f_{cu} = 25 MPa, f_y = 360 MPa,
Design using chart (D): Assume $C_1 = 3$ (for beams)

$$d = C_1 \sqrt{\frac{M_u}{f_{cu}.b}}$$



$$d = 3 \times \sqrt{\frac{300 \times 106}{25 \times 300}} = 600mm$$

$$R_1 = \frac{M_u}{f_{cu}.b.d^2}$$

$$R_1 = \frac{300 \times 10^6}{25 \times 300 \times 600^2} = 0.1111$$

from chart (D) $\Rightarrow \omega = 0.15$ $\qquad \omega = 0.15 = \mu \dfrac{f_y}{f_{cu}} = \mu \dfrac{360}{25} = 14.4\mu$

$$\mu = \frac{0.15}{14.4} = 0.104 = \frac{A_s}{b.d} = \frac{A_s}{300 \times 600} \quad \Rightarrow\Rightarrow\Rightarrow \quad A_s = 1875mm^2 = 18.75cm^2$$

Example 2:

Design example (1) using chart (C):

Assume $\quad C_1 = 3$ (for beams)

$$d = C_1\sqrt{\frac{M_u}{f_{cu}.b}} \qquad\qquad d = 3\times\sqrt{\frac{300\times106}{25\times300}} = 600mm$$

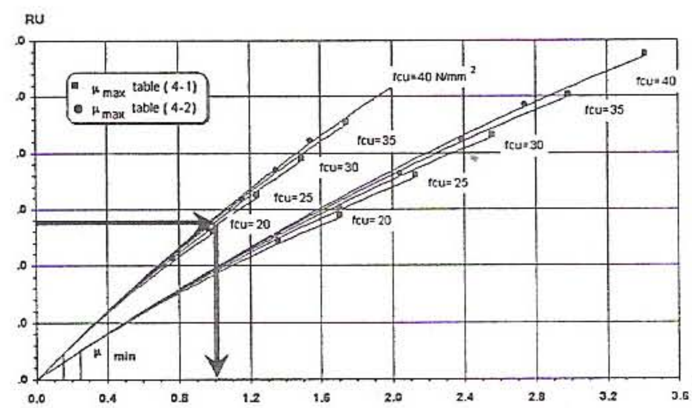
from chart (C) $\Rightarrow J = 0.743$

$$A_s = \frac{M_u}{J.f_y.d} = \frac{300\times10^6}{0.743\times360\times600} \qquad \Rightarrow\Rightarrow\Rightarrow \qquad A_s = 1870mm^2 = 18.70cm^2$$

Example 3:

Design example (1) using charts (A & B):

Assume, d=600mm



$$R_u = \frac{M_u}{b.d^2}$$

$$R_u = \frac{300\times10^6}{300\times600^2} = 2.77778\,MPa$$

from chart (A) $\Rightarrow \mu = 1.038$

$$As = \frac{1.038}{100}\times300\times600 = 1868mm^2 = 18.68cm^2$$

Example 4:

Design example(1) using the first principle method:

Assume, $\quad f_s = f_y \;\;\&\; f_c = f_{cu} \;\&\; d=600mm$

* The force equilibrium:

$$T = C \;\Rightarrow\; A_s\frac{f_y}{\gamma_s} = 0.67\frac{f_{cu}}{\gamma_c}a.b \qquad\qquad a = A_s\frac{f_y}{\gamma_s}\frac{1}{0.67\frac{f_{cu}}{\gamma_c}b} = 0.093446A_s$$

$$\frac{a}{2d} = \frac{0.093446As}{2\times600} = 7.7871\times10^{-5}\,A_s \;......................(1)$$

*** The moment equilibrium:**

$$M_u = (T \text{ or } C).(d - \frac{a}{2}) = A_s \frac{f_y}{\gamma_s}(d - \frac{a}{2})$$

$$M_u = A_s \frac{f_y}{\gamma_s} d(1 - \frac{a}{2d}) \quad \text{...}(2)$$

From equations No. (1& 2) $\quad M_u = A_s.d\frac{f_y}{\gamma_s}[1-(7.7871\times10^{-5}A_s)]$

$$300\times10^6 = 187826.087A_s - 14.6262A_s^{\ 2}$$

$$\therefore 14.6262A_s^{\ 2} - 187826.087A_s + 300\times10^6 = 0$$

$$As = 1869mm^2 = 18.69cm^2$$

$$a = 0.093446A_s = 0.093446\times1869 = 118.58mm$$

$$C = \frac{a}{0.8} = 148.226mm$$

$$\frac{C}{d} = \frac{148.226}{600} = 0.247 < \left(\frac{C_{max.}}{d} = 0.44\right)$$

*** Check for $A_{s\ min.}$**

$$\frac{1.1}{f_y}.b.d = 550mm^2 \qquad \& \qquad 1.3A_{sreq.} = 2429.7mm^2$$

Where, As min. = The least of $[\frac{1.1}{f_y}.b.d]$ or $[1.3\,A_{sreq.}] = 550mm^2$

But not less than 0.15% b.d = 270mm^2

$\therefore A_{smin.} = 550\ mm^2$

$\therefore A_{s\ design} = 1869\ mm^2 = 18.69\ cm^2$

Example 5:

Design the following example using compression steel reinforcement

Given:	M_u = 600 kN.m	f_{cu} = 25 N/mm^2	f_y = 360 N/mm^2
	t = 650 mm	b = 300 mm	

Design:

$$d = C_1\sqrt{\frac{M_u}{f_{cu}.b}} \quad \Rightarrow\Rightarrow\Rightarrow \quad C1 = 2.01 \text{ (use compresion reinf orcement)}$$

$$M_{u\,max.} = R_{max.}\,\frac{f_{cu}}{\gamma_c}\,.\,b\,.\,d^2$$

$$M_{u\,max.} = 0.194 \times \frac{25}{1.5} \times 300 \times 600^2 \times 10^{-6} = 349.20 \text{ kN.m } < 600 \text{ kN.m}$$

$$\Delta M = M_u^{\,*} - M_{u\,max.} = 600 - 349.20 = 250.80 \text{ kN.m}$$

$$\Delta M = 250.80 \times 10^6 = A_s`\,\frac{f_y}{\gamma_s}\,(d - d`) = A_s` \times \frac{360}{1.15}(600 - 50)$$

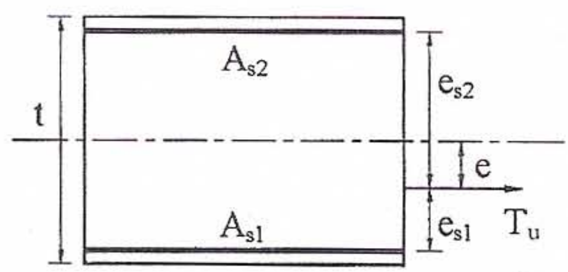
$$\therefore A_s` = 1456.67 mm^2$$

$$A_s = \mu_{max.}\,.\,b\,.\,d + A_s`$$

$$= (5.00 \times 10^{-4} \times 25 \times 300 \times 600) + (1456.67) = 3706.67 mm^2$$

Example 6:

Design of unsymmetrical rectangular section subjected to eccentric tension force using design aids for flexure.



Given: $T_u = 400$ kN $M_u = 100$ kN.m

$t = 750$ mm $b = 250$ mm

$f_{cu} = 25$ N/mm² $f_y = 240$ N/mm²

$$e = \frac{Mu}{Tu} = \frac{100}{400} = 0.25m = 250\,mm \quad \Rightarrow\Rightarrow \quad e < \frac{d - d`}{2}$$

$$e_{s1} = t/2 - e - cover = 750/2 - 250 - 40 = 85\,mm$$

$$e_{s2} = t/2 + e - cover = 750/2 + 250 - 40 = 585\,mm$$

$$A_{s1} = \frac{T_u\,e_{s2}}{e_{s1} + e_{s2}} / \frac{f_y}{\gamma_s} = 1674\,mm^2 \quad \& \quad A_{s2} = \frac{T_u\,e_{s1}}{e_{s1} + e_{s2}} / \frac{f_y}{\gamma_s} = 244\,mm^2$$

Take $A_s = 5 \varnothing 22$ & $A_s` = 2 \varnothing 16$

V) DESIGN OF SEC. SUBJECTED TO ECCENTRIC COMPRESSION FORCE:

a) Balanced conditions : Is the limit between compression and tension failure.

Balanced eccentric force $P_b = 0.67 \dfrac{f_{cu}}{\gamma_c} a_b \, b - \dfrac{A_s f_y}{\gamma_s} + \dfrac{A`_s f_y}{\gamma_s}$

$$P_b = 0.67 \frac{f_{cu}}{\gamma_c} 0.8 \left(\frac{600}{600 + \frac{f_y}{\gamma_s}} \right) b\, d - \frac{A_s f_y}{\gamma_s} + \frac{A`_s f_y}{\gamma_s}$$

$$P_b = 214.4 \left(\frac{f_{cu}}{600 + \frac{f_y}{\gamma_s}} \right) b\, d - \frac{A_s f_y}{\gamma_s} + \frac{A`_s f_y}{\gamma_s}$$

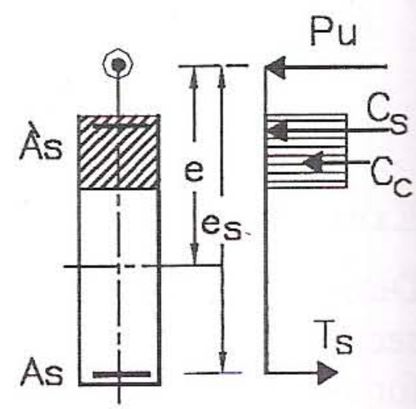
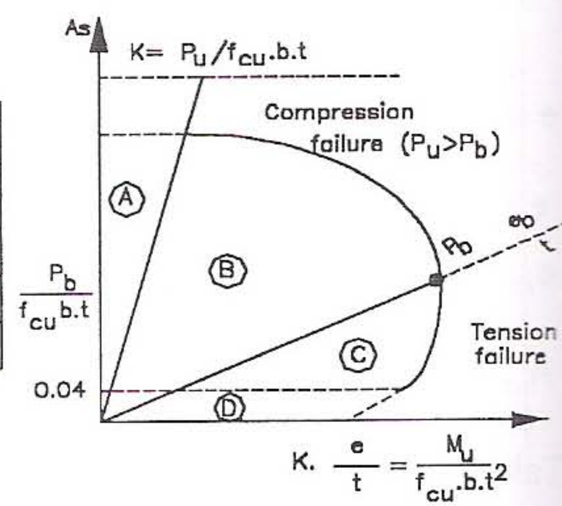


Table of K_b (balanced) $= \dfrac{P_b}{f_{cu}\, b\, t}$

(case of $A`_s = A_s$)

Type of steel (N/mm²)	280/450	360/520	400/600
ζ=0.7	0.219	0.200	0.191
ζ=0.8	0.232	0.212	0.203
ζ=0.9	0.244	0.224	0.214



Use of Interaction Diagrams

(Design charts for design of sections subjected to eccentric compression force P_u)

For different values of f_y, $\mu = A_s / b\,d$, a $\alpha = A'_s / A_s$, $\zeta = \frac{d - d'}{t}$

interaction diagrams have been calculated and are given with their computer

program in Appendix . They are used as follows :

Procedure of design of section:

i- For a given sec. calculate $k = \frac{P_u}{f_{cu}\, b\, t}$

ii- Calculate $k \frac{e}{t} = \frac{M_u}{f_{cu}\, b\, t^2}$

iii- Locate the point on the interaction diagram.

If the point is at the zone A (Eccentric force with min eccentricity $e \leq 0.05\, t$) :

Design as short column.

iv- For column with

separate stirrups : $P_u = 0.35\, f_{cu}\, A_c + 0.67\, A_s\, f_y$

If the point is at zone B (case of compression failure)

iv- Design using the interaction diagram and get ρ :

then $\mu = \rho\, f_{cu} \times 10^{-4}$

$A_s = \mu\, b\, t$, $A'_s = (\frac{A'_s}{A_s})\, \mu\, b\, t = \alpha\, \mu\, b\, t = \alpha\, A_s$

If the point is at the zone C (case of tension failure)

Section can be designed using design charts (A, B, C, D,) for simple bending as follows :

iv- Calculate $e = M_u / P_u$, $e_s = e + t/2 - \text{cover}$, $M_{us} = P_u\, e_s$

v- Use chart B , Calculate C_1 from $d = C_1 \sqrt{M_{us} / f_{cu}\, b}$

vi- From chart get J , thus $A_s = \frac{M_{us}}{j d f_y} - \frac{P_u}{f_y / \gamma_s}$ where $\gamma_s = 1.15$

OR:

v- Use curves C or D, select suitable curve for (f_y , f_{cu})

vi- Calculate k_u from $d = k_u \sqrt{M_{us} / b}$

vii- From chart get μ , thus $A_s = \mu b d - \frac{P_u}{f_y / \gamma_s}$ where $\gamma_s = 1.15$

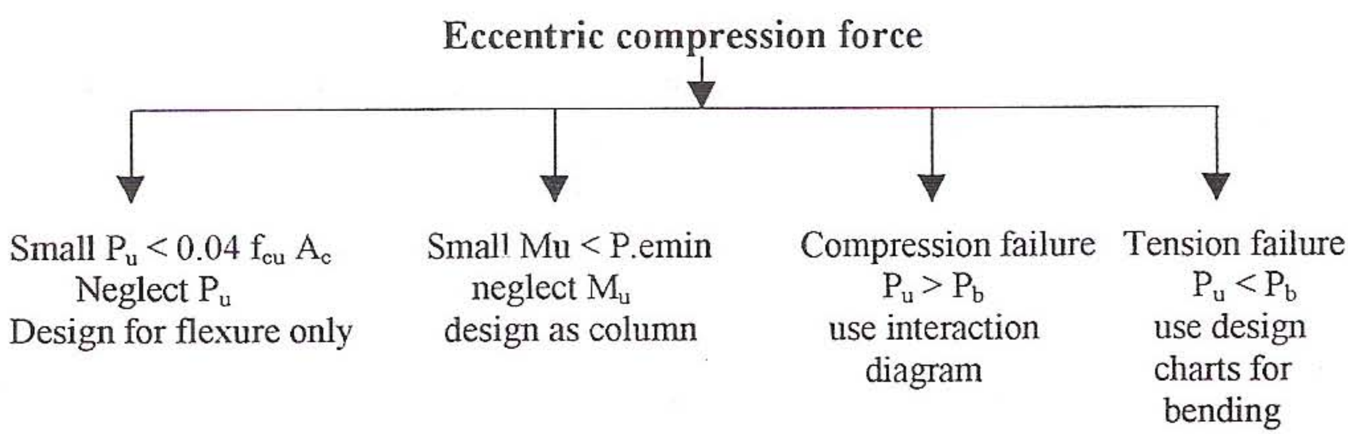
If the point is at the zone D $\left(\frac{P_u}{f_{cu}\, b\, t} \leq 0.04 \right)$

Case of small eccentric force P_u which can be neglected and section is to be designed on simple bending only .

The interaction diagrams are given for values of :

fy = 280 , 360 , 400 N/mm^2
$\alpha = A'_s / A_s = 1$, 0.8 , 0.6
$\zeta = \frac{d - d'}{t} = 0.90$, 0.8

<u>Note :</u> f_{cu} is not a parameter & is included in the chart.



Solved Examples for Sections Subjected to Eccentric Force

Example 1:

Design of a section subjected to eccentric compressive force using interaction diagrams.

Given:

$P_u = 1400$ kN $M_u = 240$ kN.m

$t = 750$ mm $b = 250$ mm

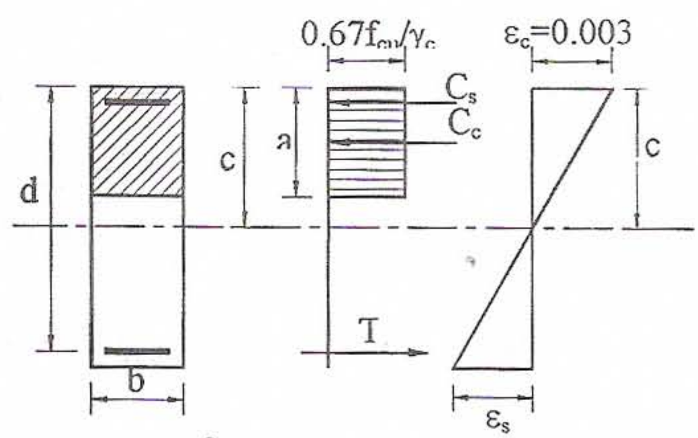
$f_{cu} = 25$ N/mm^2 $f_y = 400$ N/mm^2

$\alpha = 1.00$

Design:

$$\frac{P_u}{f_{cu}.b.t} = \frac{1400\times10^3}{25\times250\times750} = 0.30$$

$$\frac{M_u}{f_{cu}.b.t^2} = \frac{240\times10^6}{25\times250\times750^2} = 0.068$$



From the interaction diagram for $f_y = 400$ N/mm^2, ξ=0.9, α=1.0 {chart 1/18} (Page - 408)

$\Rightarrow\Rightarrow\Rightarrow\Rightarrow \rho = 1.25$

$$e = \frac{Mu}{Pu} = \frac{240}{14000} = 0.17m = 170mm$$

$$\Rightarrow\Rightarrow \quad \frac{e}{t} = \frac{170}{750} = 0.227 \text{ (comp. failure)}$$

$\mu = \rho \,.\, f_{cu} \,.\, 10^{-4} = 1.25 \times 25 \times 10^{-4} = 0.003125$

$A_s = \mu \,.\, b \,.\, t = 0.003125 \times 250 \times 750 = 586$ mm^2

$A_s` = \alpha \,.\, A_s = 1.0 \times 586 = 586$ mm^2

Take $A_s = 3 \varnothing 16$ & $A_s` = 3 \varnothing 16$

Example 2:

Design of a section subjected to eccentric compressive force using interaction diagrams.

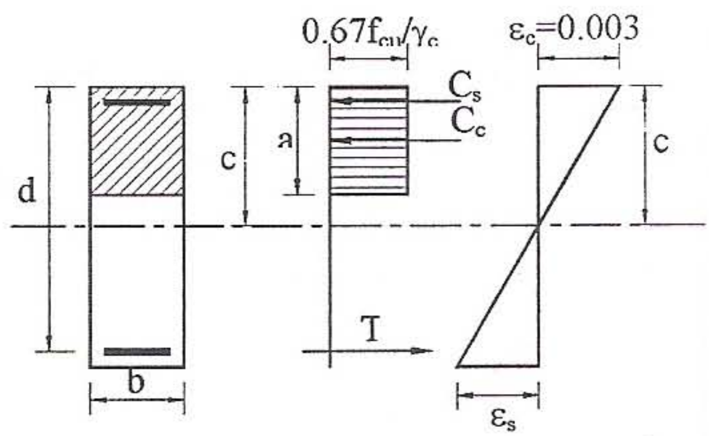
Given: $P_u = 600$ kN $\quad M_u = 320$ kN.m

$t = 750$ mm $\quad b = 250$ mm

$f_{cu} = 25$ N/mm^2 $\quad f_y = 400$ N/mm^2

$\alpha = 0.80$

Design:



$$\frac{P_u}{f_{cu}.b.t} = \frac{600 \times 10^3}{25 \times 250 \times 750} = 0.128$$

$$\frac{M_u}{f_{cu}.b.t^2} = \frac{320 \times 10^6}{25 \times 250 \times 750^2} = 0.091$$

From the interaction diagram for $f_y = 400$ N/mm^2, $\xi=0.9$, $\alpha=0.80$ {chart 7/18} (Page - 414)

$\Rightarrow\Rightarrow\Rightarrow\Rightarrow \rho = 1.5$

$$e = \frac{Mu}{Pu} = \frac{320}{600} = 0.533\text{m} = 533\text{mm}$$

$$\Rightarrow\Rightarrow \quad \frac{e}{t} = \frac{533}{750} = 0.71 \text{ (tension failure)}$$

$\mu = \rho \,.\, f_{cu} \,.\, 10^{-4} = 1.5 \times 25 \times 10^{-4} = 0.00375$

$A_s = \mu \,.\, b \,.\, t = 0.00375 \times 250 \times 750 = 704$ mm^2

$A_s` = \alpha \,.\, A_s = 0.8 \times 704 = 564$ mm^2

Take $A_s = 4 \varnothing 16$ & $A_s` = 3 \varnothing 16$

Example 3:

Design of unsymmetrical rectangular section subjected to eccentric compressive force using interaction diagrams.

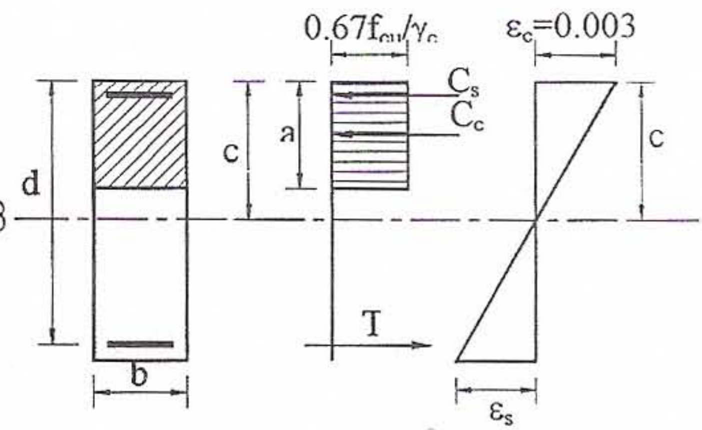
Given: $P_u = 600$ kN $\quad M_u = 320$ kN.m

$t = 750$ mm $\quad b = 250$ mm

$f_{cu} = 25$ N/mm^2 $\quad f_y = 400$ N/mm^2

$\alpha = 0.60$

Design:



$$\frac{P_u}{f_{cu}.b.t} = \frac{600\times10^3}{25\times250\times750} = 0.128$$

$$\frac{M_u}{f_{cu}.b.t^2} = \frac{320\times10^6}{25\times250\times750^2} = 0.091$$

From the interaction diagram for $f_y = 400$ N/mm^2, ξ=0.9, α=0.60 {chart 13/18} (Page - 420)

$\Rightarrow\Rightarrow\Rightarrow\Rightarrow \rho = 1.6$

$$e = \frac{Mu}{Pu} = \frac{320}{600} = 0.533\text{m} = 533\text{mm}$$

$$\Rightarrow\Rightarrow \quad \frac{e}{t} = \frac{533}{750} = 0.71 \text{ (tension failure)}$$

$\mu = \rho \, . \, f_{cu} \, . \, 10^{-4} = 1.6 \times 25 \times 10^{-4} = 0.004$

$A_s = \mu \, . \, b \, . \, t = 0.004 \times 250 \times 750 = 750$ mm^2

$A_s` = \alpha \, . \, A_s = 0.6 \times 750 = 450$ mm^2

Take $A_s = 4 \varnothing 16$ & $A_s` = 4 \varnothing 12$

ULTIMATE LIMIT DESIGN CHARTS FOR SIMPLE BENDING & ECCENTRIC FORCE (TENSION FAILURE) FOR $f_y = 240$ & $360\ N/mm^2$

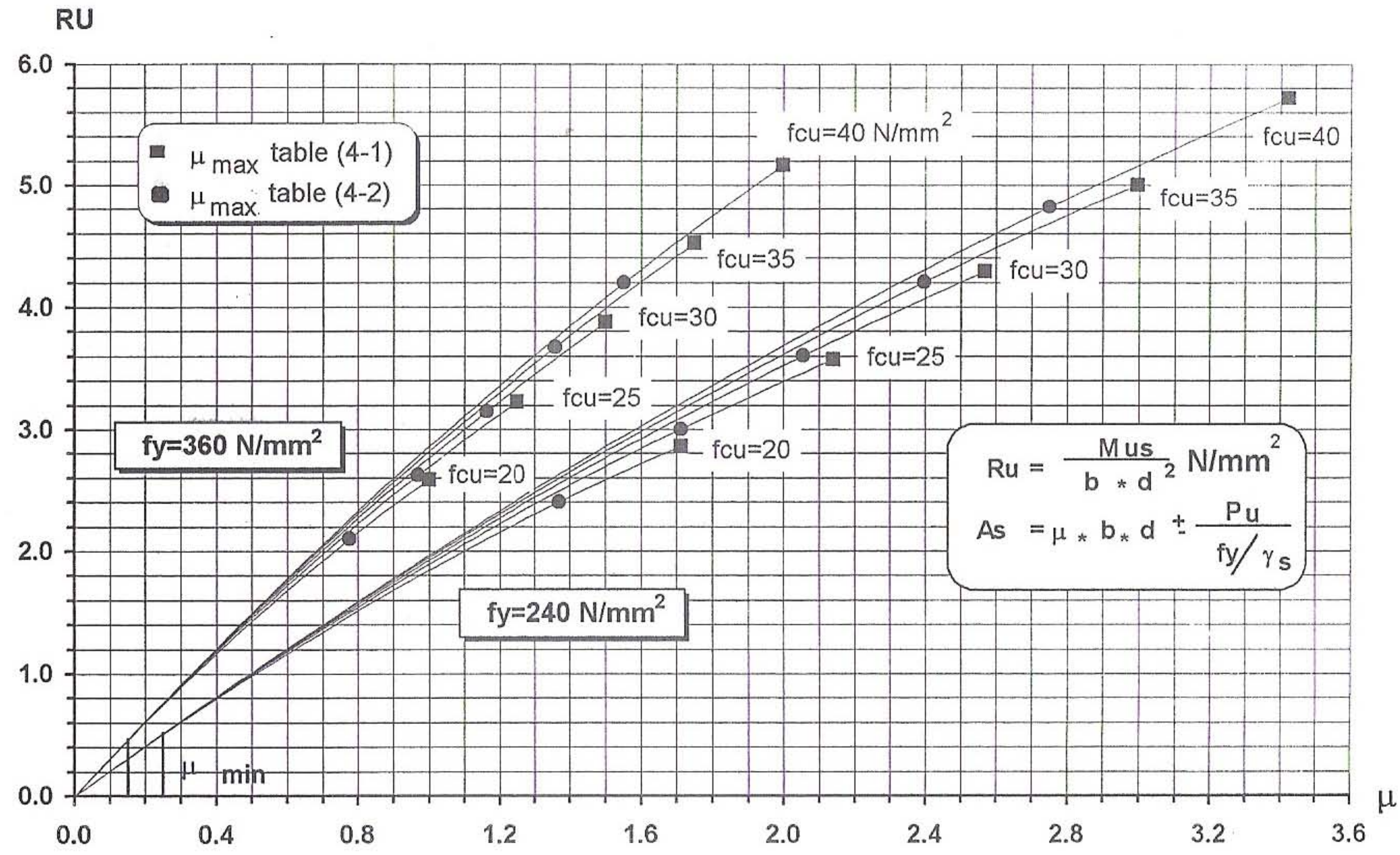


Chart (A)

$$R_u = \mu \frac{f_y}{1.15}\left[1 - 0.973\,\mu\,\frac{f_y}{f_{cu}}\right] \qquad \mu = \frac{100\,A_s}{b * d}$$

ULTIMATE LIMIT DESIGN CHARTS FOR SIMPLE BENDING & ECCENTRIC FORCE (TENSION FAILURE) FOR $f_y = 280$ & $400\ N/mm^2$

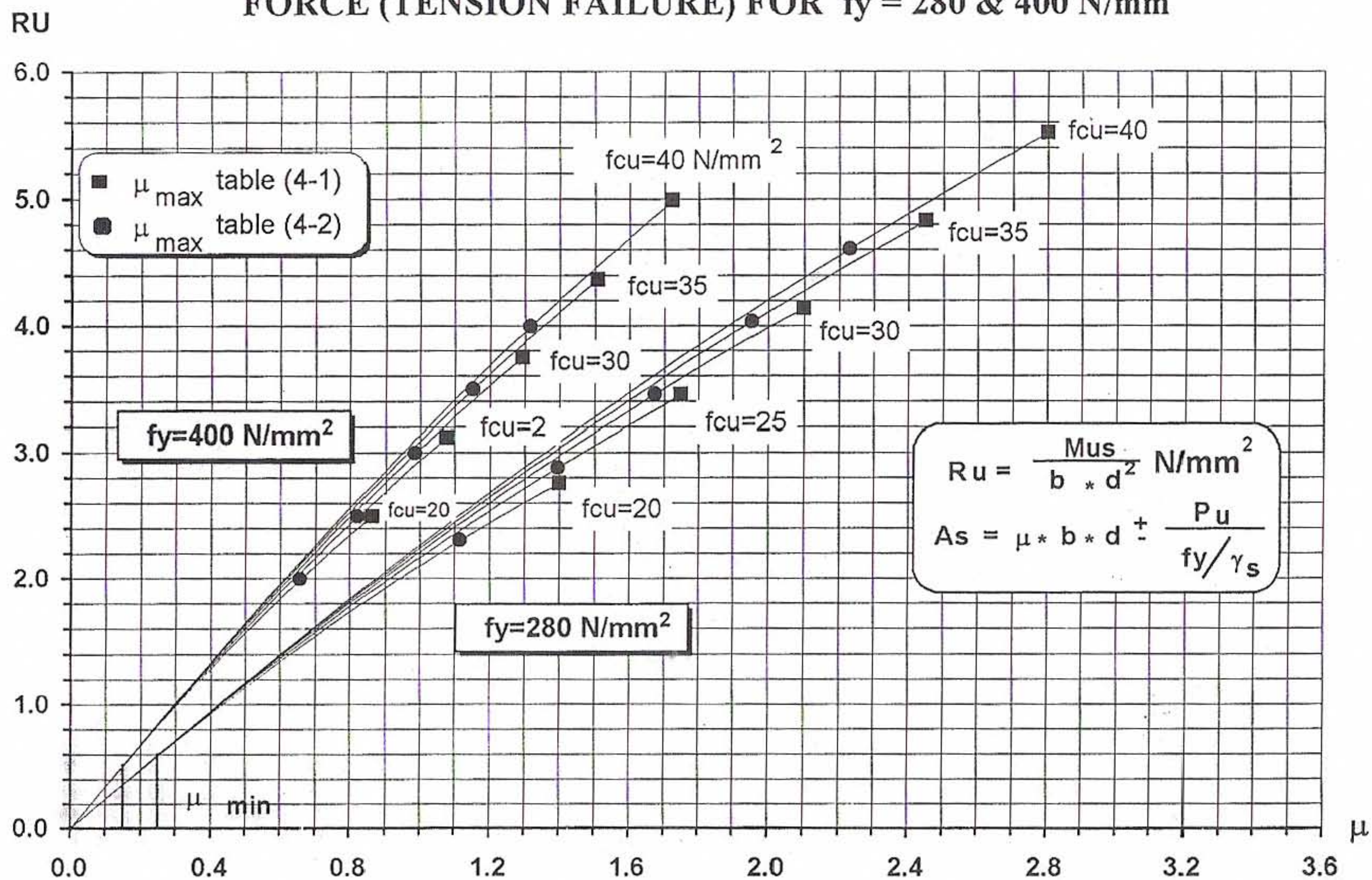


Chart (B)

$\mu_{min.} =$ **The least of** $\frac{1.1}{f_y}$ b . d & 1.3 $As_{req.}$ **But not less than** 0.25%.b.d for st. 240, 0.15%.b.d for st. 360

ULTIMATE LIMIT DESIGN CHARTS FOR SIMPLE BENDING & ECCENTRIC FORCE (TENSION FAILURE)

FOR ALL GRADES OF CONCRETE AND STEEL

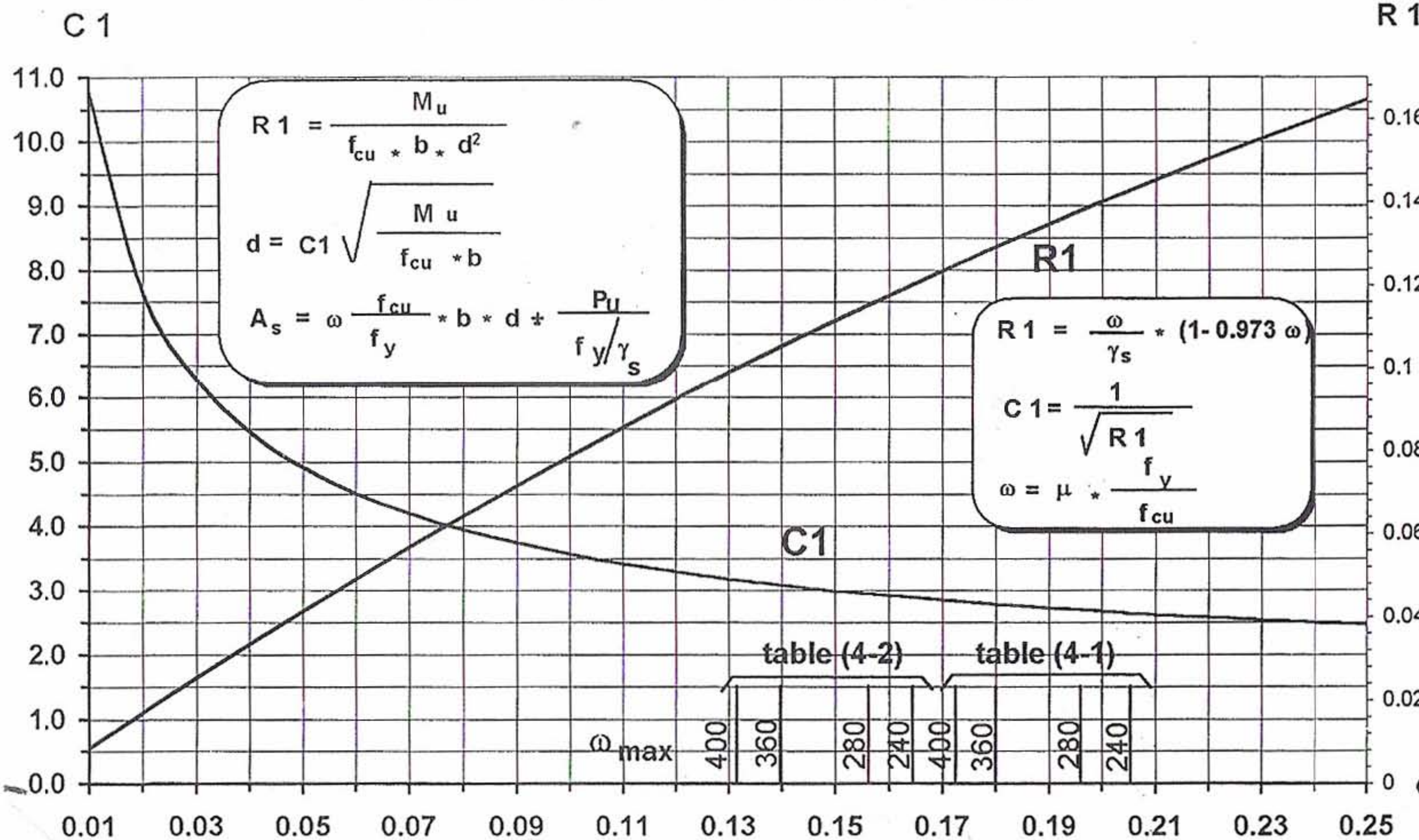


Chart (D)

$\mu_{min.}$ = The least of $\frac{1.1}{f_y}$ b . d & 1.3 $As_{req.}$ But not less than 0.25%.b.d for st. 240, 0.15%.b.d for st. 360

c/d	C1	J
0.1250	4.854	0.826
0.1375	4.640	0.821
0.1500	4.455	0.817
0.1625	4.291	0.813
0.1750	4.146	0.808
0.1875	4.016	0.804
0.2000	3.899	0.800
0.2125	3.793	0.795
0.2250	3.697	0.791
0.2375	3.608	0.786
0.2500	3.526	0.782
0.2625	3.451	0.778
0.2750	3.381	0.773
0.2875	3.316	0.769
0.3000	3.255	0.765
0.3125	3.199	0.760
0.3250	3.146	0.756
0.3375	3.096	0.752
0.3500	3.049	0.747
0.3625	3.004	0.743
0.3750	2.963	0.739
0.3875	2.923	0.734
0.4000	2.885	0.730
0.4125	2.850	0.726
0.4250	2.816	0.721
0.4375	2.784	0.717
0.4500	2.753	0.713
0.4625	2.724	0.708
0.4750	2.696	0.704
0.4875	2.670	0.700
0.5000	2.645	0.695

$$J = \frac{1}{\gamma_s}(1 - 0.4c/d)$$

$$C1 = \frac{1}{\sqrt{0.3573\, c/d\, (1 - 0.4c/d)}}$$

ULTIMATE LIMIT DESIGN CHARTS

FOR SIMPLE BENDING & ECCENTRIC FORCE (TENSION FAILURE)

FOR ALL KINDS OF STEEL AND CONCRETE

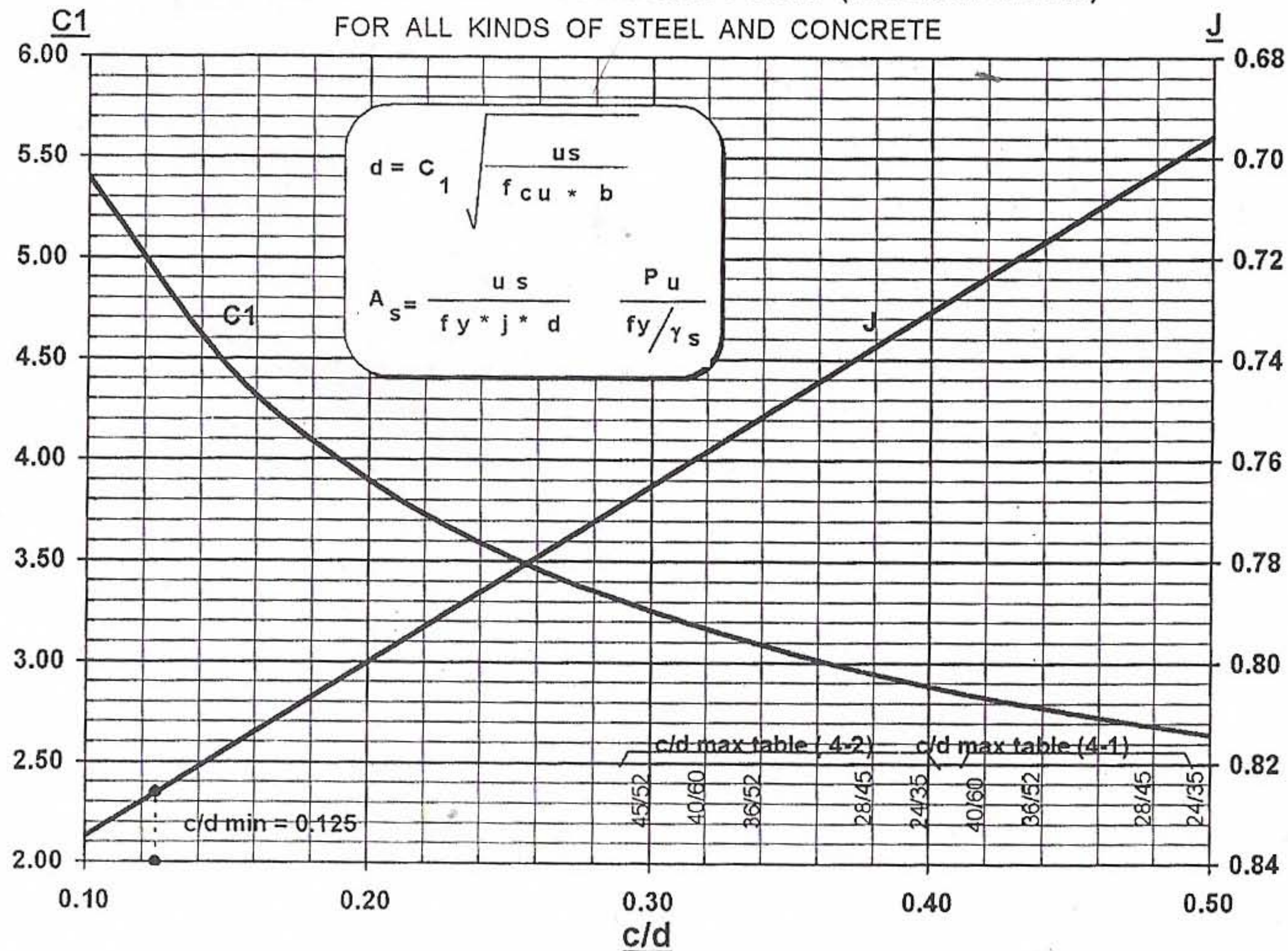


Chart (C)

ULTIMATE LIMIT DESIGN CHARTS FOR SIMPLE BENDING & ECCENTRIC FORCE (TENSION FAILURE)

FOR ALL GRADES OF STEEL

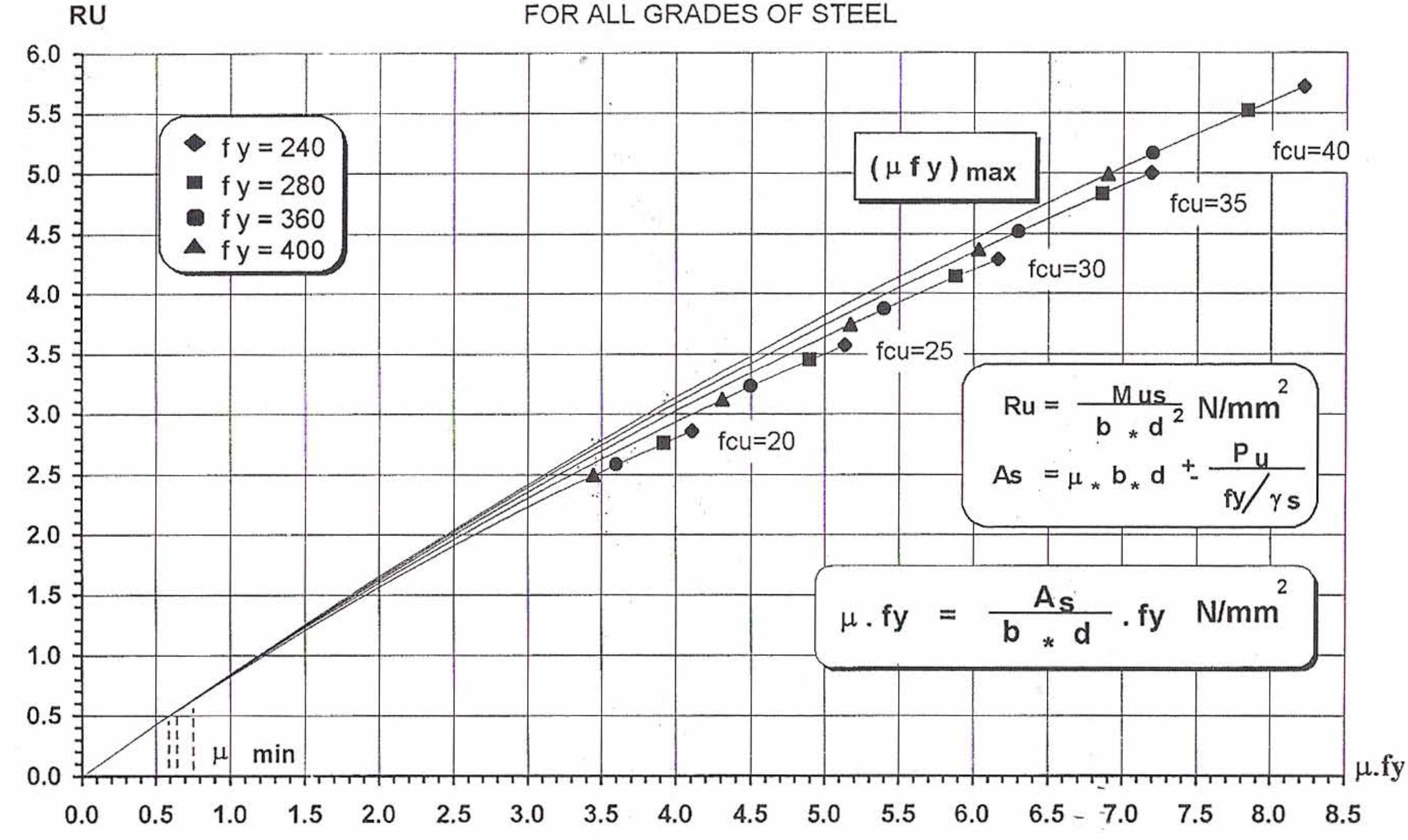


Chart (E)

$$Ru = \mu \frac{f_y}{1.15}\left[1 - 0.973\,\mu \frac{f_y}{f_{cu}}\right]$$

ULTIMATE LI0MIT DESIGN TABLES

(μ) Values for (f_y=280) N/mm^2					
R	fcu=40	fcu=35	fcu=30	fcu=25	fcu=20
0.587					0.250
0.591				0.250	0.251
0.595			0.250	0.251	0.253
0.598		0.250	0.251	0.253	0.254
0.6	0.251	0.251	0.252	0.253	0.255
0.8	0.336	0.337	0.339	0.341	0.345
1	0.423	0.425	0.427	0.431	0.437
1.2	0.511	0.513	0.517	0.523	0.531
1.4	0.599	0.603	0.609	0.616	0.629
1.6	0.690	0.695	0.702	0.712	0.730
1.8	0.781	0.788	0.797	0.811	0.834
2	0.873	0.882	0.894	0.912	0.942
2.2	0.967	0.978	0.993	1.016	1.055
2.304	1.017	1.029	1.046	1.071	1.116
2.4	1.063	1.076	1.095	1.123	1.173
2.6	1.159	1.175	1.198	1.234	1.297
2.759	1.237	1.256	1.283	1.324	1.400
2.8	1.258	1.277	1.305	1.348	
2.88	1.298	1.318	1.348	1.395	
3	1.358	1.380	1.414	1.467	
3.2	1.459	1.486	1.526	1.590	
3.4	1.563	1.594	1.641	1.718	
3.449	1.588	1.621	1.670	1.750	
3.456	1.592	1.625	1.674		
3.6	1.668	1.705	1.760		
3.8	1.775	1.818	1.883		
4	1.885	1.934	2.010		
4.033	1.903	1.953	2.031		
4.138	1.962	2.016	2.100		
4.2	1.996	2.053			
4.4	2.111	2.176			
4.6	2.227	2.302			
4.609	2.232	2.307			
4.8	2.346	2.432			
4.828	2.363	2.450			
5	2.469				
5.2	2.594				
5.4	2.723				
5.518	2.800				

(μ) Values for (f_y=400) N/mm^2					
R	fcu=40	fcu=35	fcu=30	fcu=25	fcu=20
0.505					0.150
0.51			0.150	0.150	0.151
0.514	0.150	0.150	0.151	0.151	0.152
0.55	0.161	0.161	0.162	0.162	0.163
0.6	0.175	0.176	0.177	0.177	0.179
0.8	0.235	0.236	0.237	0.239	0.241
1	0.296	0.297	0.299	0.302	0.306
1.2	0.357	0.359	0.362	0.366	0.372
1.4	0.420	0.422	0.426	0.431	0.440
1.6	0.483	0.486	0.491	0.499	0.511
1.8	0.547	0.551	0.558	0.568	0.584
1.995	0.610	0.616	0.624	0.637	0.658
2	0.611	0.617	0.626	0.638	0.660
2.2	0.677	0.685	0.695	0.711	0.739
2.4	0.744	0.753	0.766	0.786	0.821
2.495	0.776	0.786	0.800	0.823	0.862
2.6	0.812	0.823	0.839	0.864	
2.8	0.880	0.894	0.913	0.944	
2.994	0.948	0.964	0.987	1.024	
3	0.950	0.966	0.990	1.027	
3.119	0.993	1.010	1.036	1.077	
3.2	1.022	1.040	1.068		
3.4	1.094	1.116	1.149		
3.49	1.127	1.151	1.186		
3.6	1.168	1.193	1.232		
3.744	1.222	1.250	1.293		
3.8	1.243	1.273			
3.99	1.316	1.350			
4	1.319	1.354			
4.2	1.398	1.437			
4.365	1.463	1.508			
4.4	1.477				
4.6	1.559				
4.8	1.642				
4.99	1.724				

Assume; b & d

$R_u = M_u / b.d^2$ N/mm^2

From table Find (μ) value

$A_s = (\mu/100) \therefore (b.d)$

$\mu_{min.}$ = The least of $\frac{1.1}{f_y}$ b . d & 1.3 $A_{s\,req.}$ **But not less than** 0.25%.b.d for st. 280, 0.15%.b.d for st. 400

ULTIMATE LIMIT DESIGN TABLE FOR SECTIONS SUBJECT TO SIMPLE BENDING WITH COMPRESSION REINFORCEMENT

(According to table 4 - 1)

M*u = Max. bending moment without compression rft.

$\overline{Mu}$ = Allowed bending moment using compretion rft.

$$\mu' = A'_s / A_c \quad , \quad \mu\% = \mu^*_{max} + \mu'$$

			$\frac{M^*_u max}{b\ d^2}$	d'/d	$\mu' = 0.2$			$\mu' = 0.3$				$\mu' = 0.4$			
f_y	f_{cu}	M*max		0.05	0.075	0.1	0.15	0.05	0.075	0.1	0.15	0.05	0.075	0.1	0.15
240	20	17.02	2.85	32.4	32.3	32.2	32.0	34.4	34.3	34.1	33.8	36.4	36.2	36.0	35.6
	22.5	19.15	3.20	36.0	35.9	35.8	35.6	38.0	37.8	37.7	37.3	40.0	39.7	39.5	39.1
	25	21.27	3.56	39.5	39.4	39.3	39.1	41.5	41.4	41.2	40.9	43.5	43.3	43.1	42.7
	27.5	23.40	3.91	43.1	43.0	42.9	42.7	45.1	44.9	44.8	44.5	47.1	46.9	46.6	46.2
	30	25.53	4.27	46.7	46.6	46.4	46.2	48.6	48.5	48.3	48.0	50.6	50.4	50.2	49.8
280	20	13.98	2.76	32.2	32.1	32.0	31.7	34.5	34.3	34.1	33.8	36.8	36.6	36.3	35.8
	22.5	15.73	3.10	35.6	35.5	35.4	35.2	38.0	37.8	37.6	37.2	40.3	40.0	39.8	39.3
	25	17.48	3.45	39.1	39.0	38.8	38.6	41.4	41.2	41.0	40.7	43.7	43.5	43.2	42.7
	27.5	19.23	3.79	42.5	42.4	42.3	42.0	44.8	44.7	44.5	44.1	47.2	46.9	46.7	46.2
	30	20.98	4.14	46.0	45.9	45.7	45.5	48.3	48.1	47.9	47.6	50.6	50.4	50.1	49.6
360	20	10.05	2.59	31.9	31.7	31.6	31.2	34.8	34.6	34.4	33.9	37.8	37.5	37.2	36.6
	22.5	11.30	2.92	35.1	35.0	34.8	34.5	38.1	37.9	37.6	37.1	41.1	40.7	40.4	39.8
	25	12.56	3.24	38.4	38.2	38.0	37.7	41.3	41.1	40.9	40.4	44.3	44.0	43.7	43.0
	27.5	13.82	3.56	41.6	41.4	41.3	41.0	44.6	44.3	44.1	43.6	47.5	47.2	46.9	46.3
	30	15.07	3.89	44.8	44.7	44.5	44.2	47.8	47.6	47.3	46.9	50.8	50.5	50.2	49.5
400	20	8.71	2.52	31.8	31.6	31.4	31.1	35.1	34.8	34.6	34.0	38.4	38.0	37.7	37.0
	22.5	9.80	2.83	34.9	34.7	34.6	34.2	38.2	38.0	37.7	37.2	41.5	41.2	40.8	40.1
	25	10.89	3.15	38.1	37.9	37.7	37.4	41.4	41.1	40.9	40.3	44.7	44.3	44.0	43.3
	27.5	11.98	3.46	41.2	41.0	40.9	40.5	44.5	44.3	44.0	43.5	47.8	47.5	47.1	46.4
	30	13.07	3.78	44.4	44.2	44.0	43.7	47.7	47.4	47.1	46.6	51.0	50.61	50.3	49.6

ULTIMATE LIMIT DESIGN TABLE FOR SECTIONS SUBJECT TO SIMPLE BENDING WITH COMPRESSION REINFORCEMENT

(Allowing for moment redistribution of ± 10%)

(According to table 4 - 2)

$$\mu' = A'_s / A_c \quad , \quad \mu\% = \mu^*_{max} + \mu'$$

f_y	f_{cu}	M*max	$M^*_u max$ / $b\,d^2$	d'/d	μ' = 0.2			μ' = 0.3				μ' = 0.4			
				0.05	0.075	0.1	0.15	0.05	0.075	0.1	0.15	0.05	0.075	0.1	0.15
240	20	13.72	2.41	28.0	27.9	27.8	27.6	30.0	29.8	29.7	29.4	32.0	31.8	31.6	31.1
	22.5	15.44	2.71	31.0	30.9	30.8	30.6	33.0	32.8	32.7	32.4	35.0	34.8	34.6	34.23
	25	17.15	3.01	34.0	33.9	33.8	33.6	36.0	35.9	35.7	35.4	38.0	37.8	37.6	7.2
	27.5	18.87	3.31	37.0	36.9	36.8	36.6	39.0	38.9	38.7	38.4	41.0	40.8	40.6	40.2
	30	20.58	3.61	40.0	39.9	39.8	39.6	42.0	41.9	41.7	41.4	44.0	43.8	43.6	43.2
280	20	11.15	2.30	27.7	27.5	27.4	27.2	30.0	29.8	29.6	29.2	32.3	32.0	31.8	31.3
	22.5	12.55	2.59	30.5	30.4	30.3	30.1	32.9	32.7	32.5	32.1	35.2	34.9	34.7	34.2
	25	13.94	2.88	33.4	33.3	33.2	32.9	35.7	35.6	35.4	35.0	38.0	37.8	37.6	37.1
	27.5	15.34	3.17	36.3	36.2	36.1	35.8	38.6	38.4	38.2	37.9	40.9	40.7	40.4	40.0
	30	16.73	3.46	39.2	39.1	38.9	38.7	41.5	41.3	41.1	40.8	43.8	43.6	43.3	42.8
360	20	7.84	2.12	27.1	27.0	26.8	26.5	30.1	29.9	29.6	29.2	33.1	32.8	32.4	31.8
	22.5	8.82	2.38	29.8	29.6	29.5	29.1	32.7	32.5	32.3	31.8	35.7	35.4	35.1	34.5
	25	9.80	2.65	32.4	32.3	32.1	31.8	35.4	35.2	34.9	34.5	38.4	38.1	37.7	37.1
	27.5	10.78	2.91	35.1	34.9	34.8	34.4	38.0	37.8	37.6	37.1	41.0	40.7	40.4	39.8
	30	11.76	3.18	37.7	37.6	37.4	37.1	40.7	40.5	40.2	39.8	43.7	43.4	43.0	42.4
400	20	6.72	2.03	26.9	26.8	26.6	26.2	30.2	30.0	29.7	29.2	33.5	33.2	32.9	32.2
	22.5	7.56	2.29	29.5	29.3	29.1	28.8	32.8	32.5	32.3	31.7	36.1	35.7	35.4	34.7
	25	8.40	2.54	32.0	31.8	31.7	31.3	35.3	35.1	34.8	34.3	38.6	38.3	37.9	37.2
	27.5	9.24	2.80	34.6	34.4	34.2	33.9	37.9	37.6	37.3	36.8	41.2	40.8	40.5	39.8
	30	10.08	3.05	37.1	36.9	36.8	36.4	40.4	40.1	39.9	39.4	43.7	43.4	43.0	42.3

INTERACTION

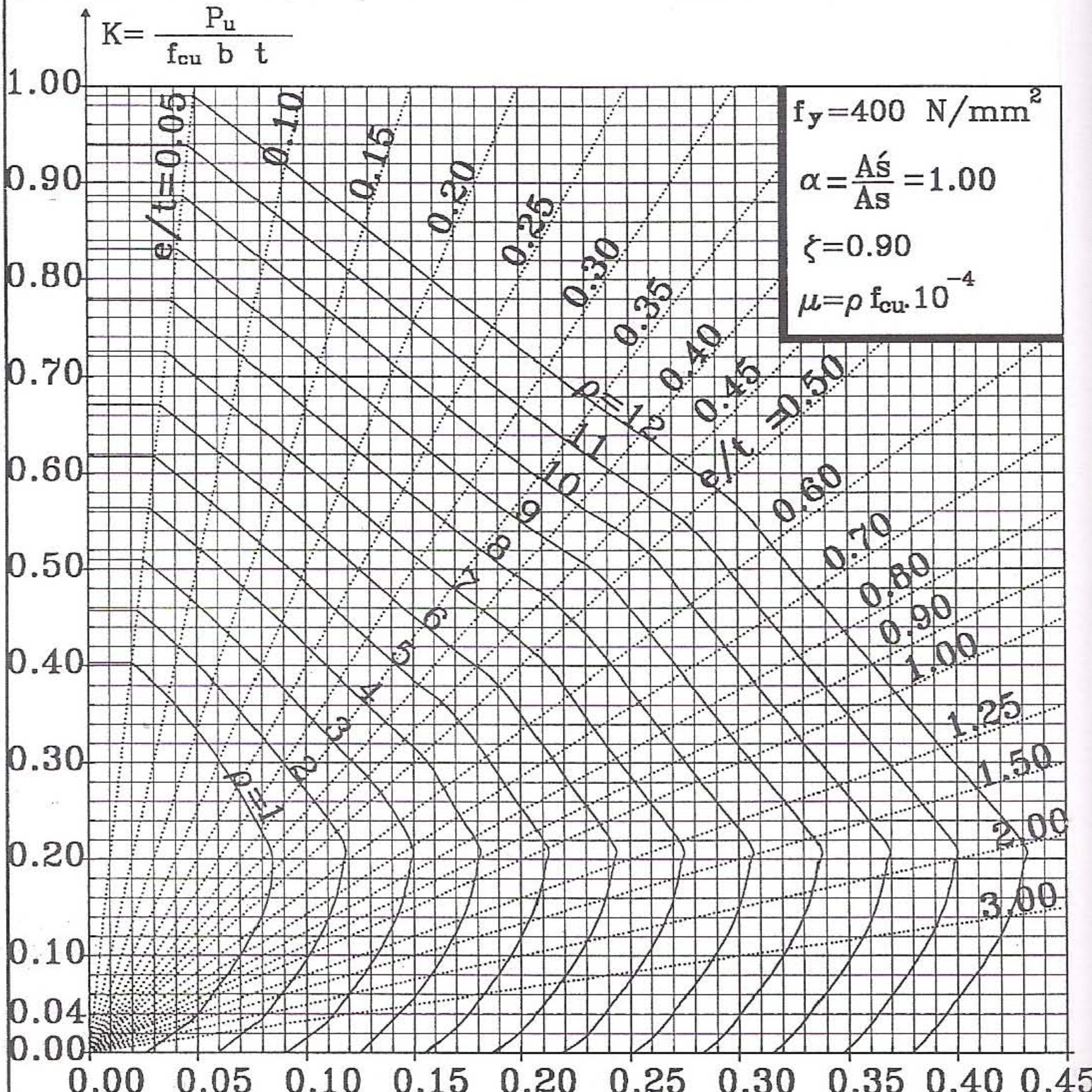
DIAGRAMS

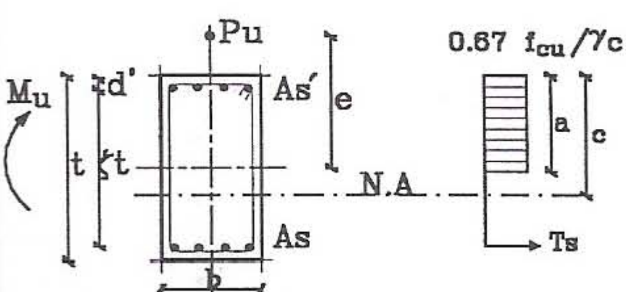
FOR
DESIGN OF SECTIONS
SUBJECTED TO ECCENTRIC
ULTIMATE COMPRESSION
FORCE

El-Behairy R.C. Design Handbook

INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES





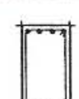
$\mu = \rho \, f_{cu} \cdot 10^{-4}$

$K \dfrac{e}{t} = \dfrac{M_u}{f_{cu} \; b \; t^2}$

$As = \mu \; b \; t$

$As' = \alpha \; As$

$\zeta = \dfrac{d - d'}{t}$

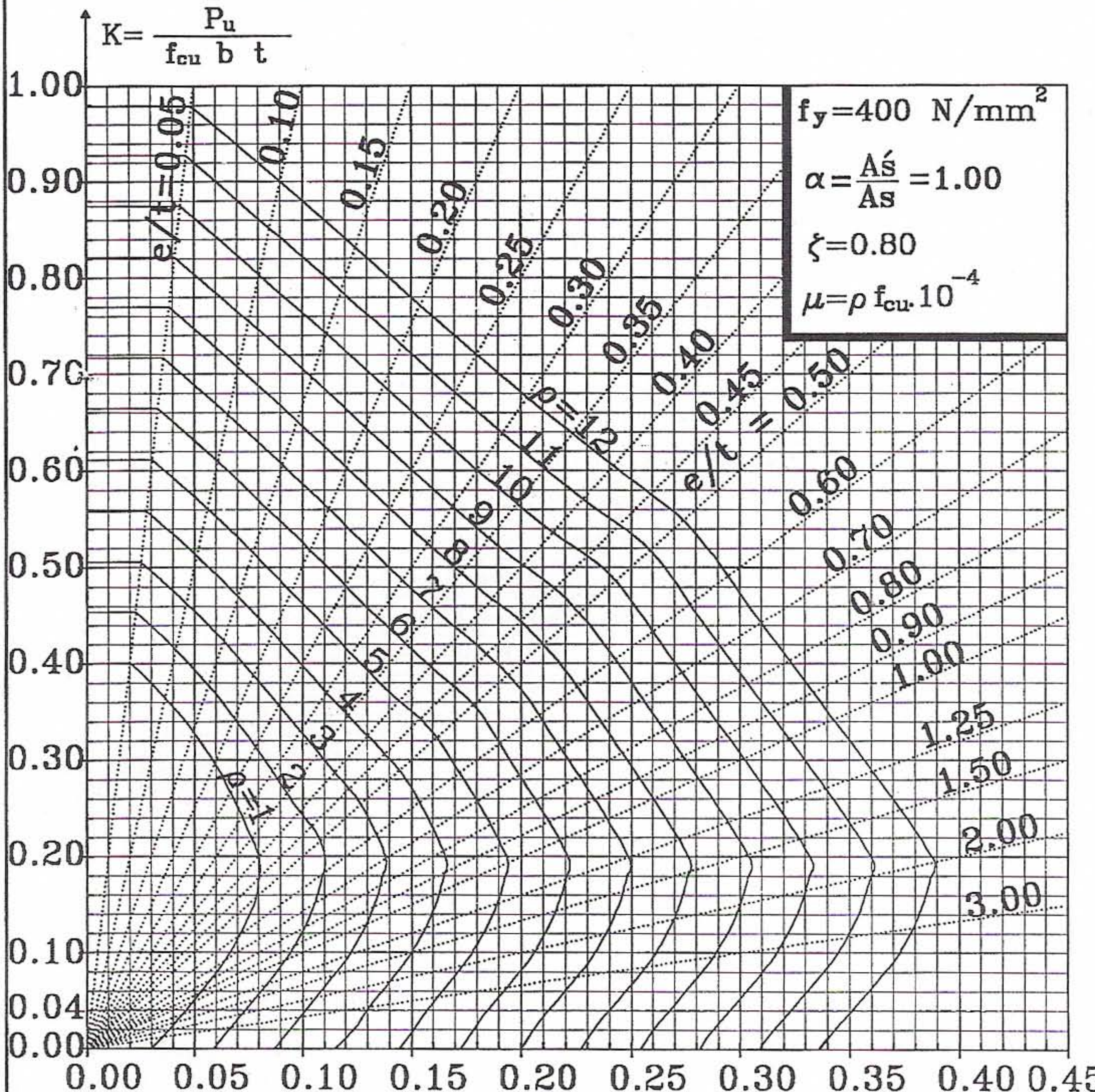


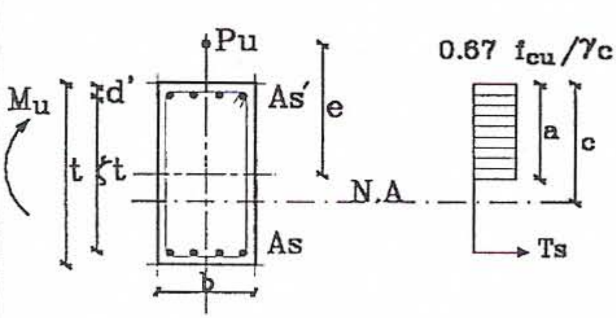
R–Sec.

1/18

INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES





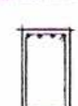
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$As = \mu\ b\ t$

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$\zeta = \dfrac{d-d'}{t}$

$K\dfrac{e}{t} = \dfrac{M_u}{f_{cu}\ b\ t^2}$



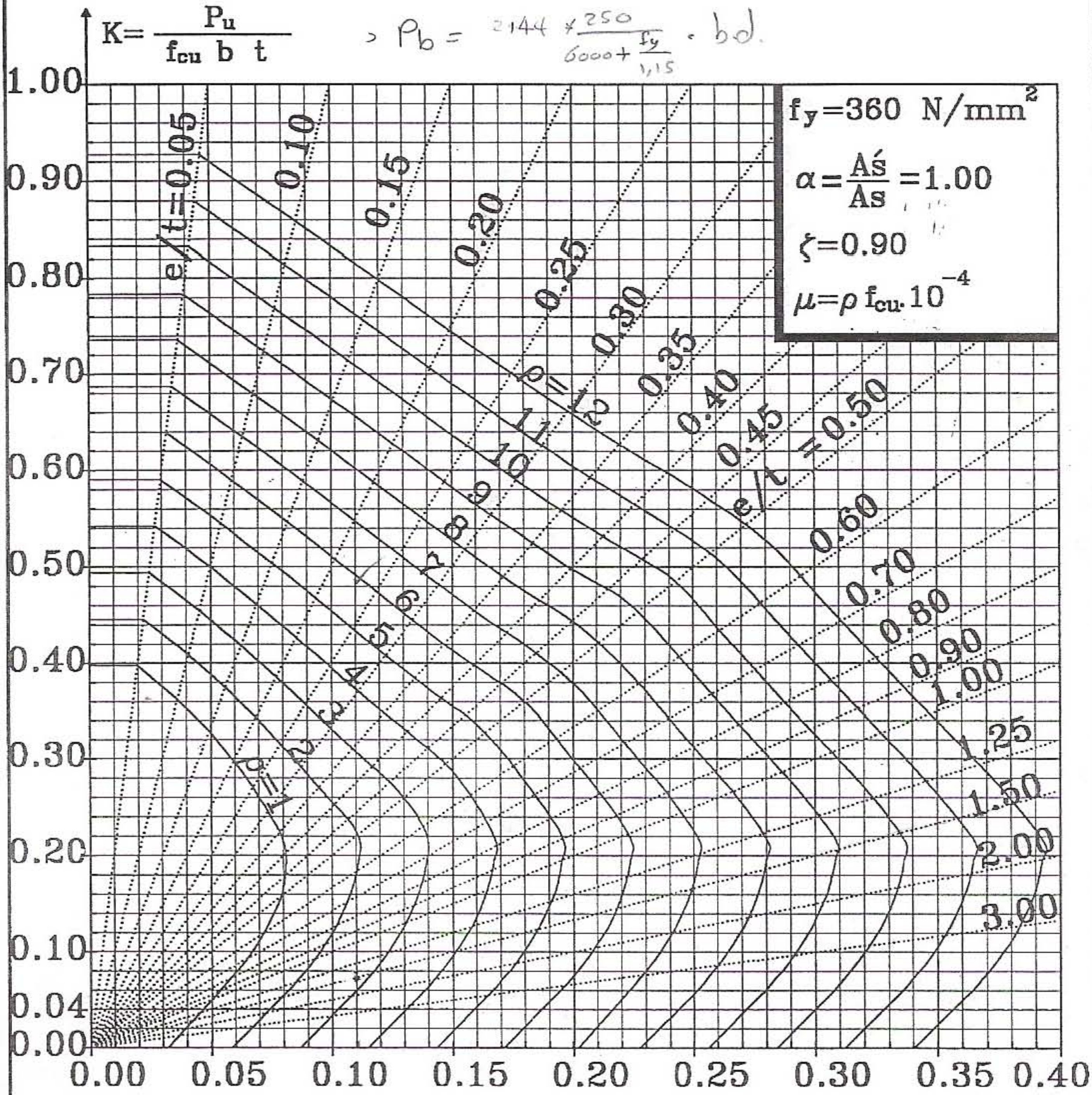
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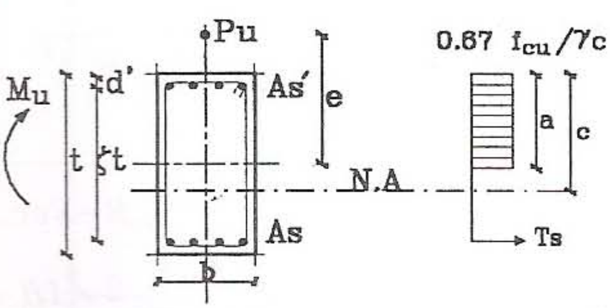
2 / 18

El–Behairy R.C.Design Handbook "Interaction Diagrams"

INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES





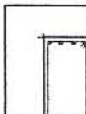
$\mu = \rho \, f_{cu} \cdot 10^{-4}$

$As = \mu \, b \, t$

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$K \dfrac{e}{t} = \dfrac{M_u}{f_{cu} \, b \, t^2}$

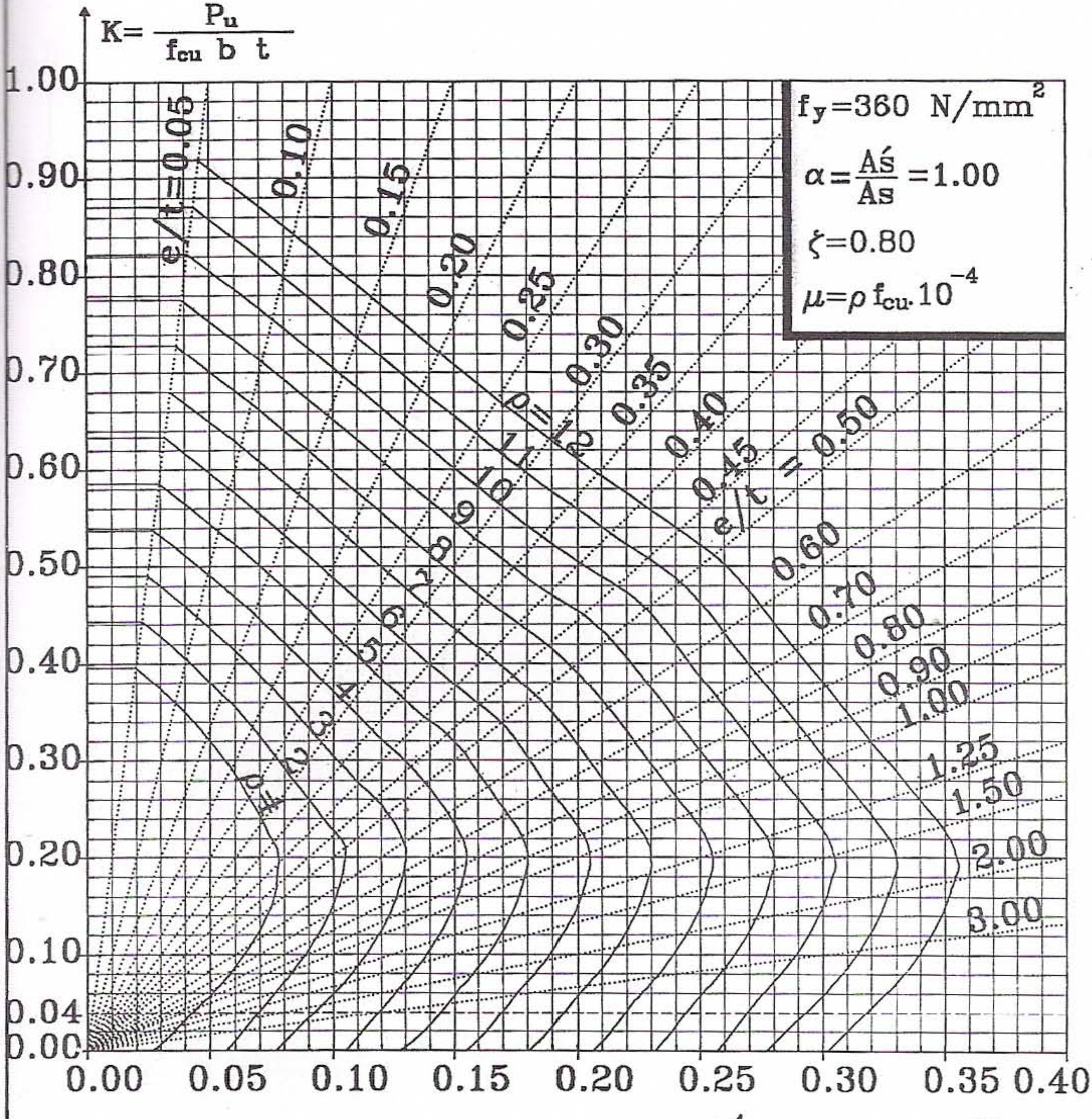


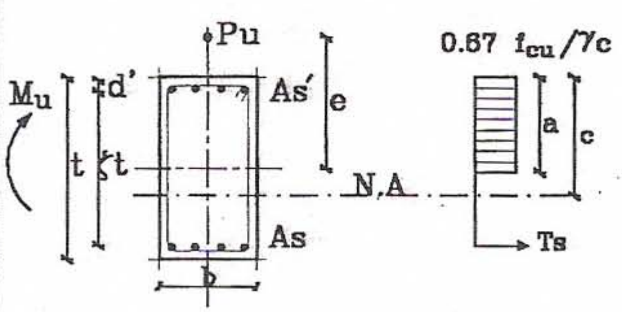
R−Sec.

3/18

INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES





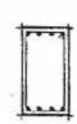
$\mu = \rho\, f_{cu} \cdot 10^{-4}$

$K\,\dfrac{e}{t} = \dfrac{M_u}{f_{cu}\, b\, t^2}$

$As = \mu\, b\, t$

$As' = \alpha\, As$

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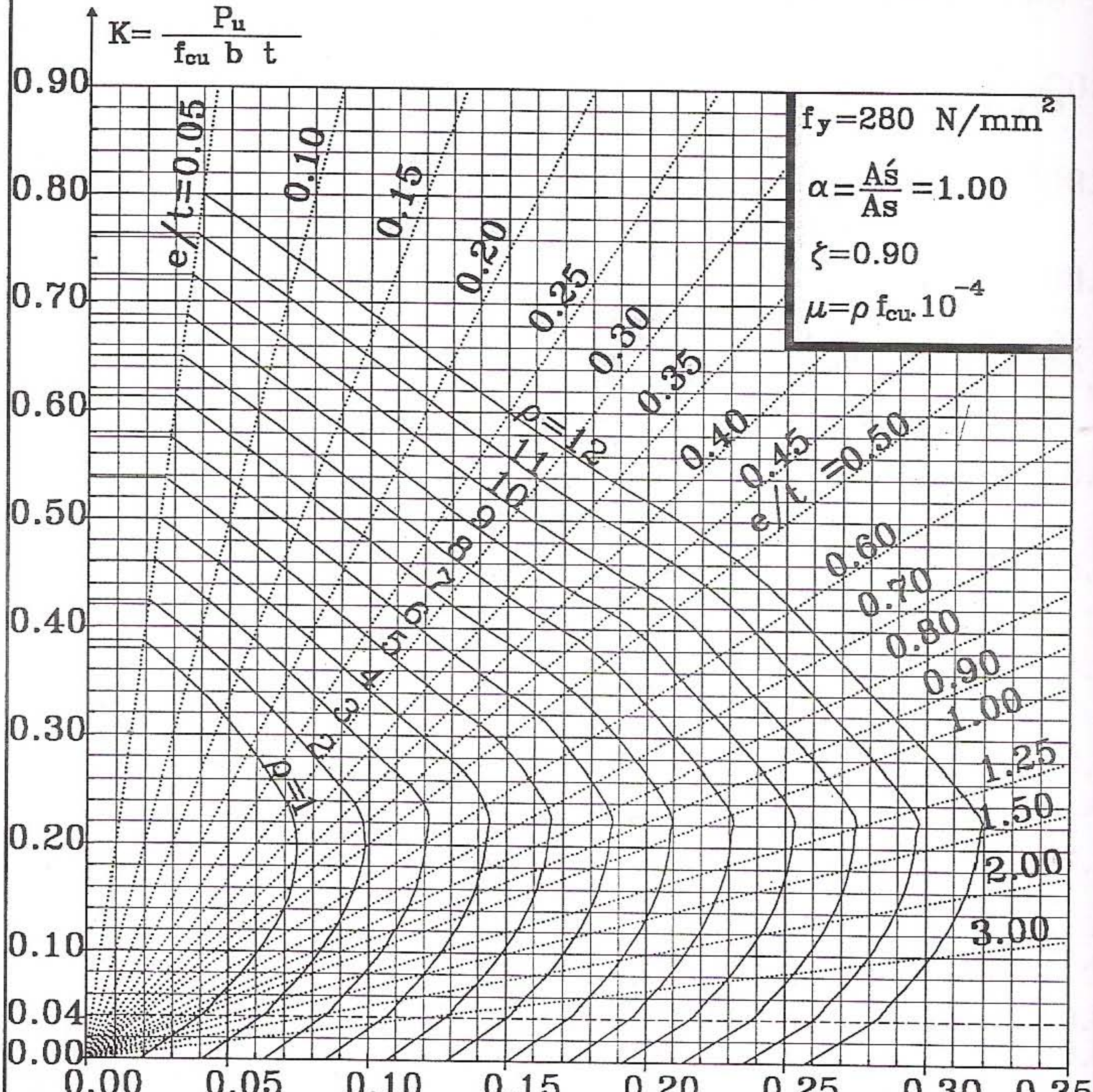


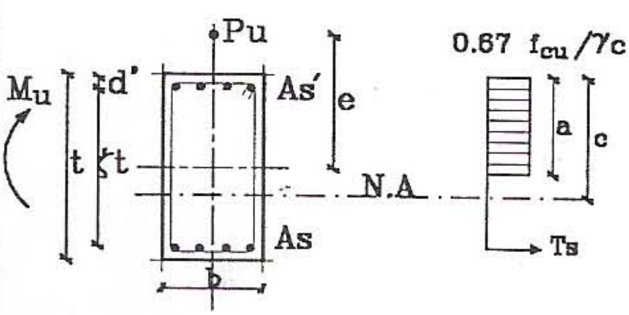
R–Sec.

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INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES





$\mu = \rho \, f_{cu} \cdot 10^{-4}$

$As = \mu \; b \; t$

$A\acute{s} = \alpha \; As$

$\zeta = \dfrac{d - d'}{t}$

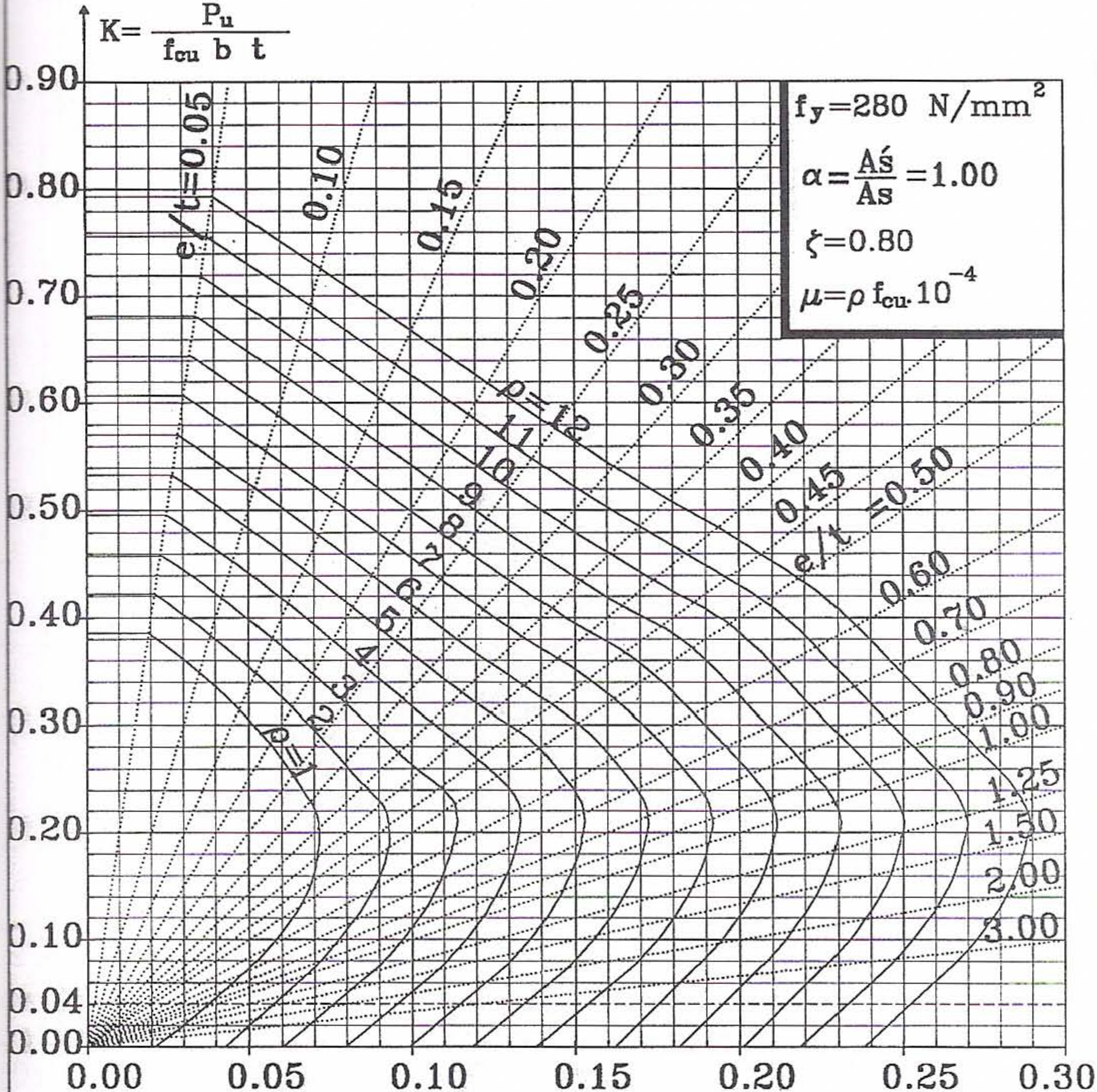
$K \dfrac{e}{t} = \dfrac{M_u}{f_{cu} \; b \; t^2}$

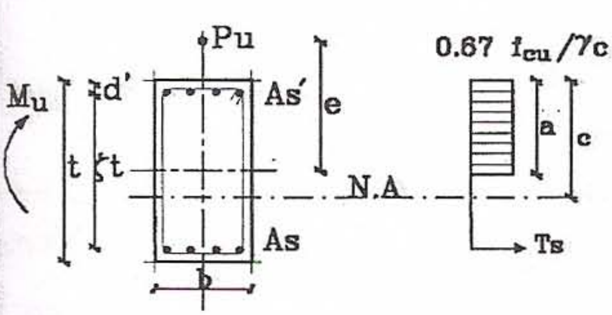
R–Sec.

5/18

INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES





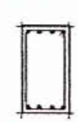
$\mu = \rho \, f_{cu} \cdot 10^{-4}$

$As = \mu \cdot b \; t$

$A\acute{s} = \alpha \; As$

$\zeta = \dfrac{d - d'}{t}$

$K \dfrac{e}{t} = \dfrac{M_u}{f_{cu} \; b \; t^2}$



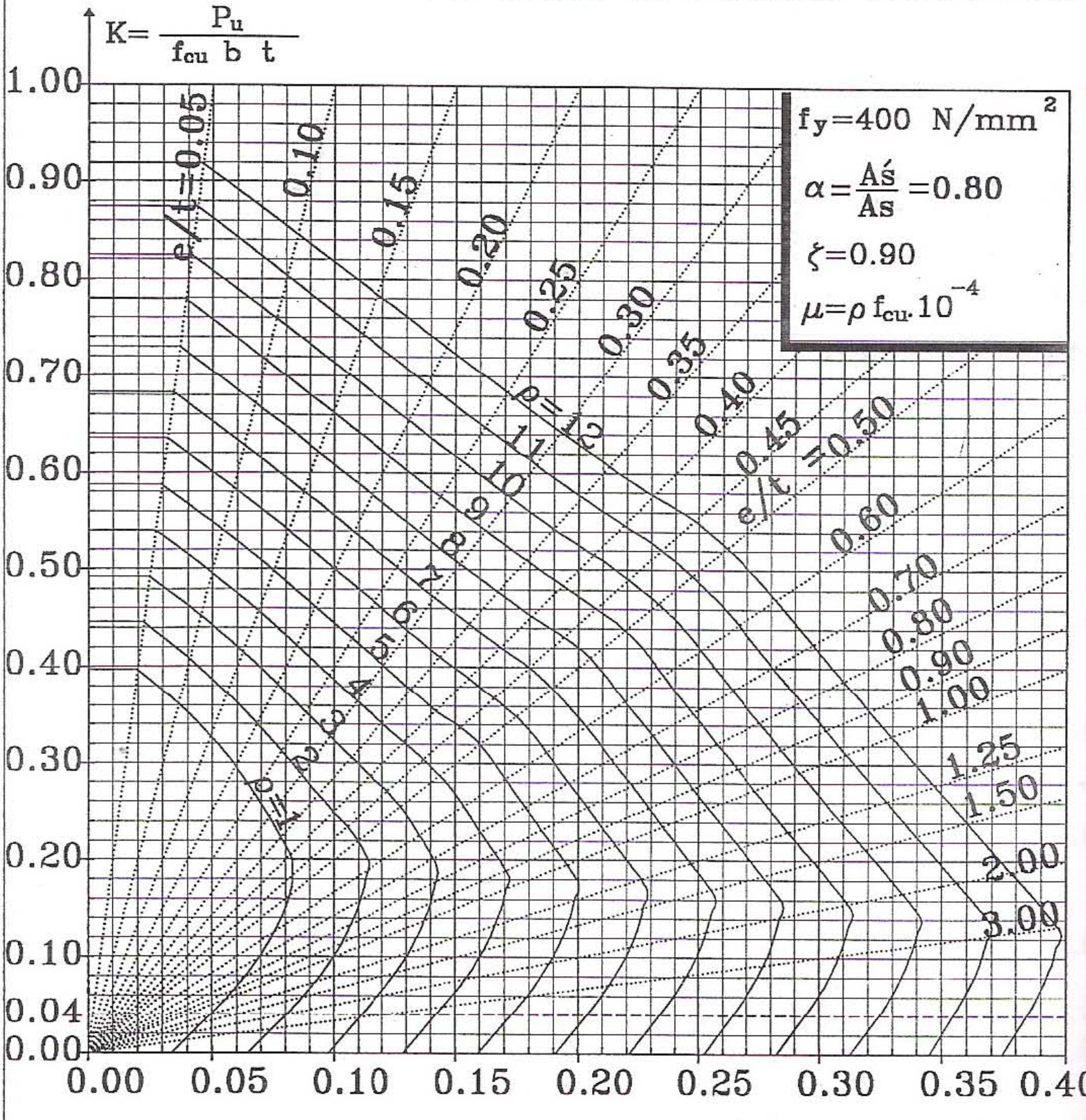
R–Sec.

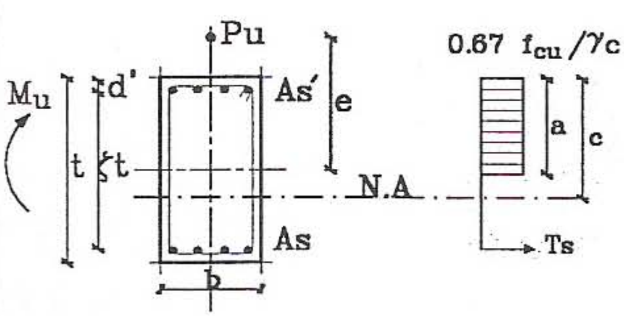
6/18

El–Behairy R.C.Design Handbook "Interaction Diagrams"

INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES





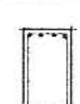
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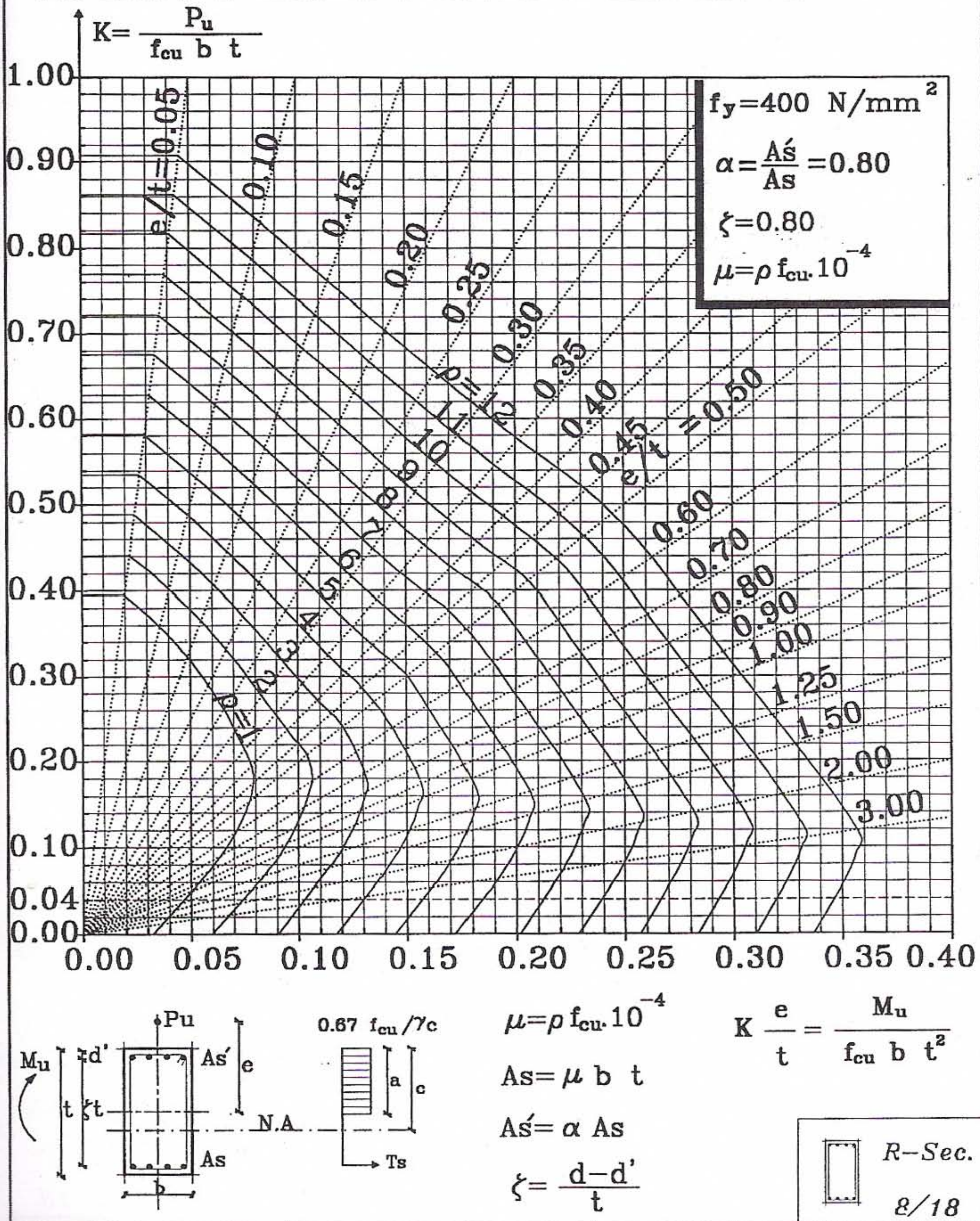


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INTERACTION DIAGRAMS

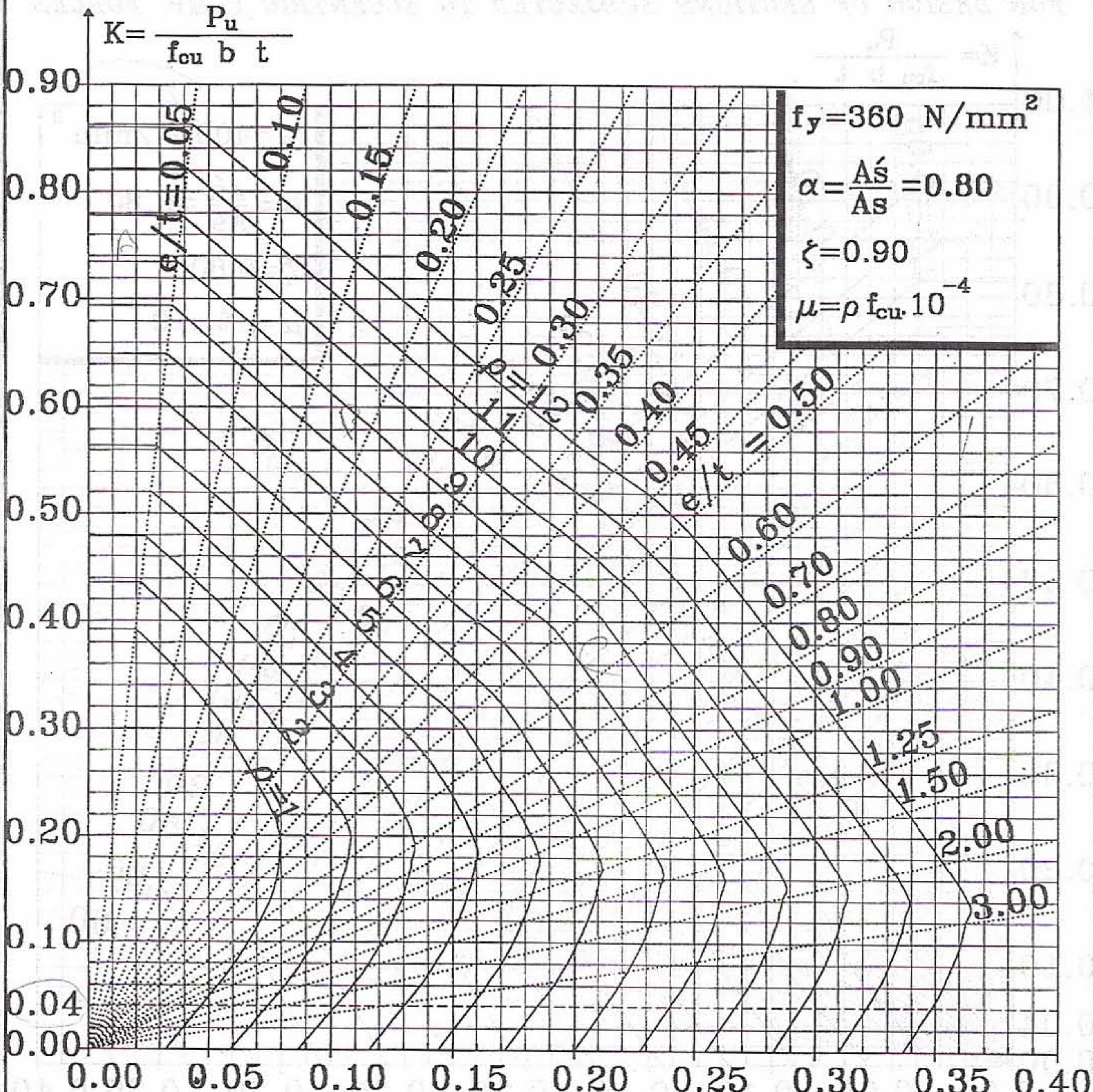
FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES

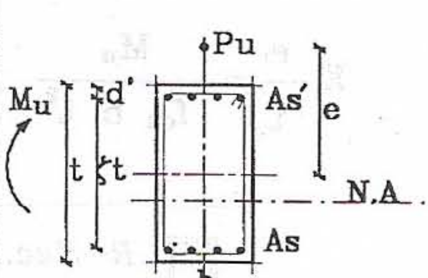


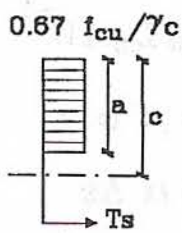
El–Behairy R.C.Design Handbook "Interaction Diagrams"

INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES







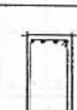
$$\mu = \rho\ f_{cu} \cdot 10^{-4}$$

$$K\ \frac{e}{t} = \frac{M_u}{f_{cu}\ b\ t^2}$$

$$As = \mu\ b\ t$$

$$A\acute{s} = \alpha\ As$$

$$\zeta = \frac{d - d'}{t}$$

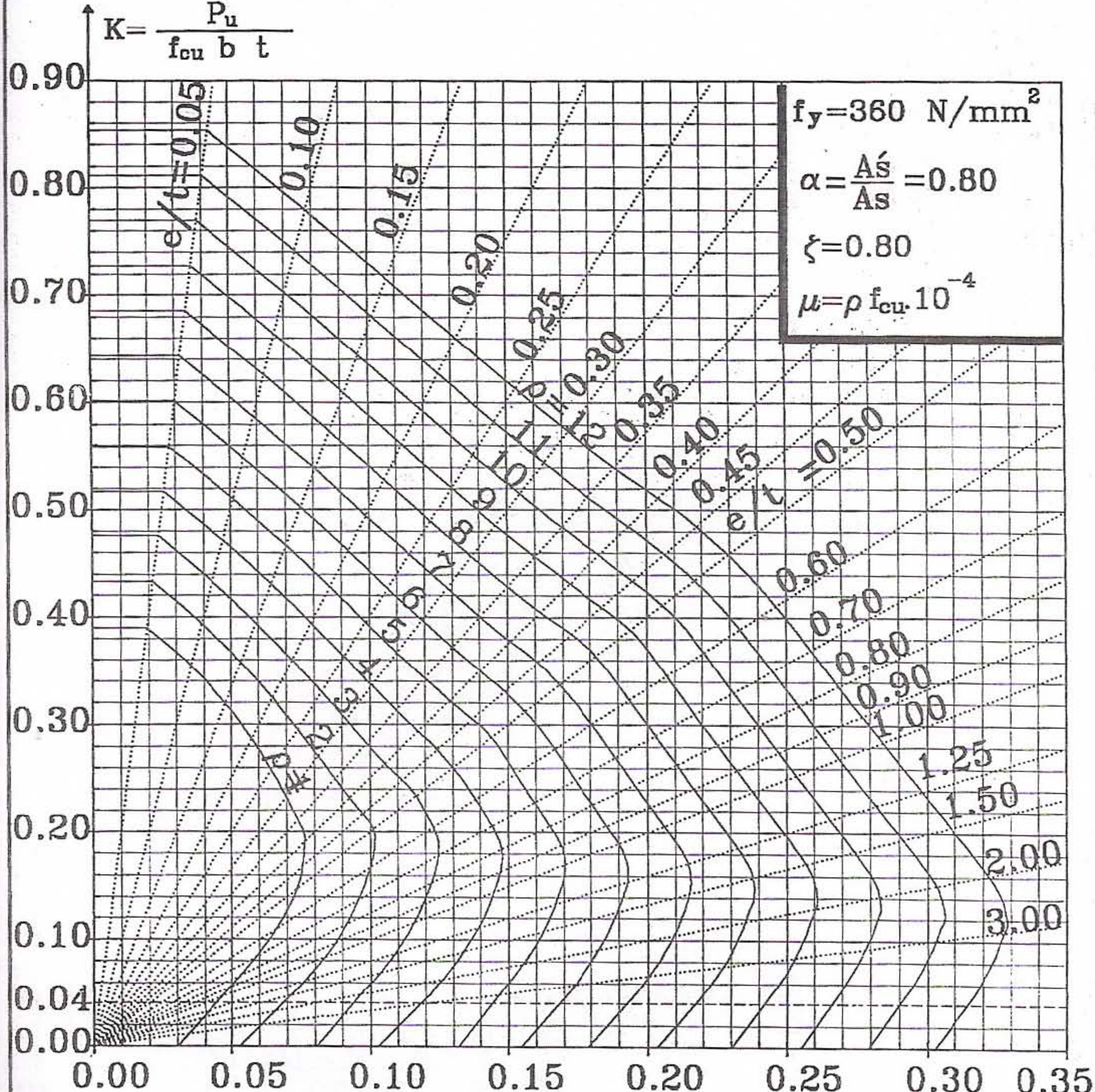


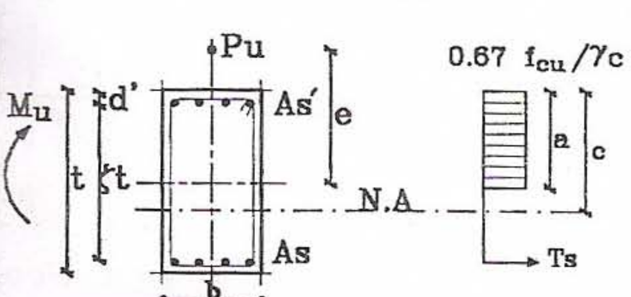
R–Sec.

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INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES





$\mu = \rho \, f_{cu} . 10^{-4}$

$As = \mu \; b \; t$

$As' = \alpha \; As$

$\zeta = \dfrac{d - d'}{t}$

$K \dfrac{e}{t} = \dfrac{M_u}{f_{cu} \; b \; t^2}$

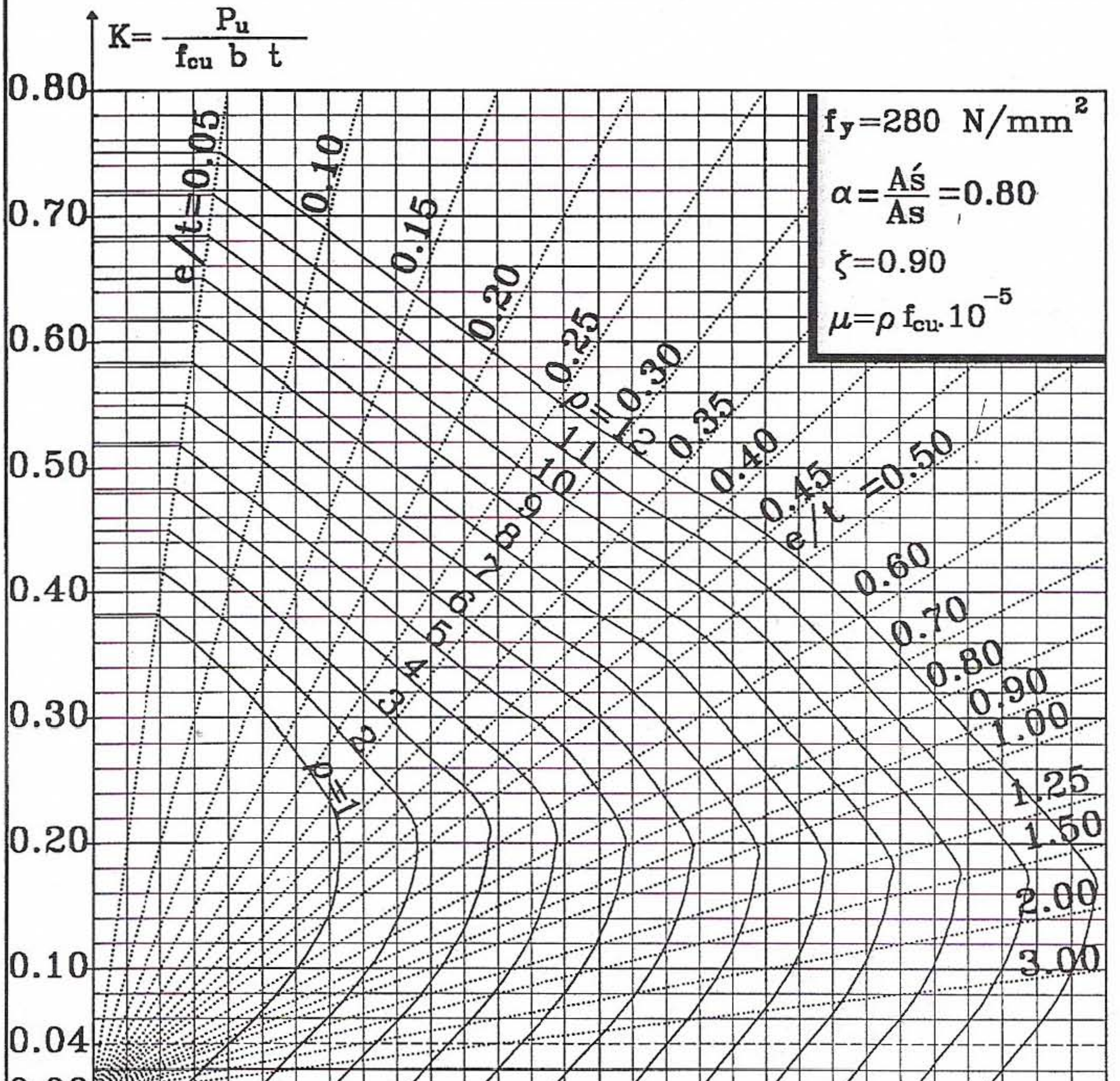


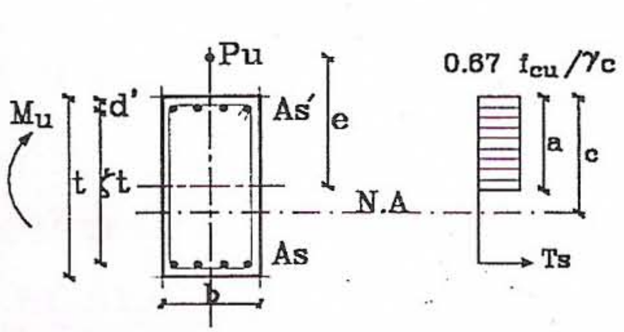
R–Sec.

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INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES





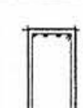
$\mu = \rho\, f_{cu} \cdot 10^{-4}$

$K \dfrac{e}{t} = \dfrac{M_u}{f_{cu}\ b\ t^2}$

$A_s = \mu\ b\ t$

$A_s' = \alpha\ A_s$

$\zeta = \dfrac{d - d'}{t}$

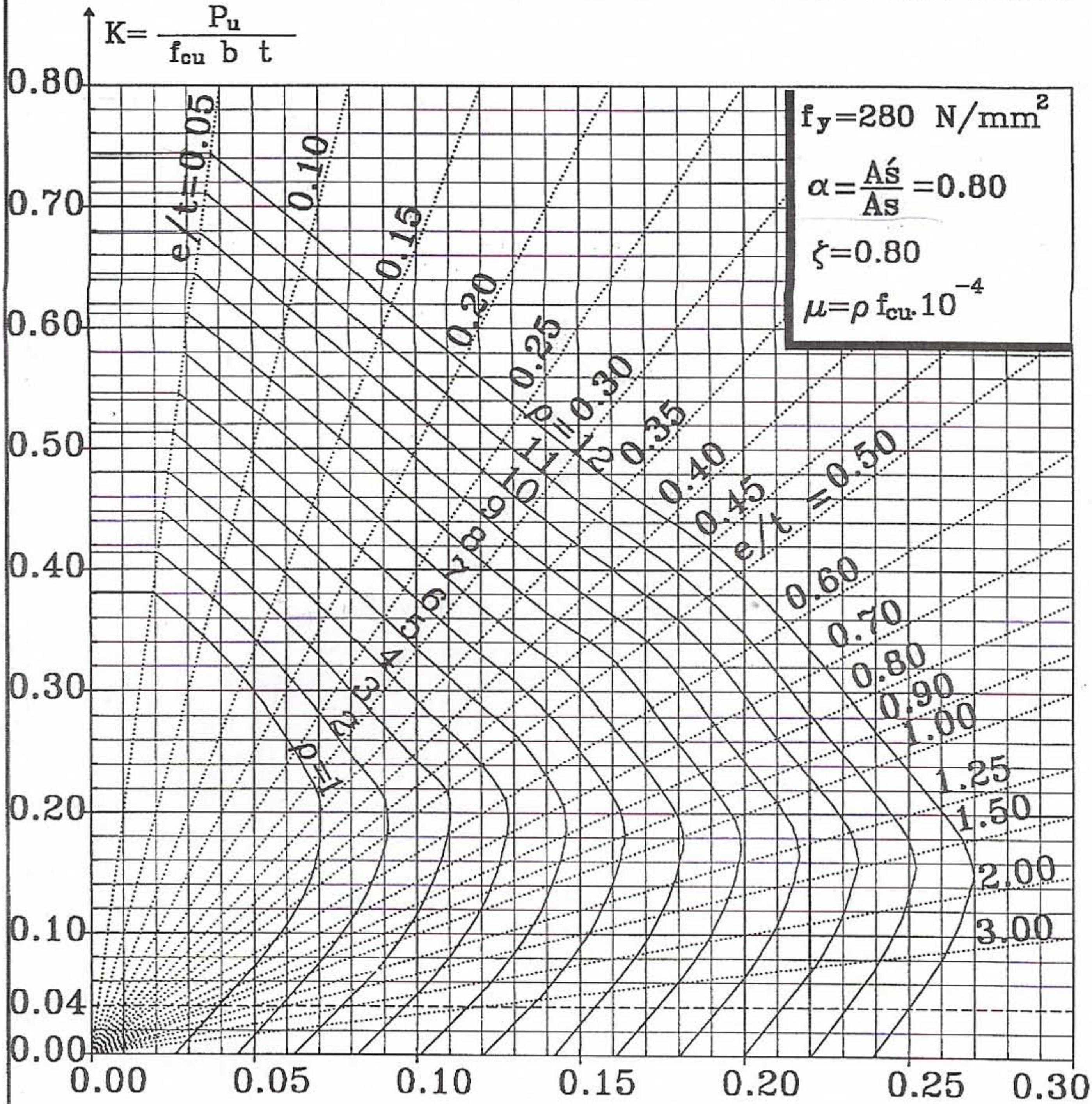


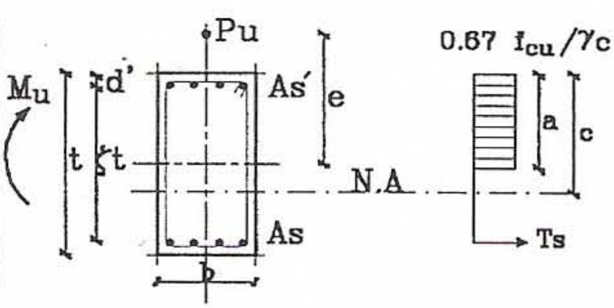
R–Sec.

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INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES





$\mu = \rho\, f_{cu} \cdot 10^{-4}$

$As = \mu\ b\ t$

$As' = \alpha\ As$

$\zeta = \frac{d - d'}{t}$

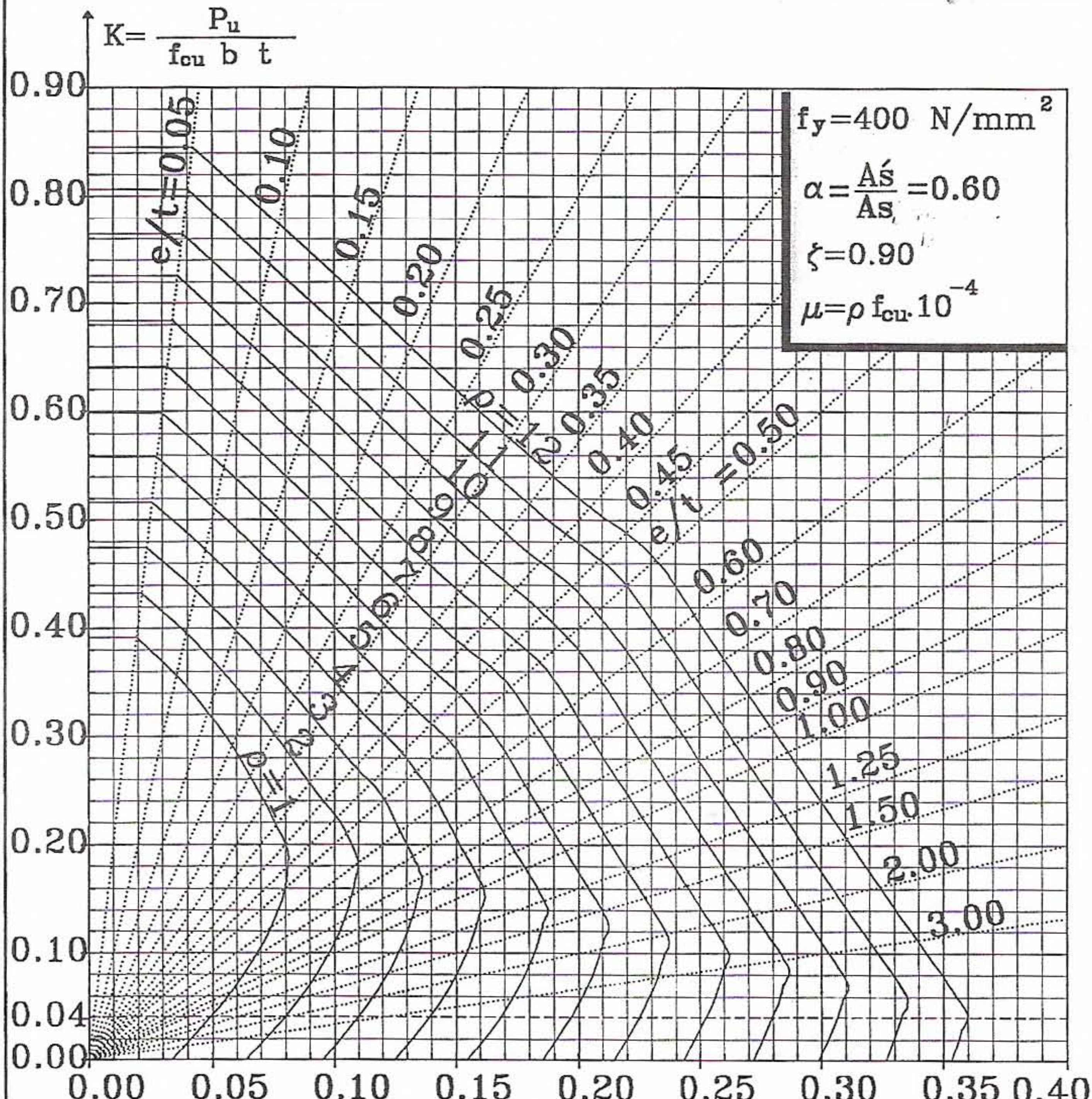
$K \frac{e}{t} = \frac{M_u}{f_{cu}\ b\ t^2}$

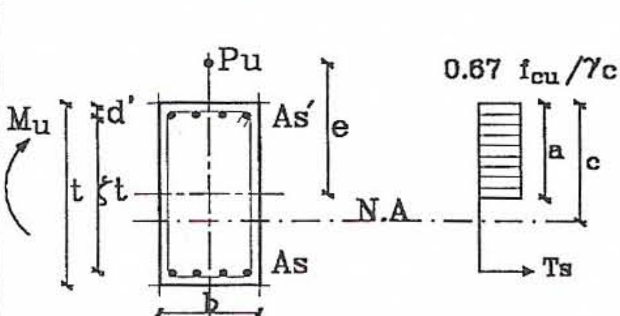
R–Sec.

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INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES





$\mu = \rho\, f_{cu} \cdot 10^{-4}$

$As = \mu\ b\ t$

$A\acute{s} = \alpha\ As$

$\zeta = \frac{d - d'}{t}$

$K\ \frac{e}{t} = \frac{M_u}{f_{cu}\ b\ t^2}$



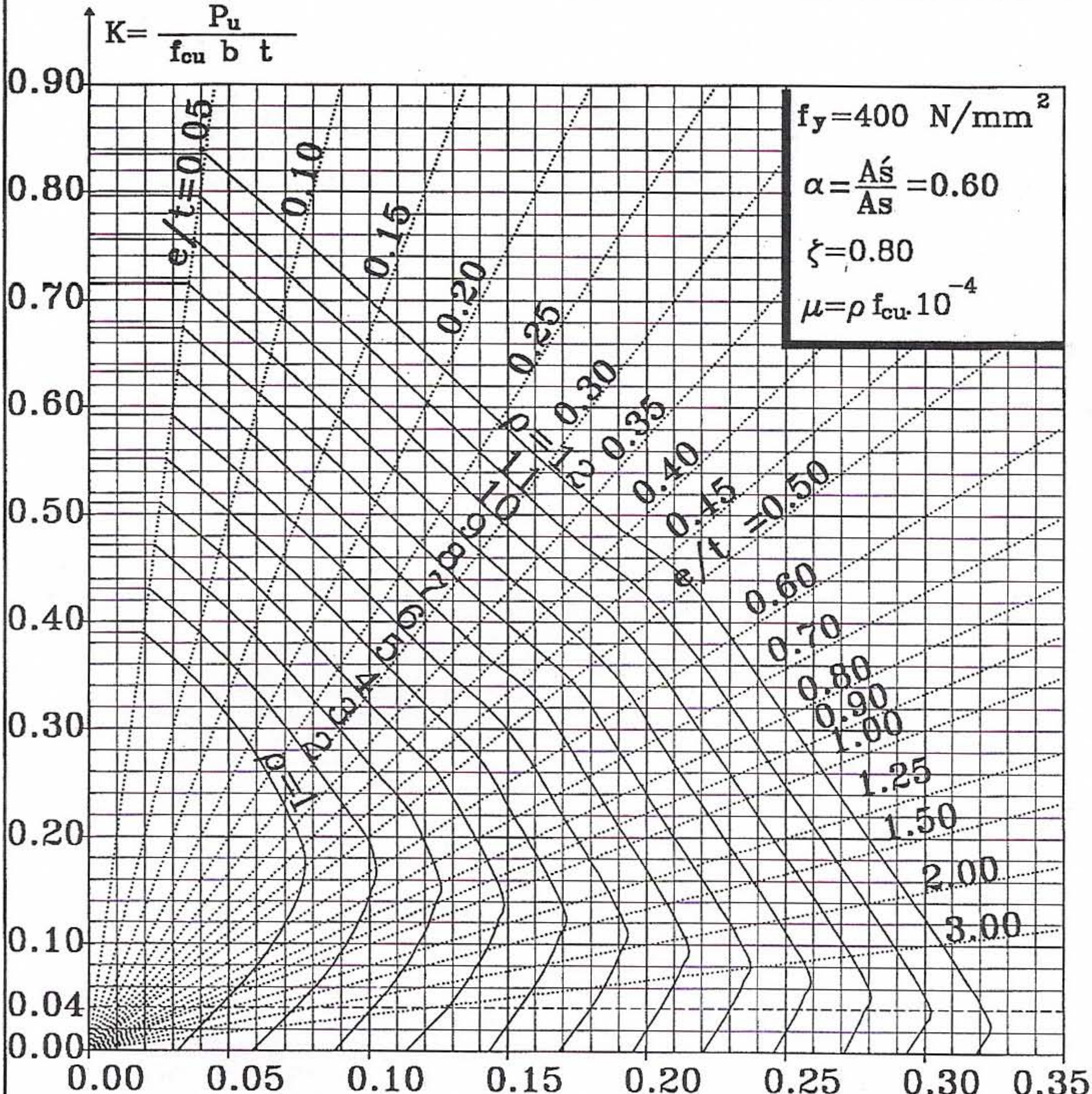
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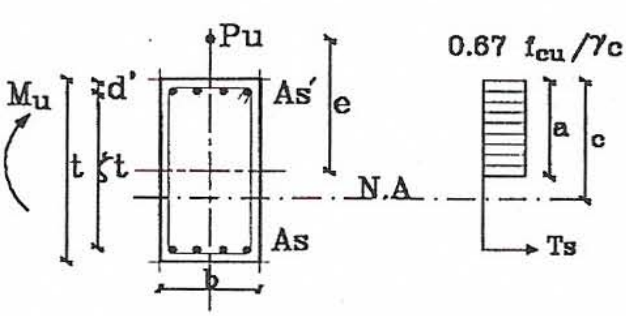
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El–Behairy R.C.Design Handbook "Interaction Diagrams"

INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES





$\mu = \rho \, f_{cu} . 10^{-4}$

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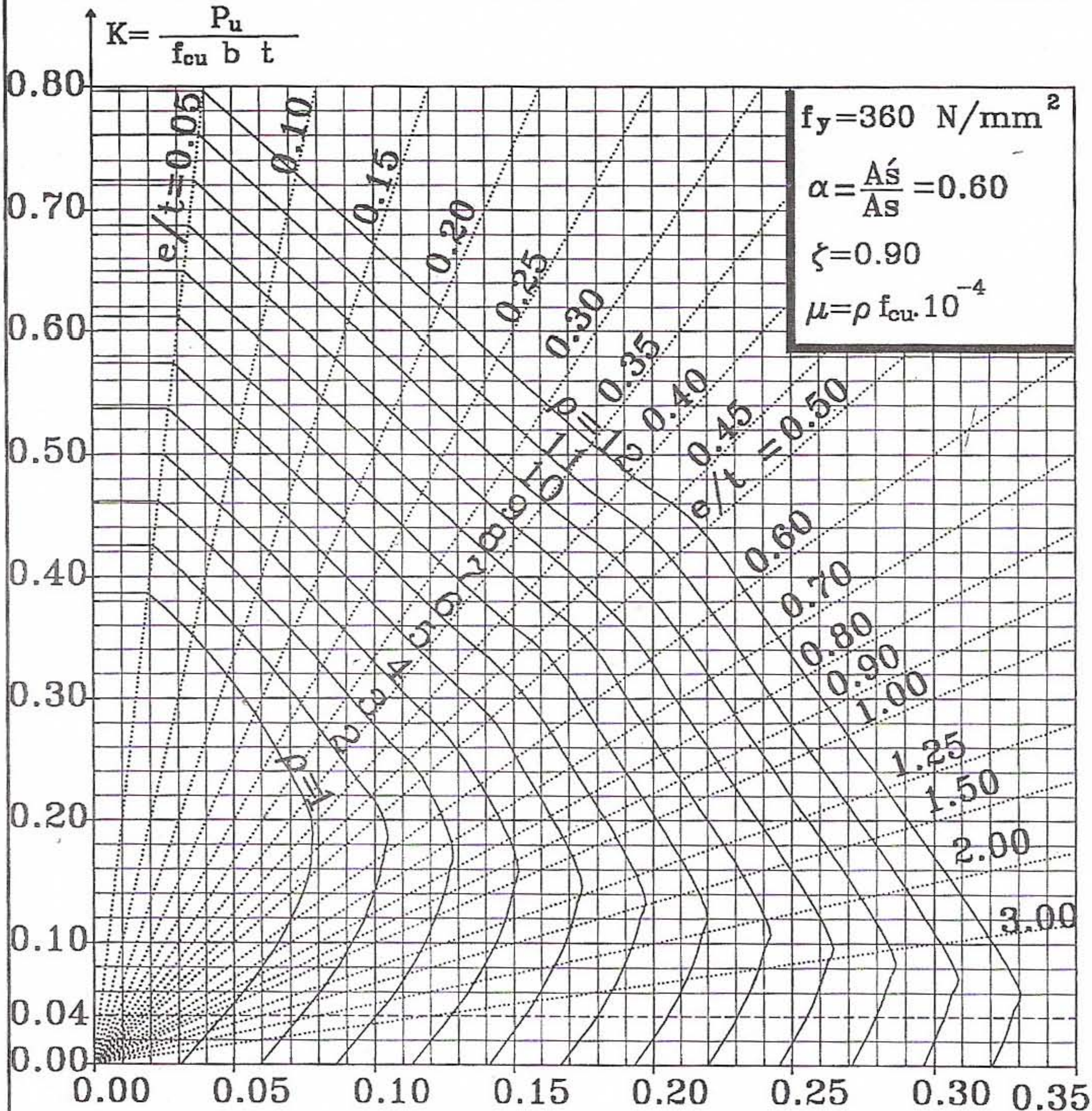


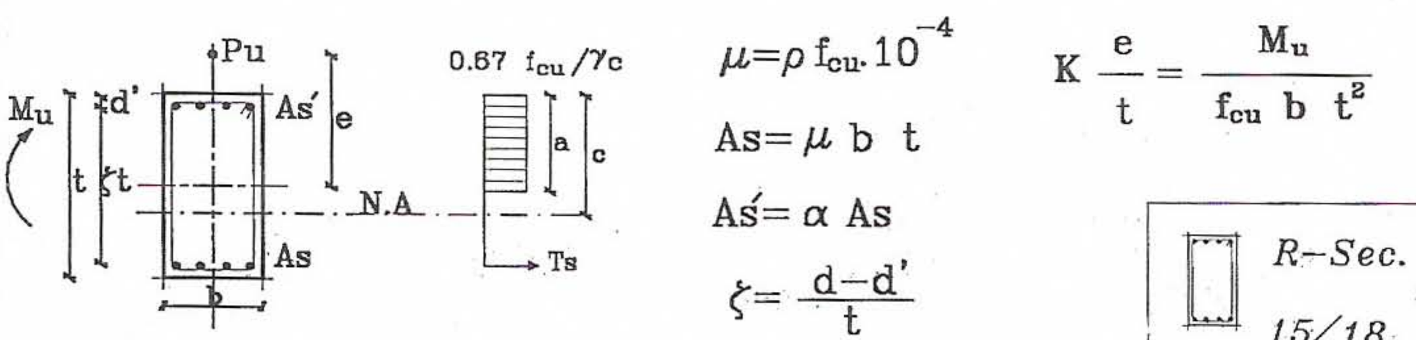
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INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES





$\mu = \rho \, f_{cu} . 10^{-4}$

$As = \mu \; b \; t$

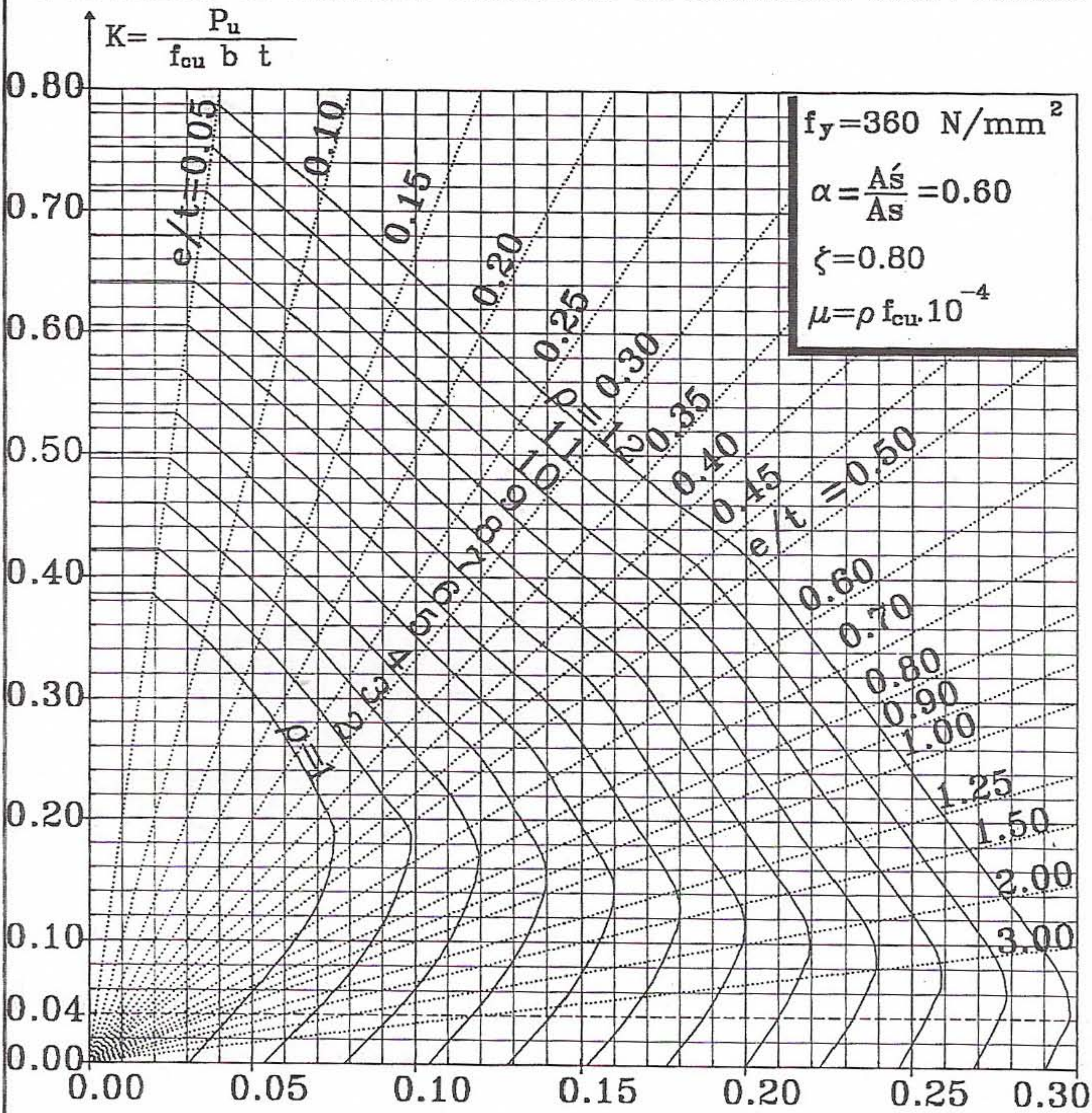
$A\acute{s} = \alpha \; As$

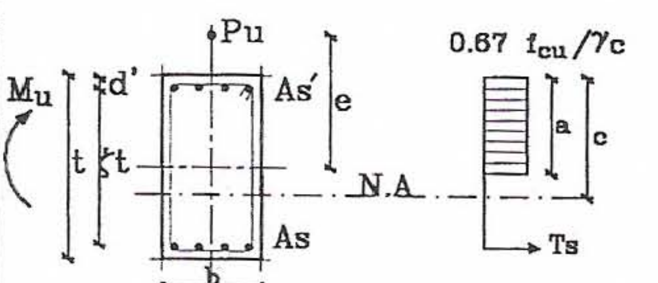
$\zeta = \frac{d - d'}{t}$

$K \frac{e}{t} = \frac{M_u}{f_{cu} \; b \; t^2}$

INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES





$\mu=\rho\, f_{cu}.10^{-4}$

$K\,\dfrac{e}{t}=\dfrac{M_u}{f_{cu}\ b\ t^2}$

$As=\mu\ b\ t$

$As'=\alpha\ As$

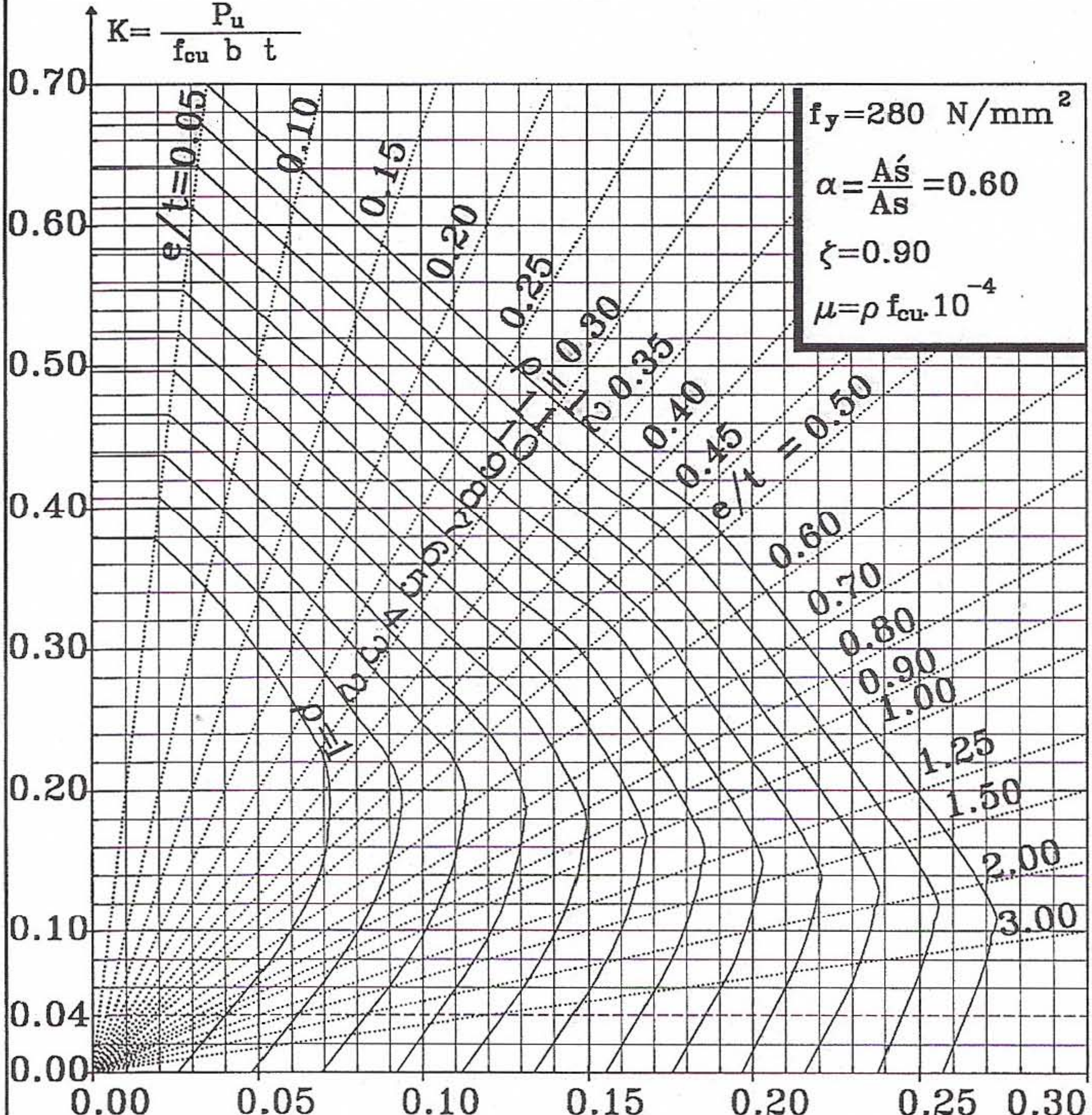
$\zeta=\dfrac{d-d'}{t}$

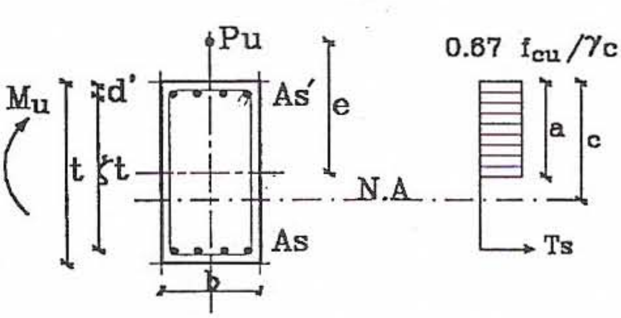
R–Sec.

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INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES





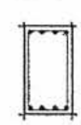
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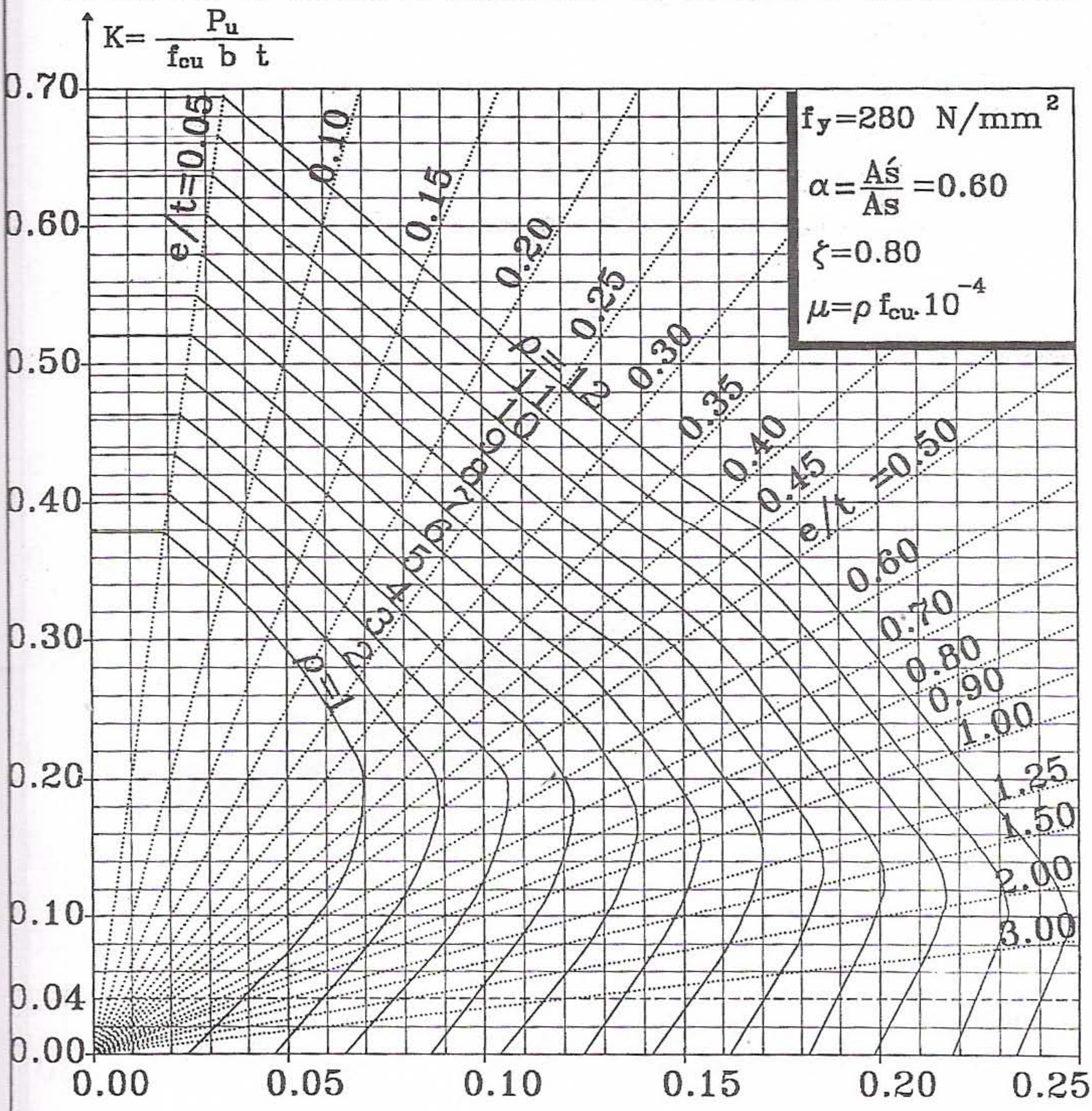


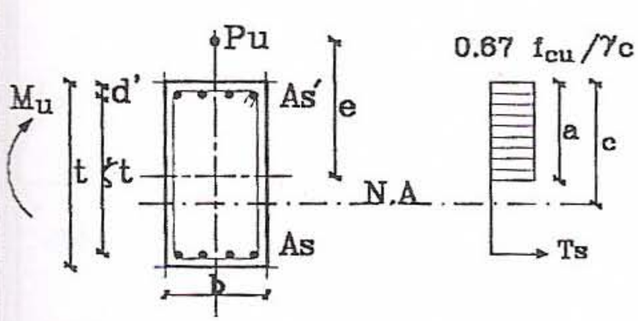
R–Sec.

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INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES





$\mu = \rho\ f_{cu} \cdot 10^{-4}$

$As = \mu\ b\ t$

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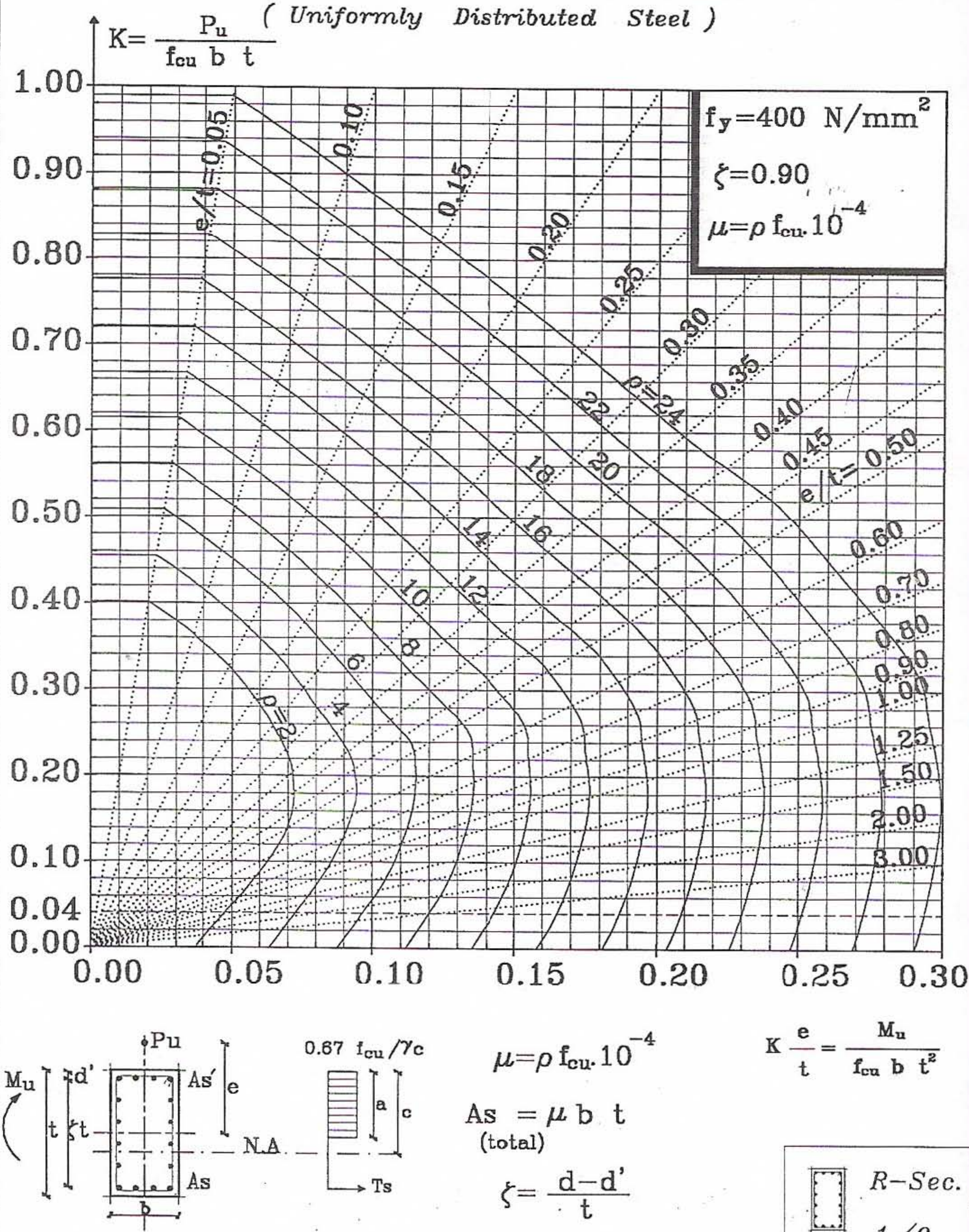
R–Sec.

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INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES

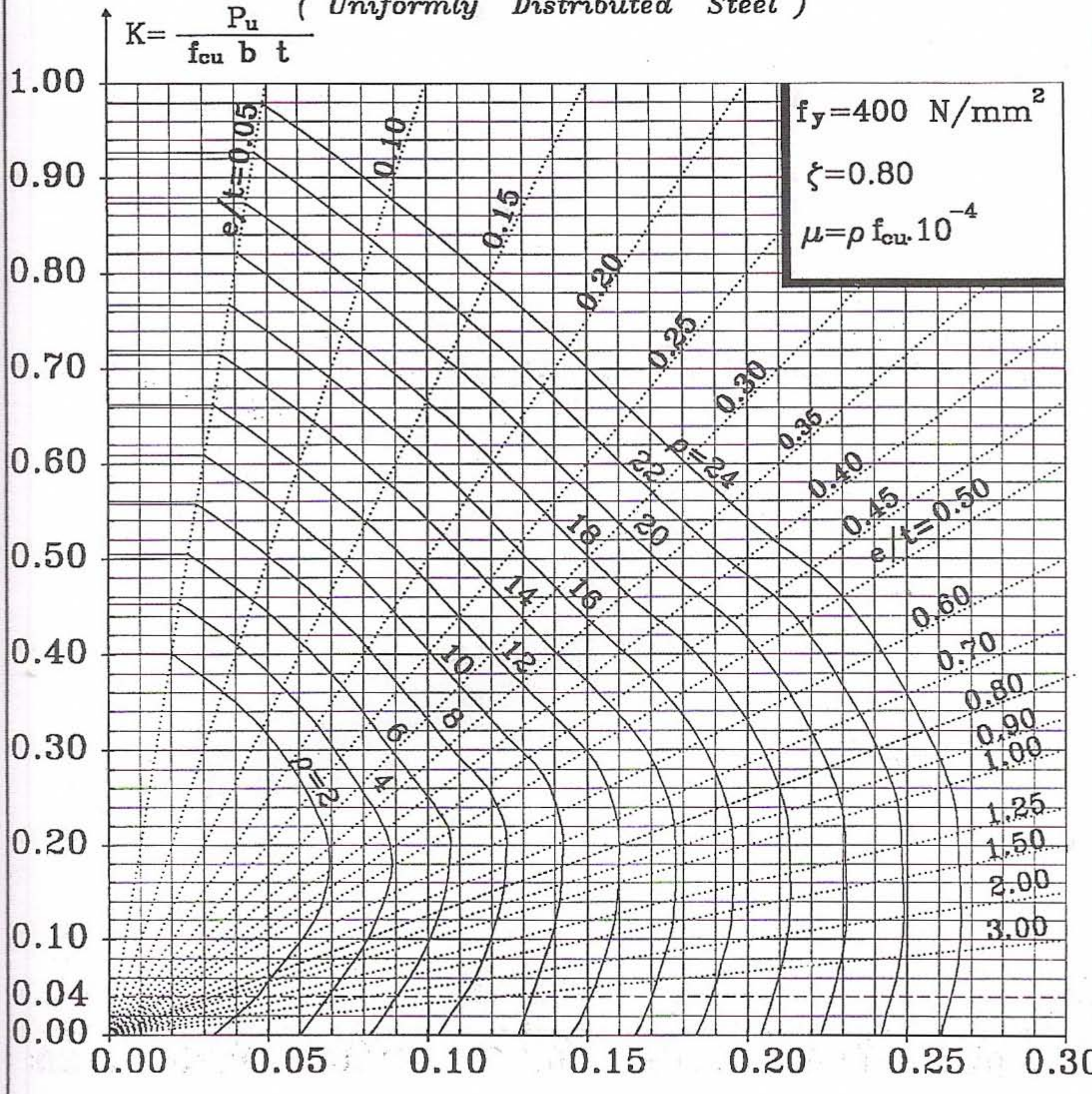
(Uniformly Distributed Steel)

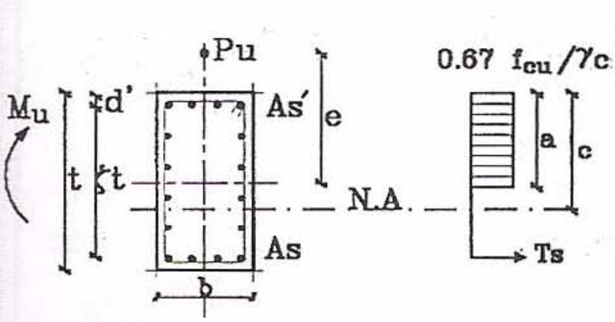


INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES

(Uniformly Distributed Steel)



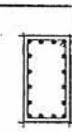


$\mu=\rho\ f_{cu}\cdot 10^{-4}$

$K\ \frac{e}{t}=\frac{M_u}{f_{cu}\ b\ t^2}$

$A_{s\ (total)}=\mu\ b\ t$

$\zeta=\frac{d-d'}{t}$



R–Sec.

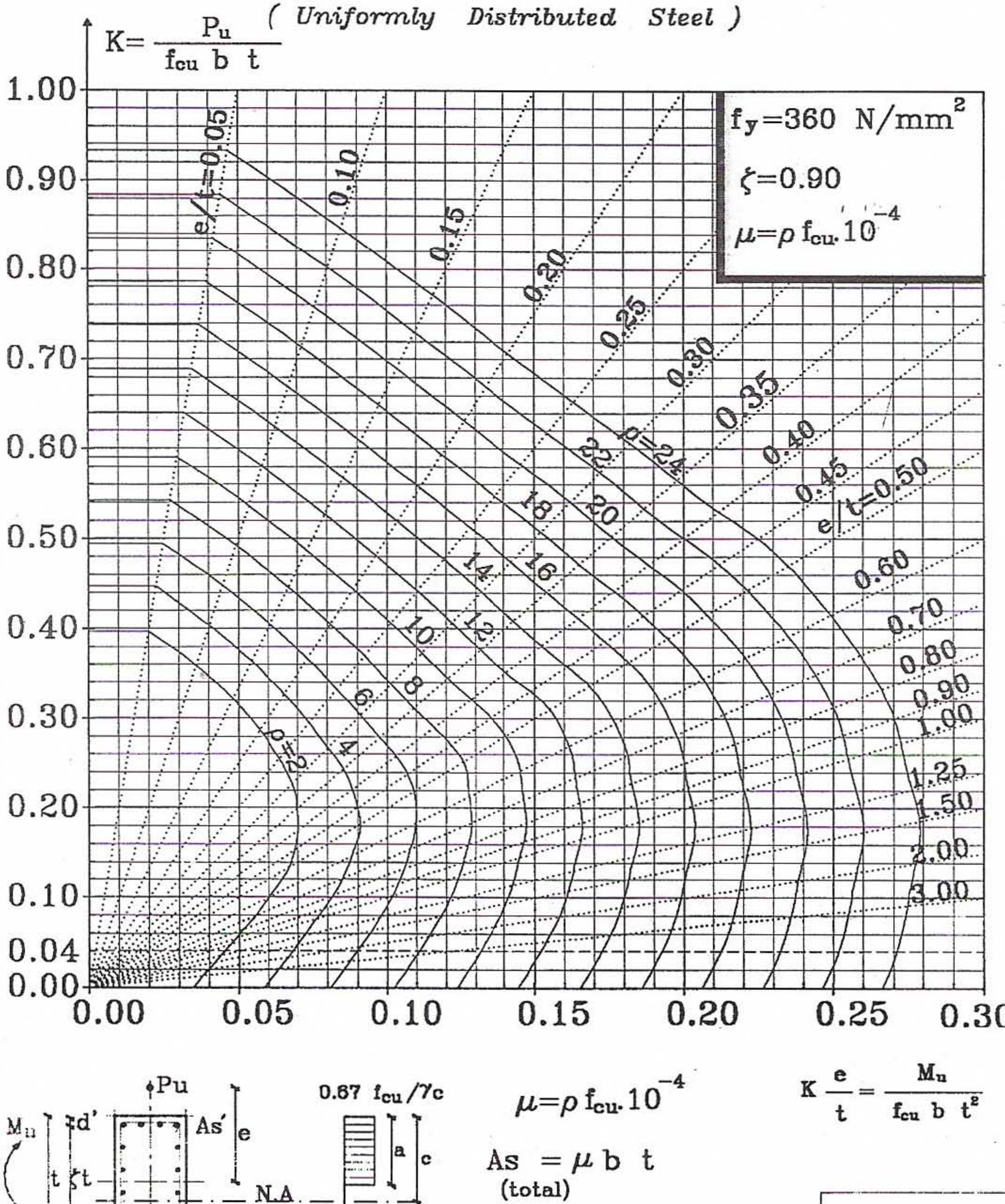
2/6

El–Behairy R.C.Design Handbook "Interaction Diagrams"

INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES

(Uniformly Distributed Steel)

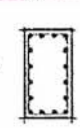


$\mu = \rho \, f_{cu} . 10^{-4}$

$K \frac{e}{t} = \frac{M_u}{f_{cu} \, b \, t^2}$

$A_s \underset{(total)}{} = \mu \, b \, t$

$\zeta = \frac{d - d'}{t}$



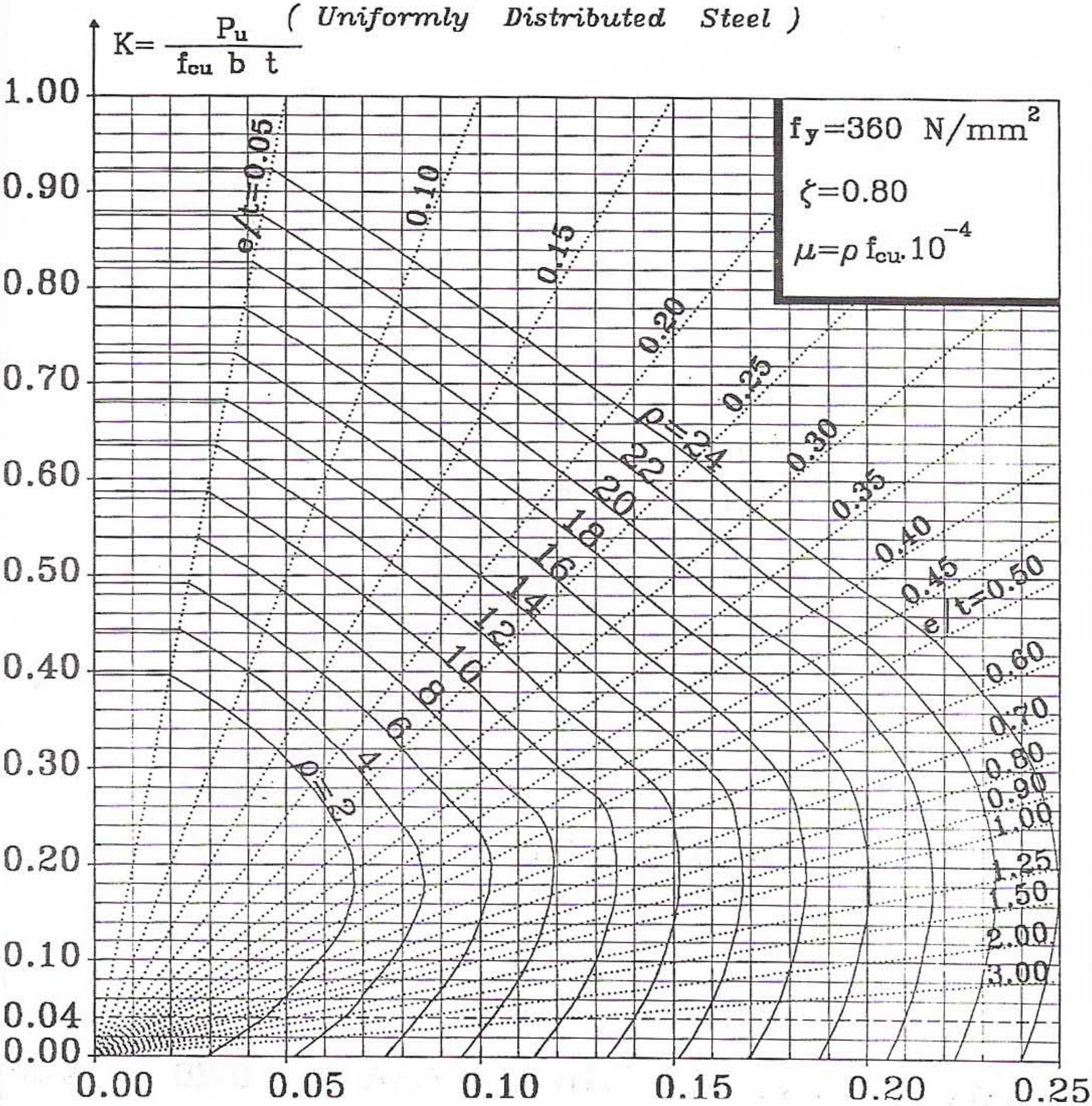
R–Sec.

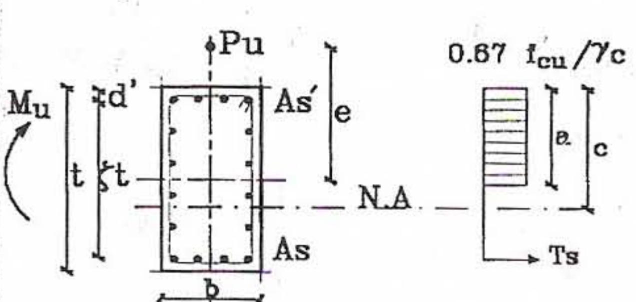
3/6

INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES

(Uniformly Distributed Steel)





$\mu = \rho\, f_{cu} \cdot 10^{-4}$

$A_s = \mu\ b\ t$ (total)

$\zeta = \frac{d - d'}{t}$

$K\ \frac{e}{t} = \frac{M_u}{f_{cu}\ b\ t^2}$

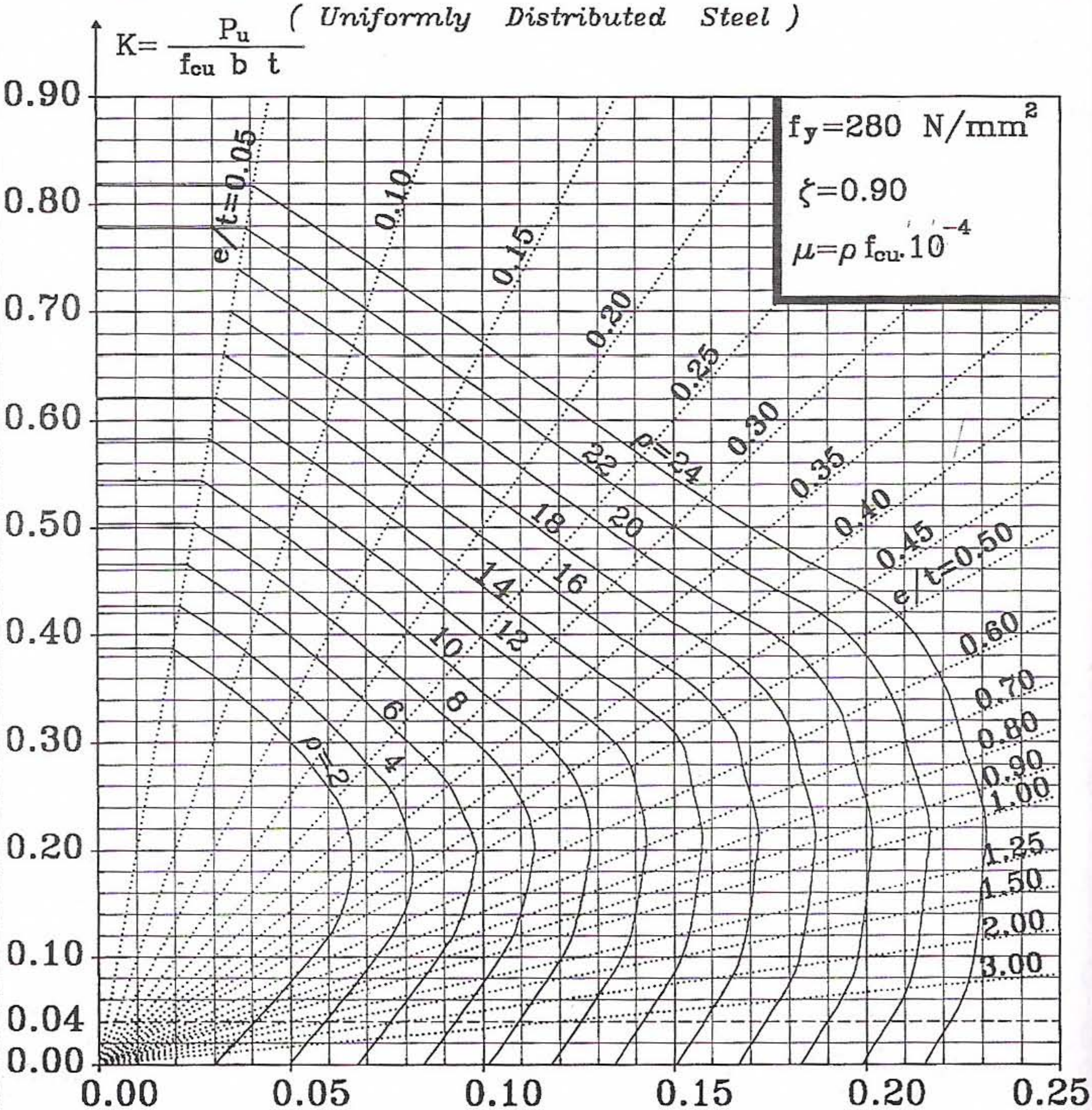
R–Sec.

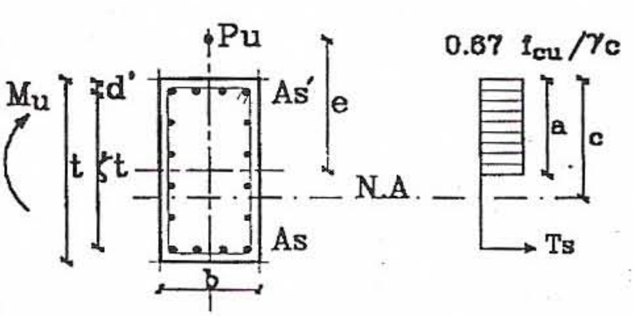
4/6

INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES

(Uniformly Distributed Steel)



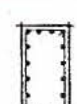


$\mu = \rho \, f_{cu} \cdot 10^{-4}$

$A_s = \mu \, b \, t$ (total)

$\zeta = \frac{d - d'}{t}$

$K \frac{e}{t} = \frac{M_u}{f_{cu} \, b \, t^2}$



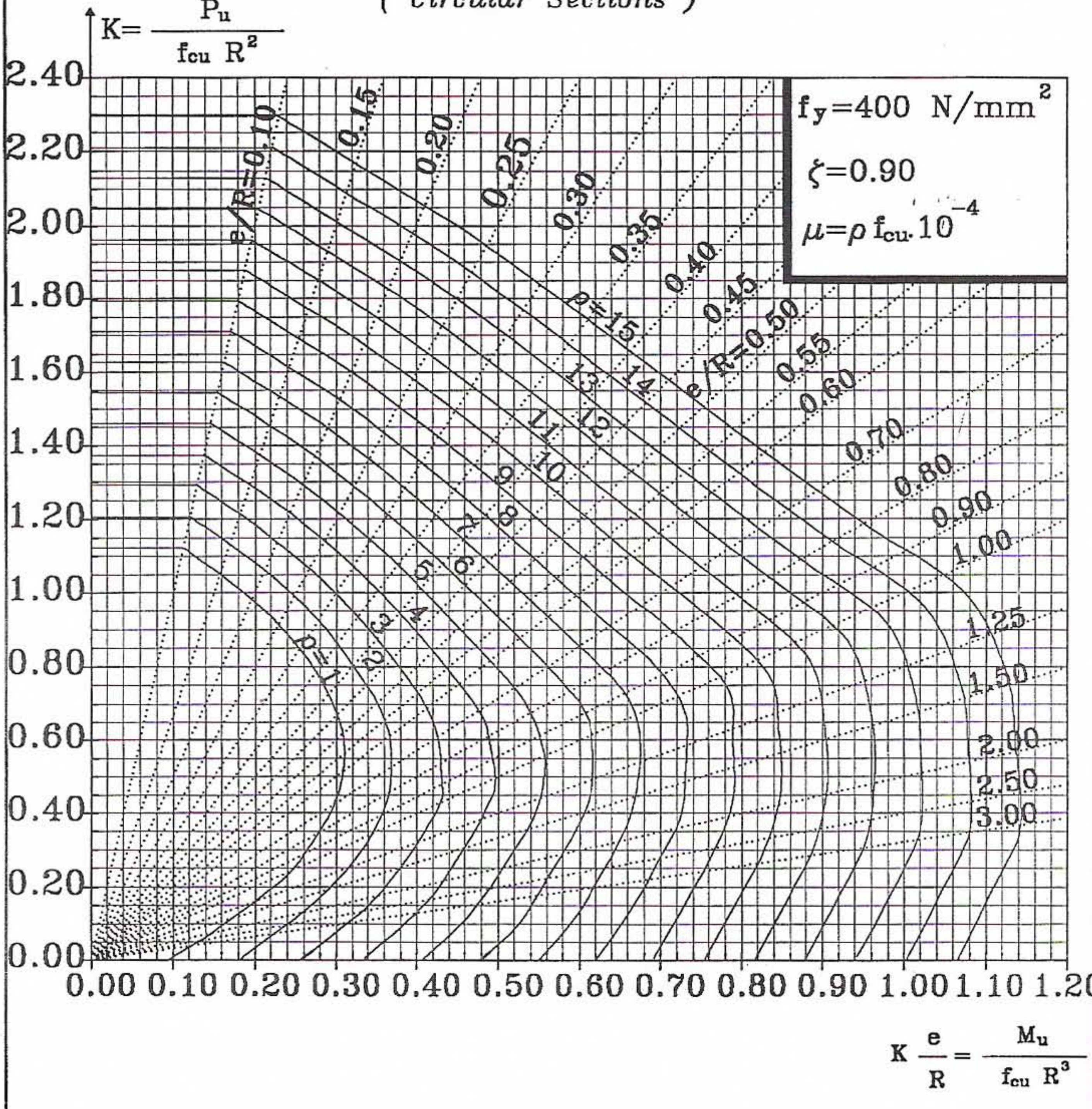
R–Sec.

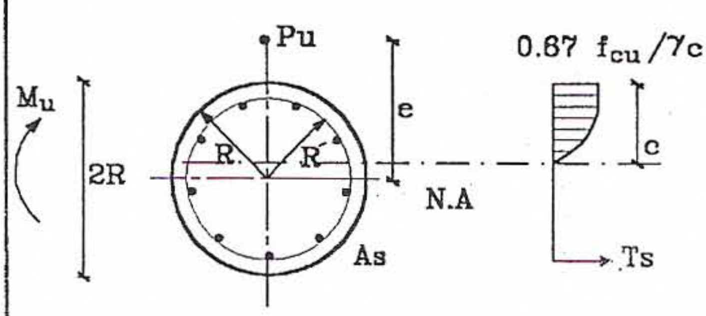
5/6

INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES

(Circular Sections)

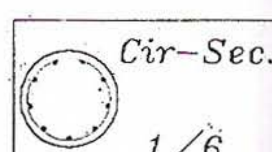




$$\zeta = \frac{R'}{R}$$

$$\mu = \rho\, f_{cu}\, 10^{-4}$$

$$A_{s\,(total)} = \mu\, \pi\, R^2$$

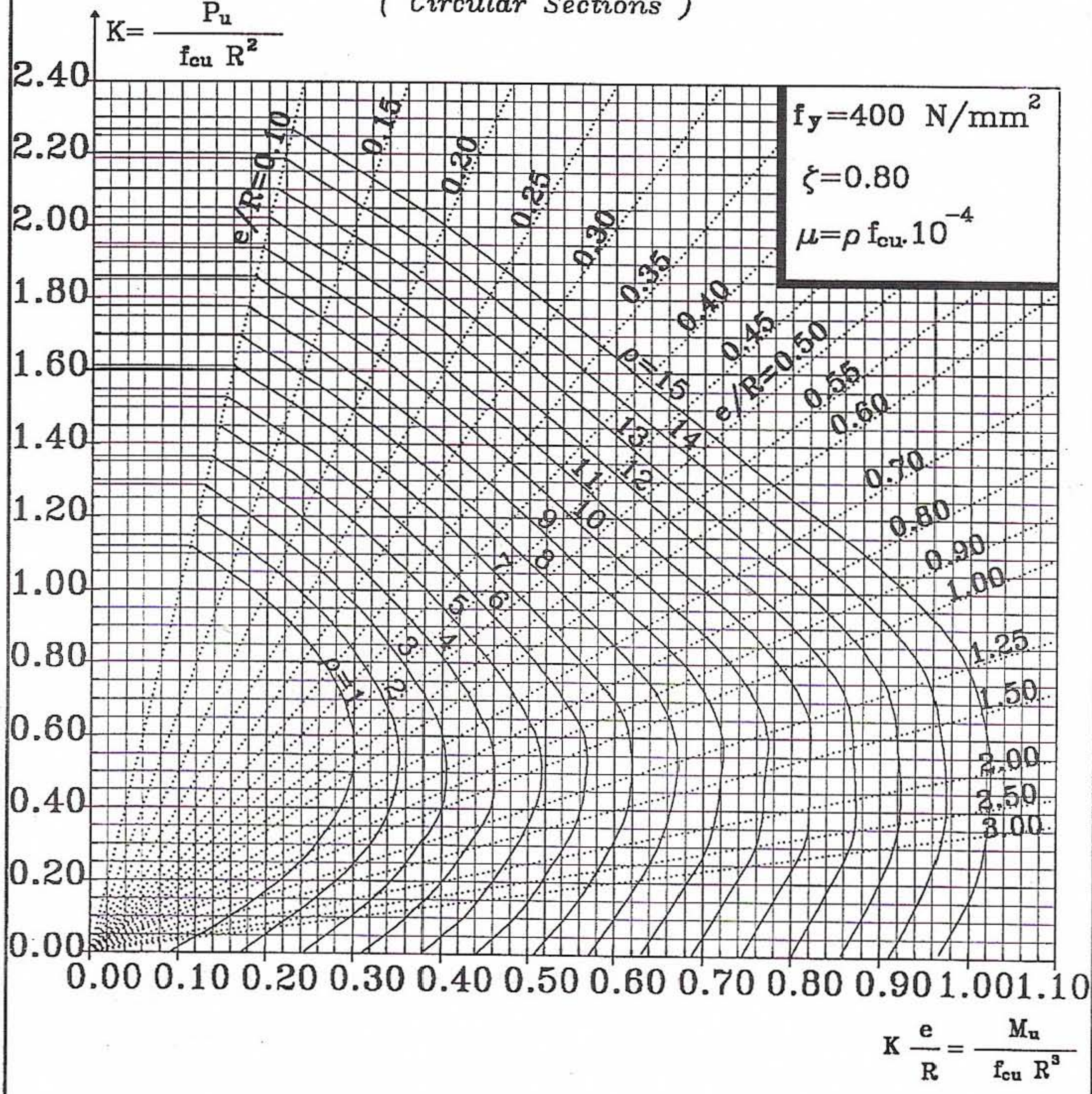


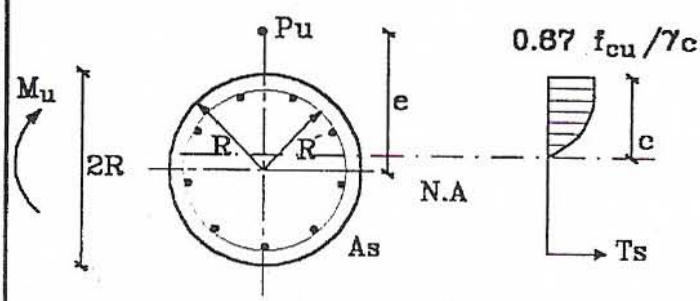
El–Behairy R.C.Design Handbook "Interaction Diagrams"

INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES

(Circular Sections)

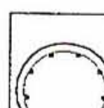




$$\zeta = \frac{R'}{R}$$

$$\mu = \rho\ f_{cu}\ 10^{-4}$$

$$A_{s\,(total)} = \mu\ \pi\ R^2$$



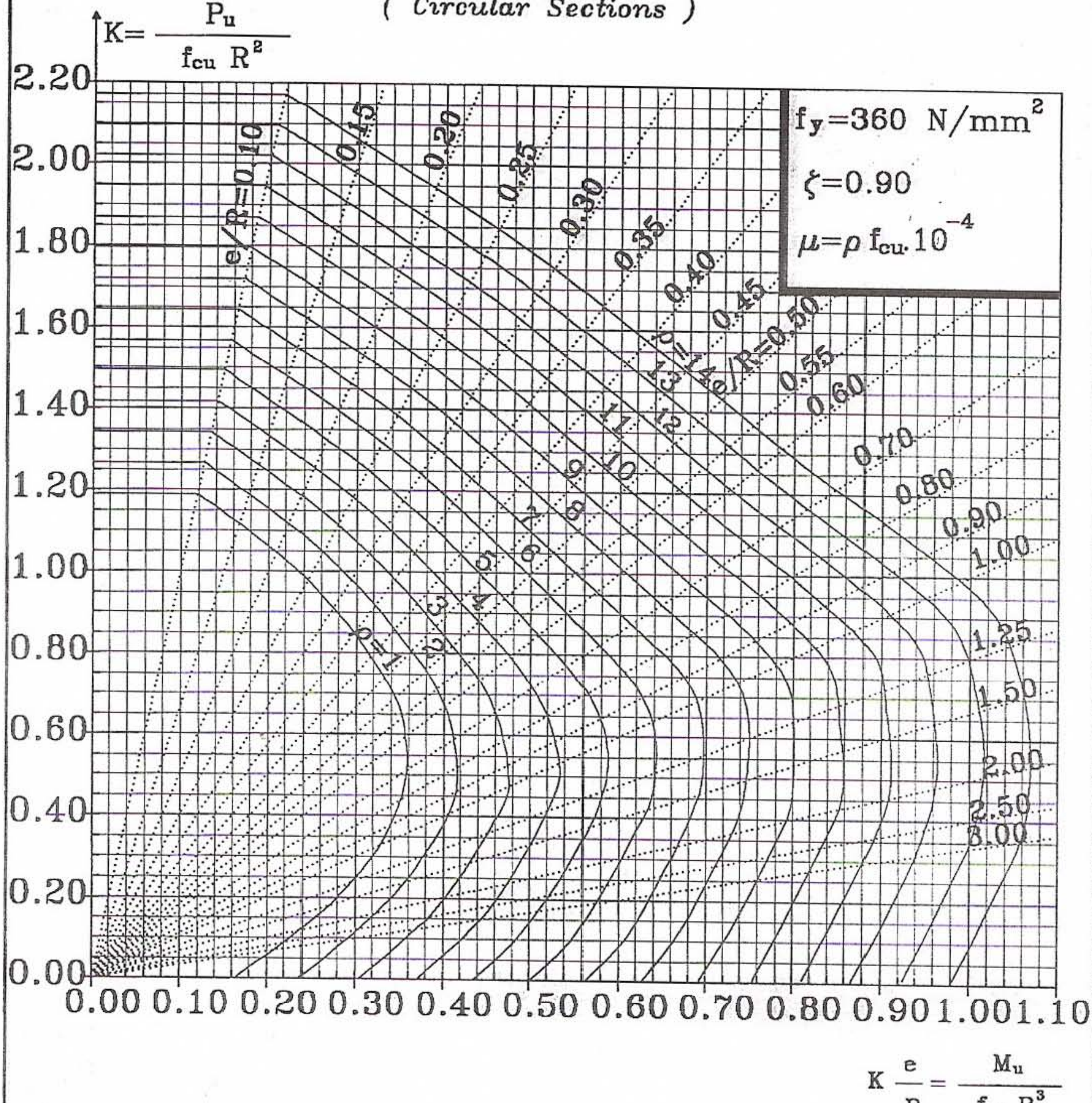
Cir–Sec.

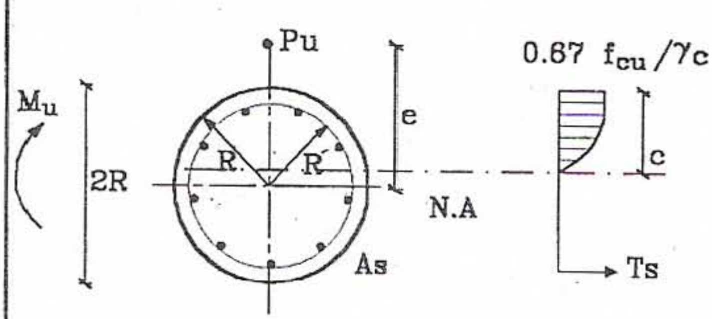
2/6

INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES

(Circular Sections)

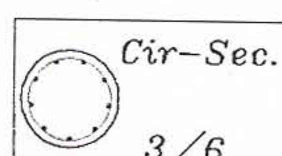




$\zeta = \frac{R'}{R}$

$\mu = \rho\, f_{cu}\, 10^{-4}$

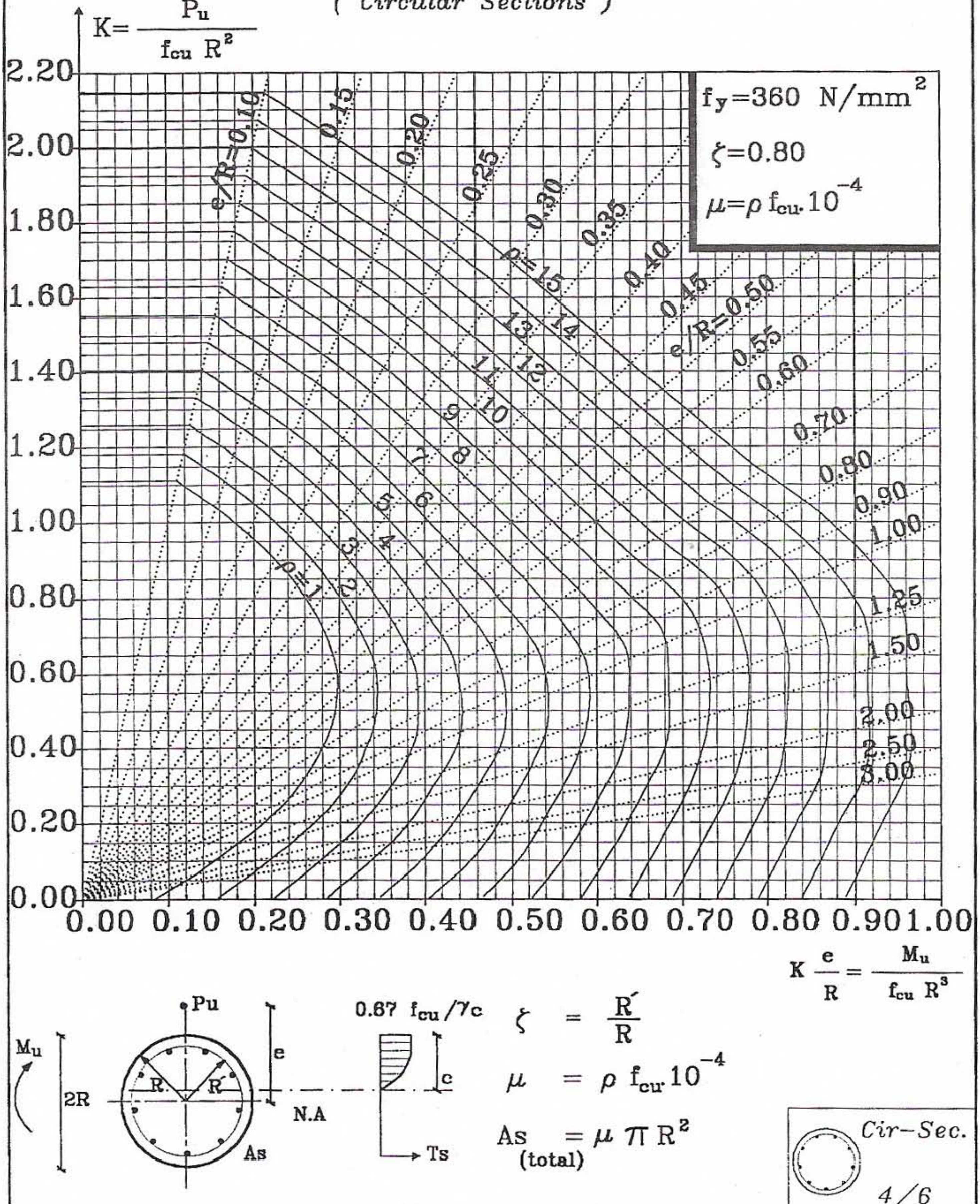
$As_{(total)} = \mu\, \pi\, R^2$



INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES

(Circular Sections)

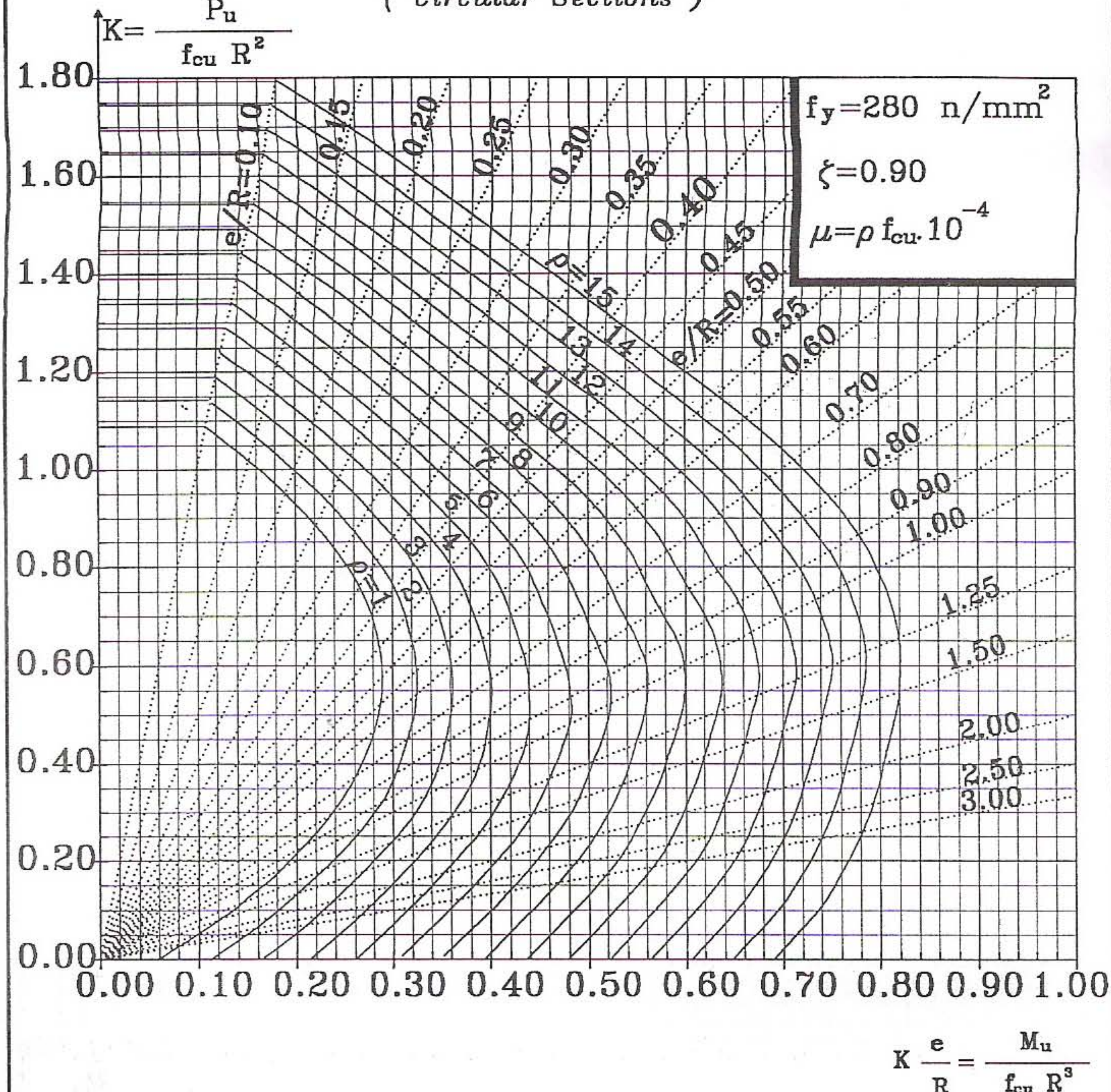


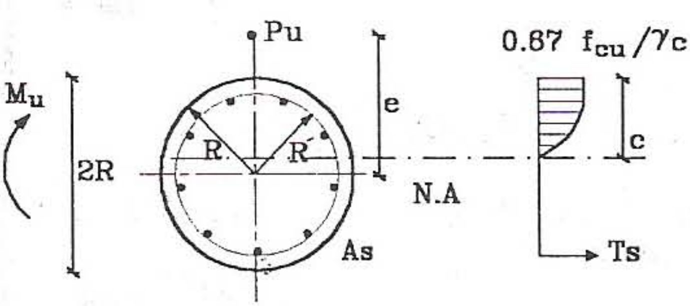
El-Behairy R.C.Design Handbook "Interaction Diagrams"

INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES

(Circular Sections)





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Cir–Sec.

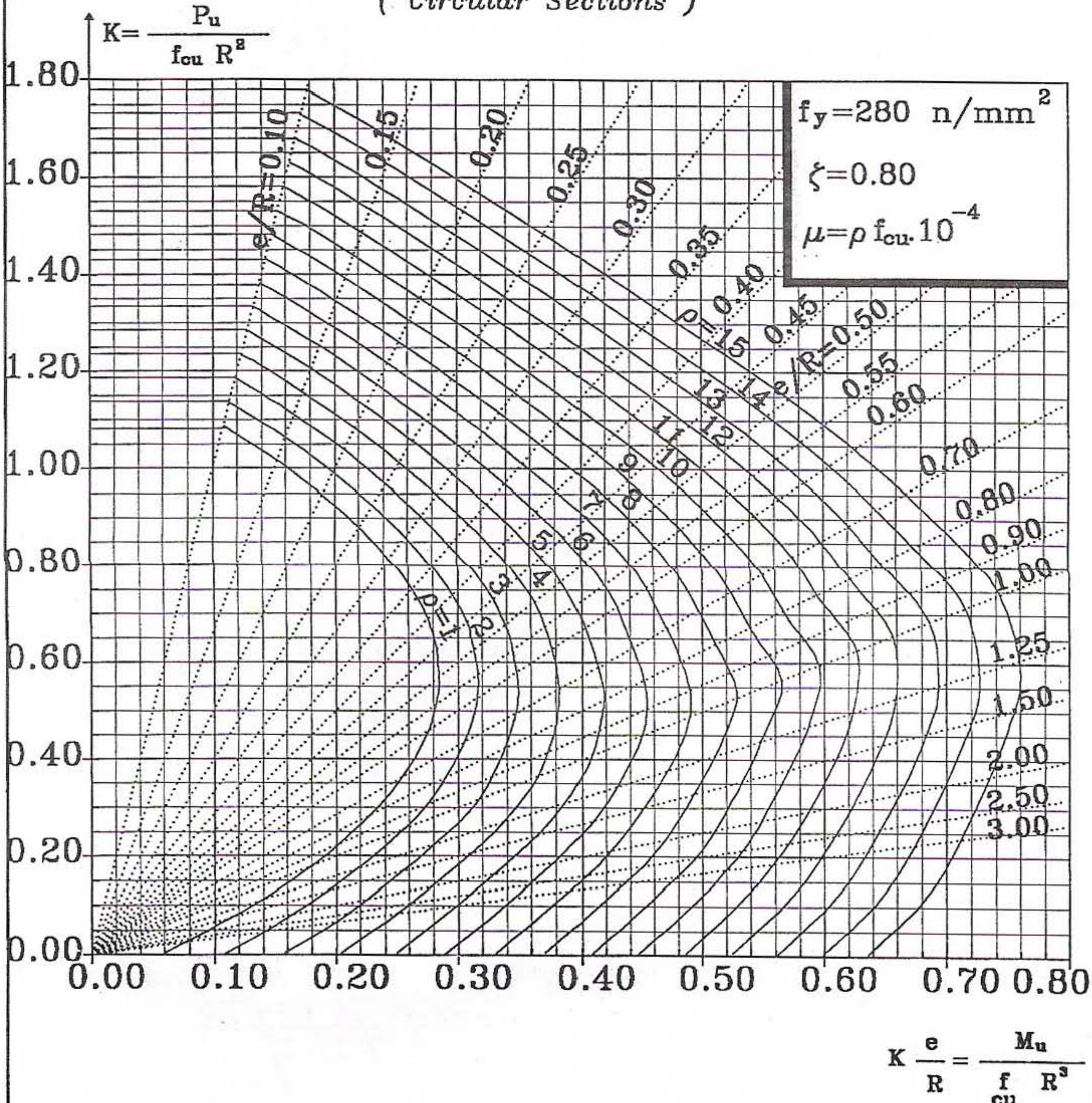
5/6

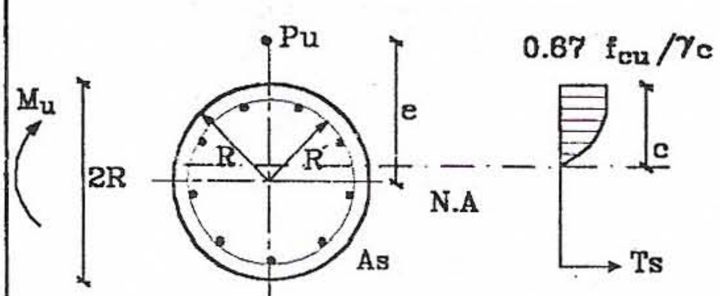
El–Behairy R.C.Design Handbook "Interaction Diagrams"

INTERACTION DIAGRAMS

FOR DESIGN OF SECTIONS SUBJECTED TO ECCENTRIC COMP. FORCES

(Circular Sections)

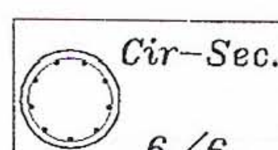




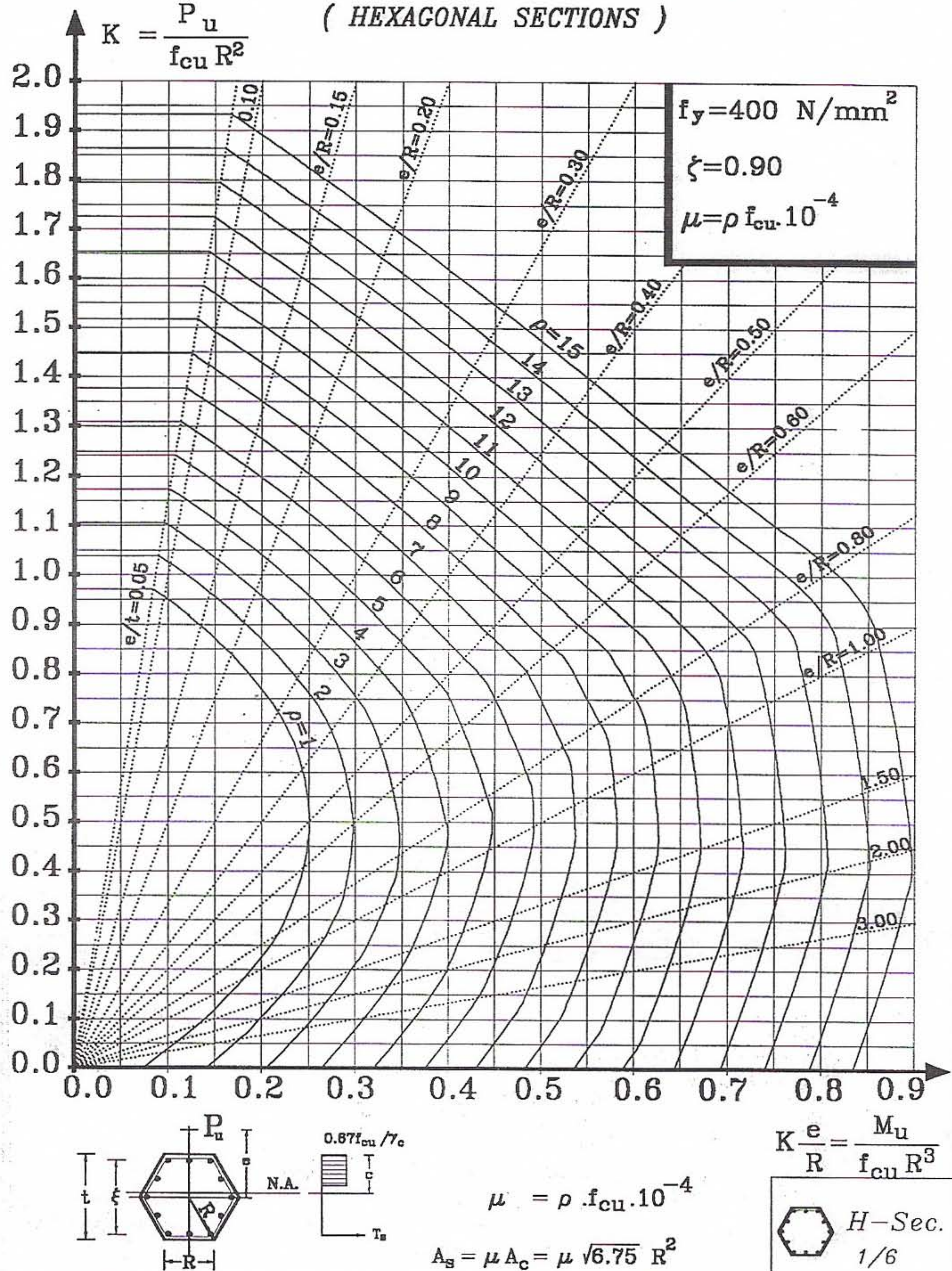
$$\zeta = \frac{R'}{R}$$

$$\mu = \rho\ f_{cu} \cdot 10^{-4}$$

$$A_{s\,(total)} = \mu\,\pi\,R^2$$



U.L.D. CHARTS FOR SECTIONS SUBJECTED TO ECCENTRIC FORCES
(HEXAGONAL SECTIONS)

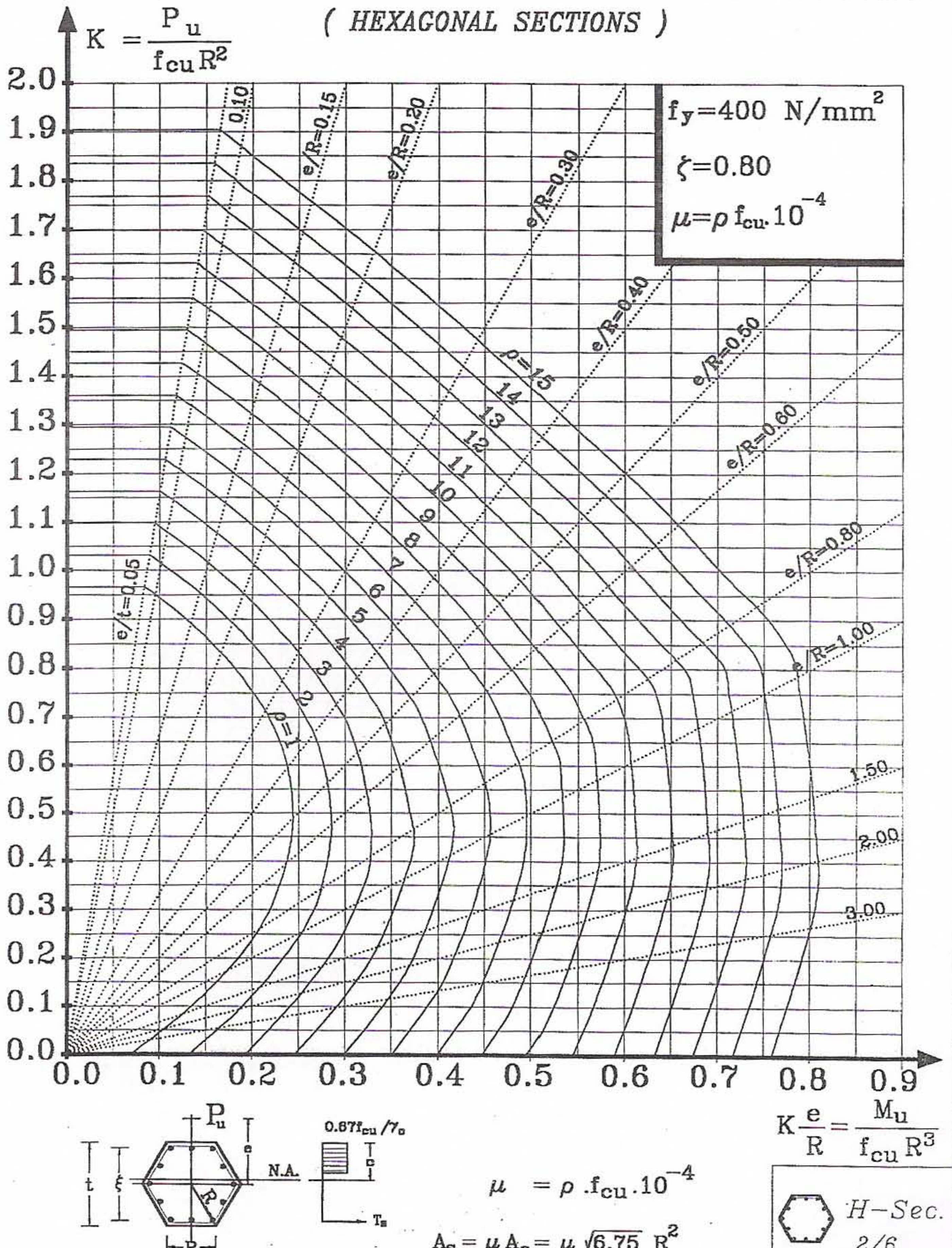


$\mu = \rho \cdot f_{cu} \cdot 10^{-4}$

$A_s = \mu A_c = \mu \sqrt{6.75}\ R^2$

El−Behairy R.C.Design Handbook "Interaction Diagrams"

U.L.D. CHARTS FOR SECTIONS SUBJECTED TO ECCENTRIC FORCES
(HEXAGONAL SECTIONS)

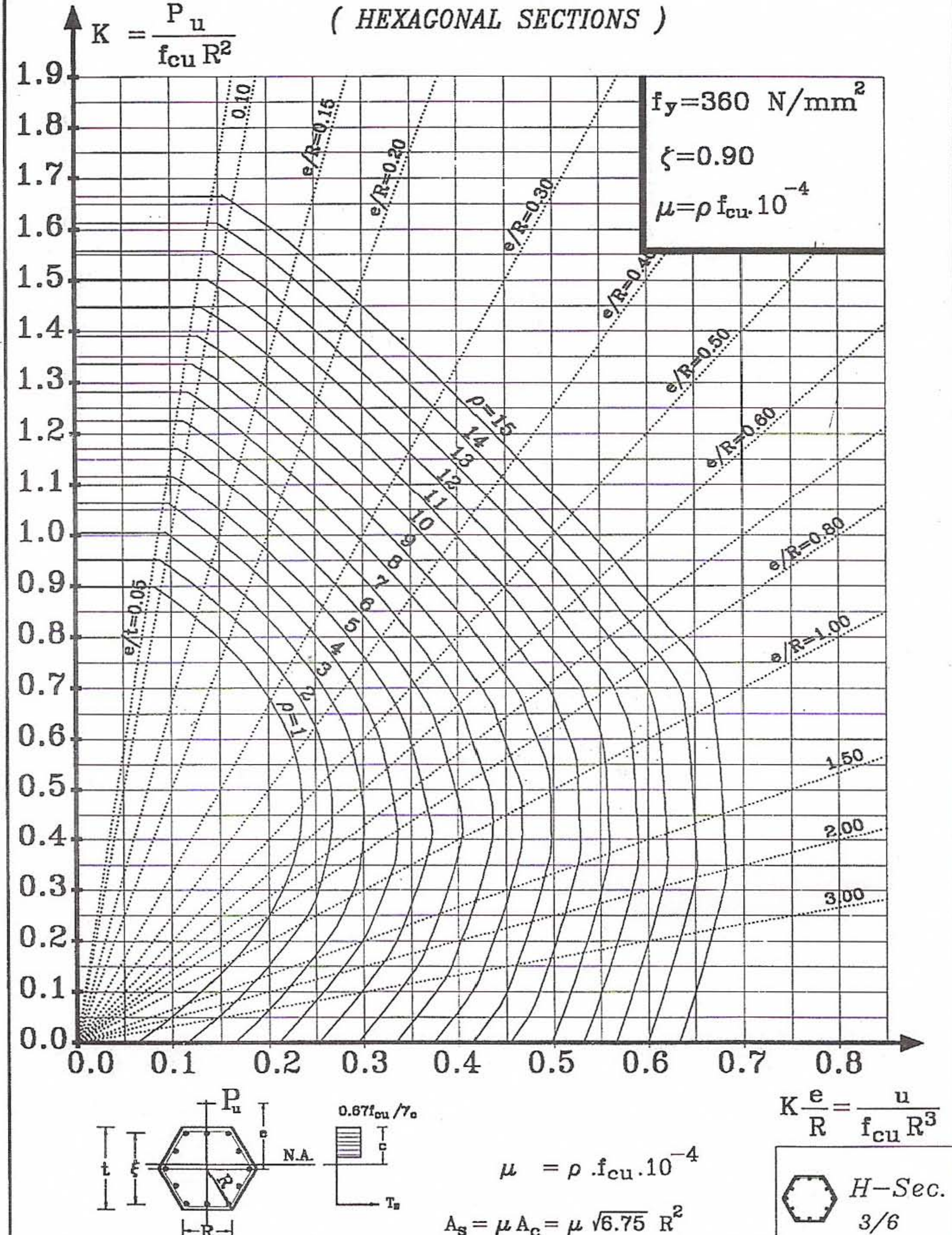


$\mu = \rho . f_{cu} . 10^{-4}$

$A_s = \mu A_c = \mu \sqrt{6.75}\ R^2$

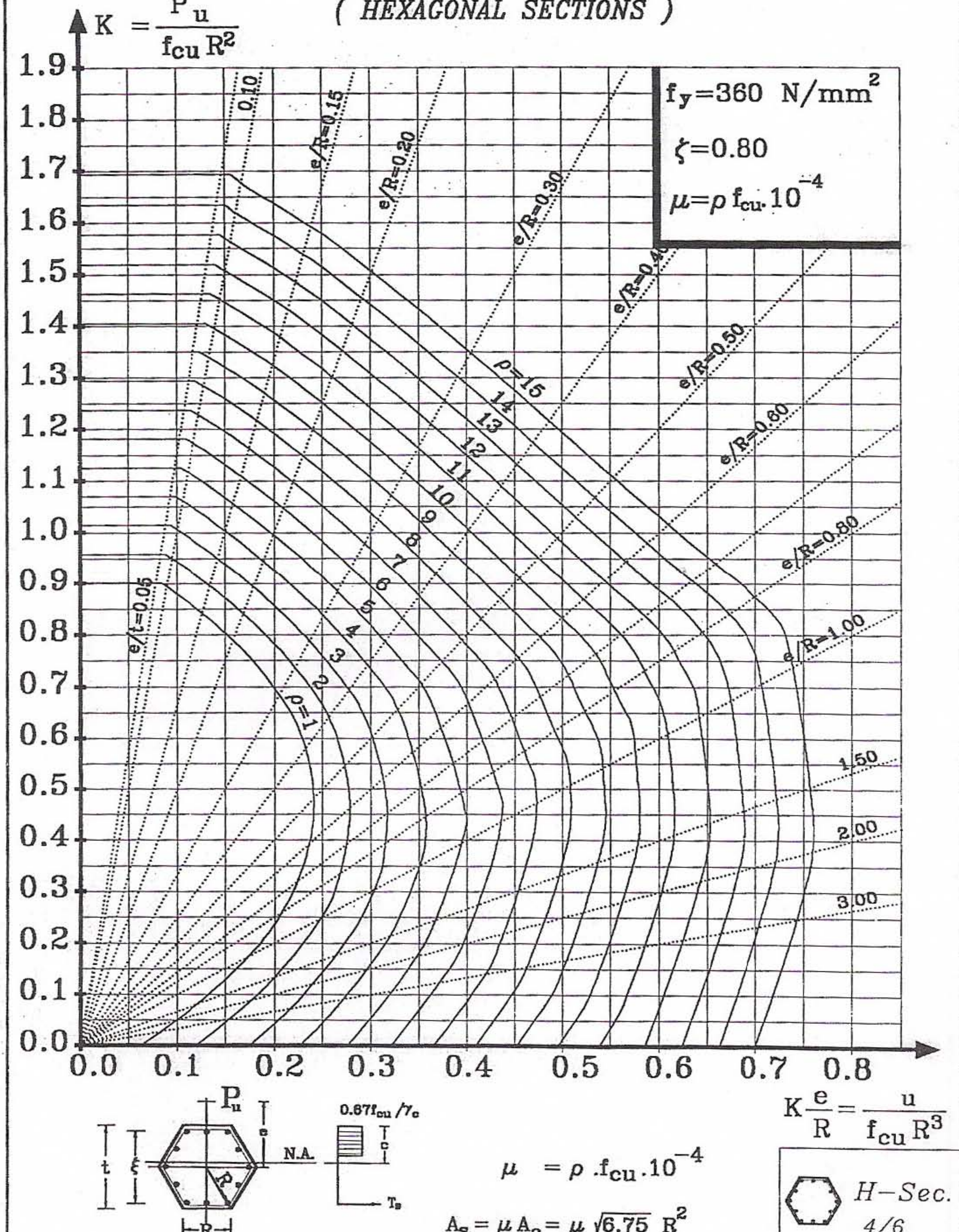
H–Sec. 2/6

U.L.D. CHARTS FOR SECTIONS SUBJECTED TO ECCENTRIC FORCES
(HEXAGONAL SECTIONS)

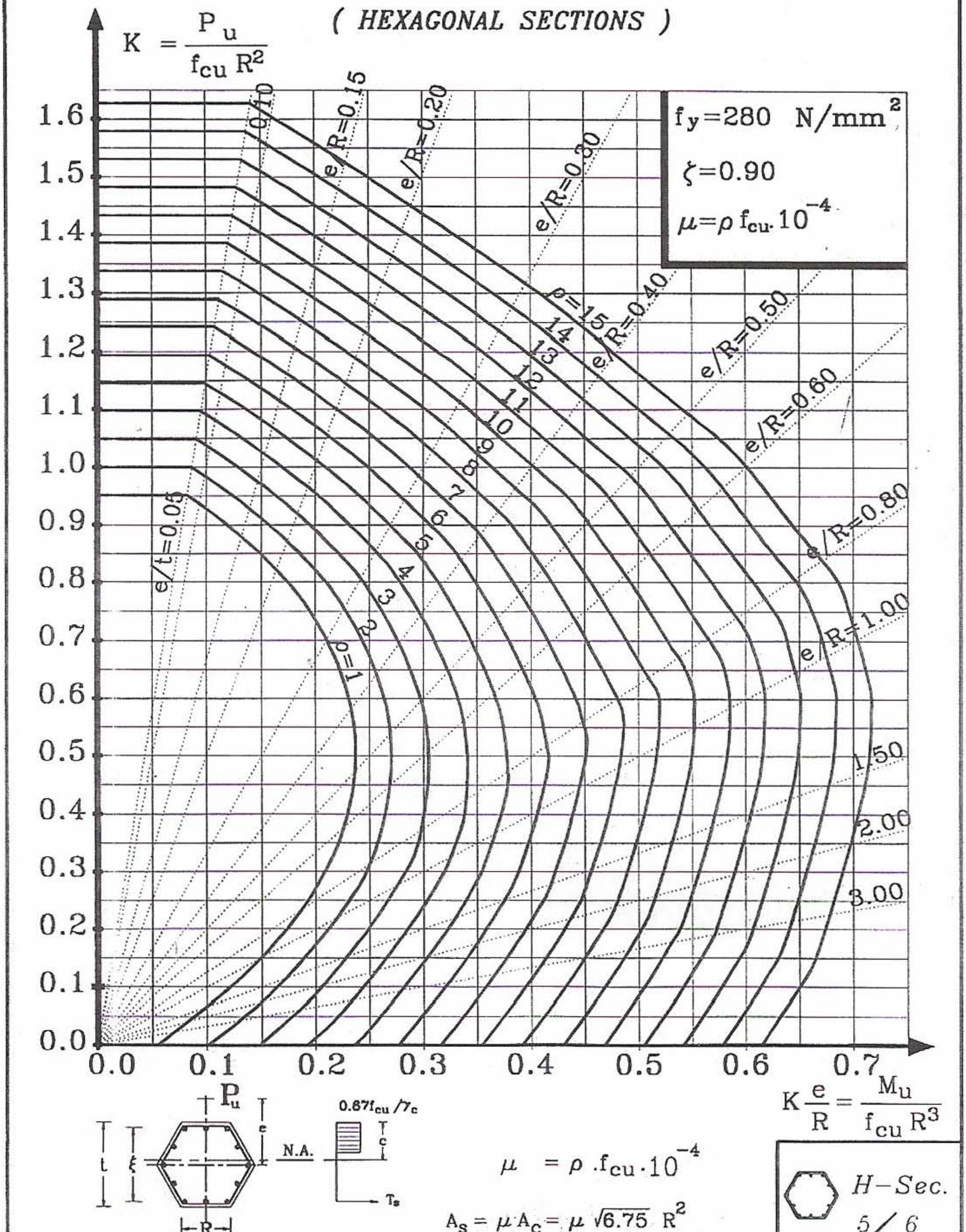


El−Behairy R.C.Design Handbook "Interaction Diagrams"

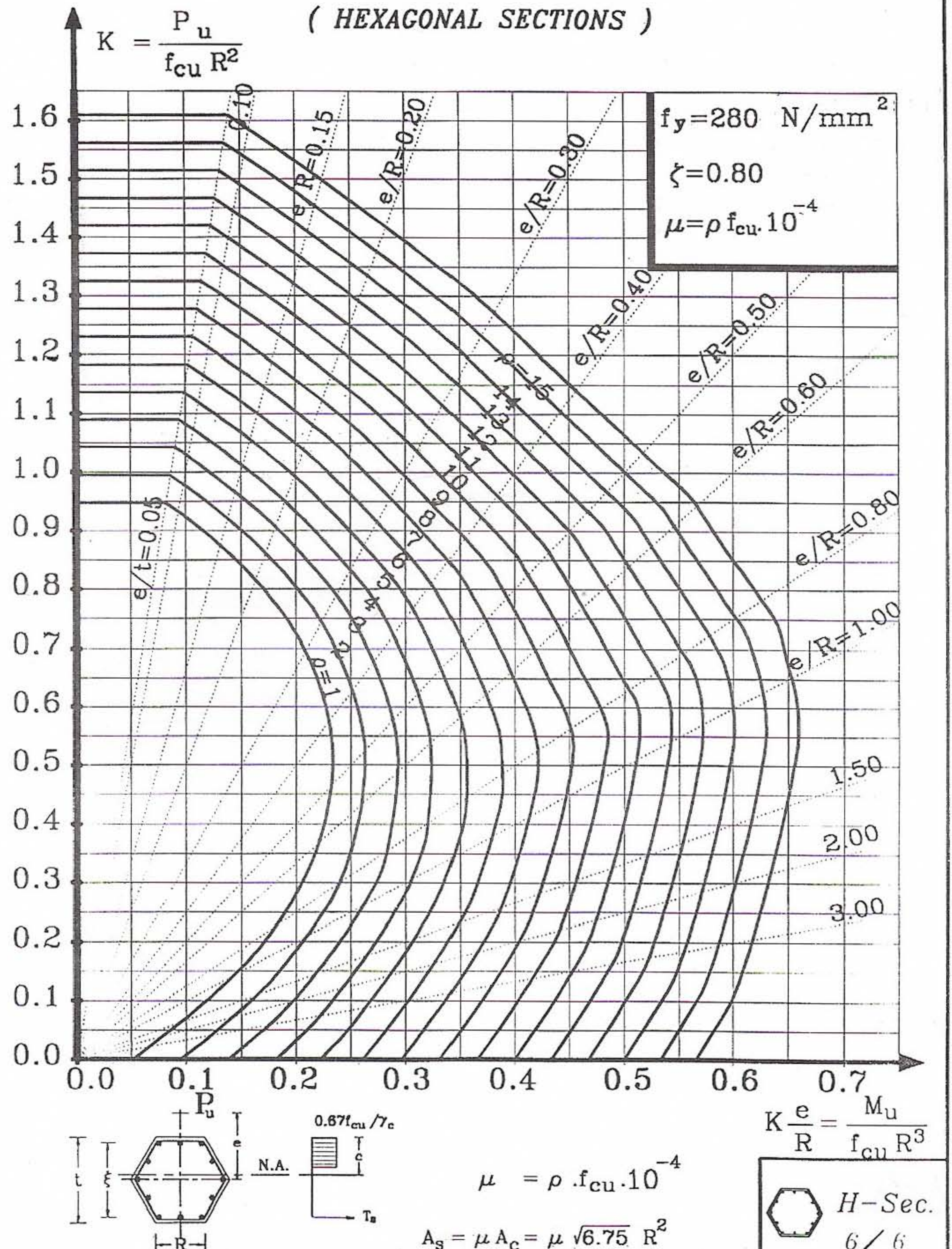
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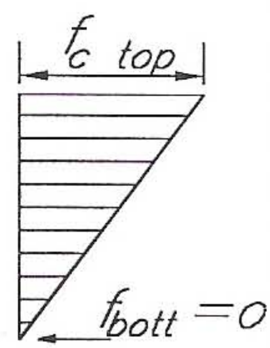


El−Behairy R.C.Design Handbook "Interaction Diagrams"

DESIGN AIDES
FOR
PRESTRESSED CONCRETE

Notes to Leonhardt`s Curves for dimensioning of Prestresses Concrete Sections :

K_1 - Curves : for obtaining the required depth :

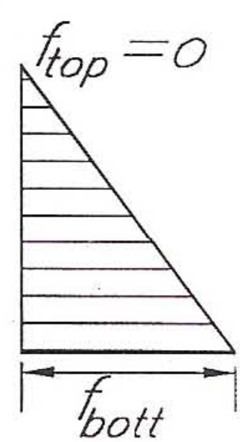


$$d = K_1 \sqrt{\frac{M_{max}}{b.\ f_c\ top}}$$

based on the assumbtion that f_{bottom} =o (Fullpre-stressing). f_{ctop} is the max compr. stress.

K_2- Curves : The Final Prestressing Force is :

$F_\infty = K_2 . f_{ctop} . b.d.$ The initial prestressing Force $Fo = F_\infty / \omega$ ($\omega = 0.95 \sim 0.8$)



K_3- Curves : for no tensile stresses in the compr.

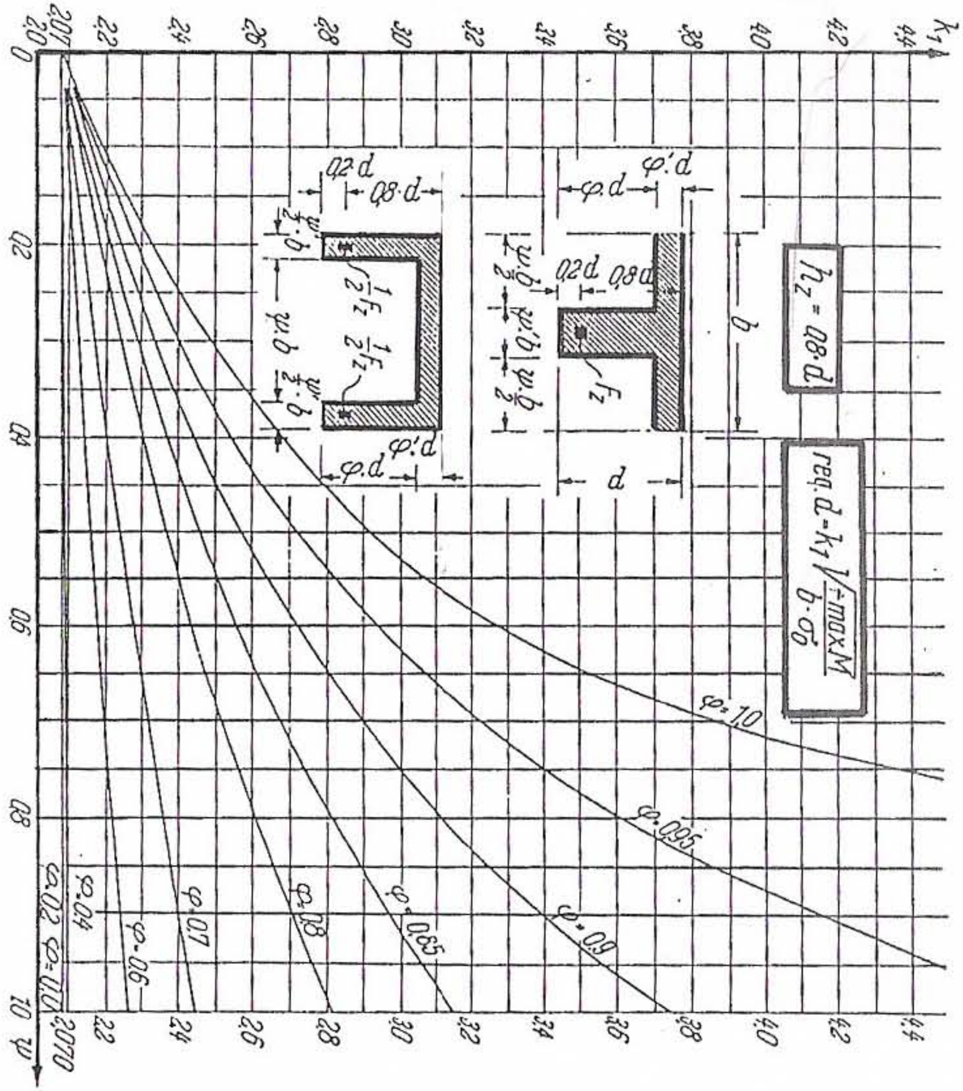
zone, the min value of the B.M. is :
M_g or min. $M_{g+p} = K_3 . F_o . d$

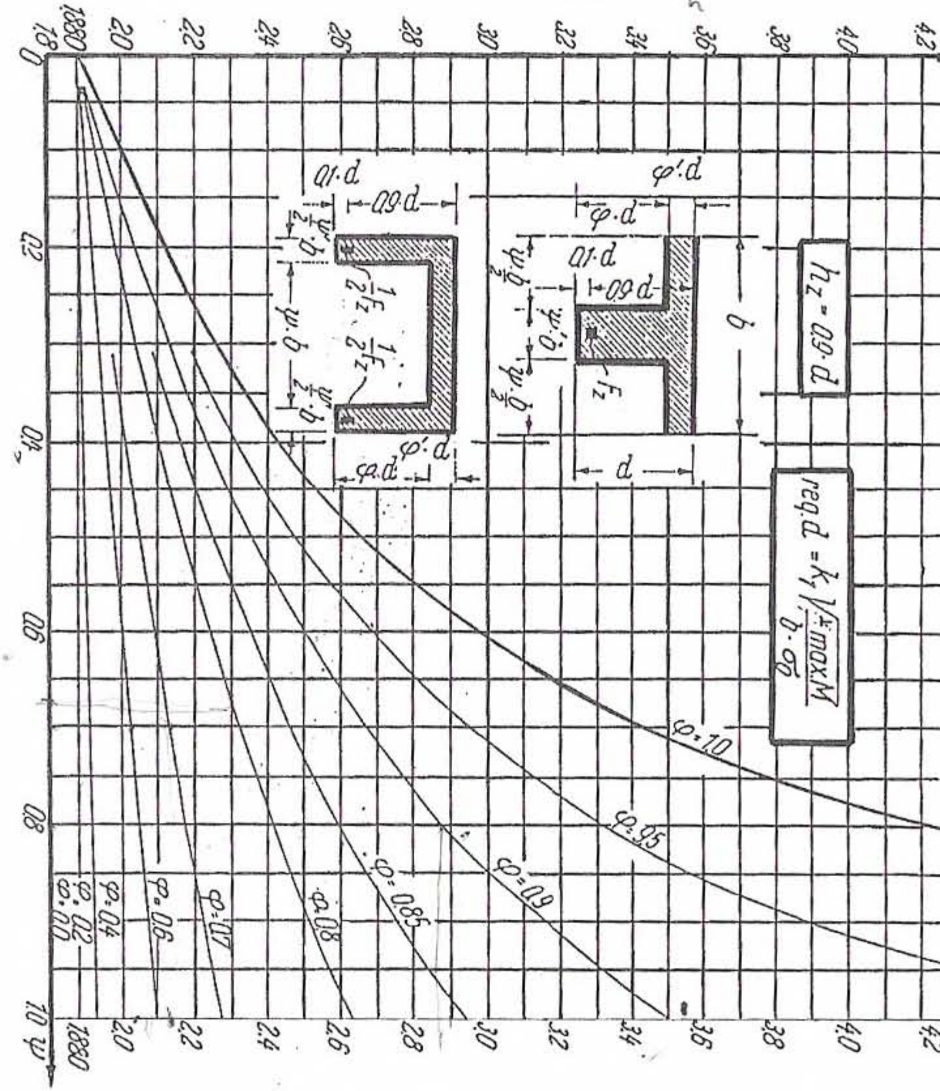
K_4- Curves : due to min M & Fo, the max. compr.

stresses in the tension zone : max. $f_{bott} = K_4 . Fo / b.d$ must not exceed the allowable value.

I. LEONHARDT 's Curves For Dimensioning of Prestressed Concrete T–Sections

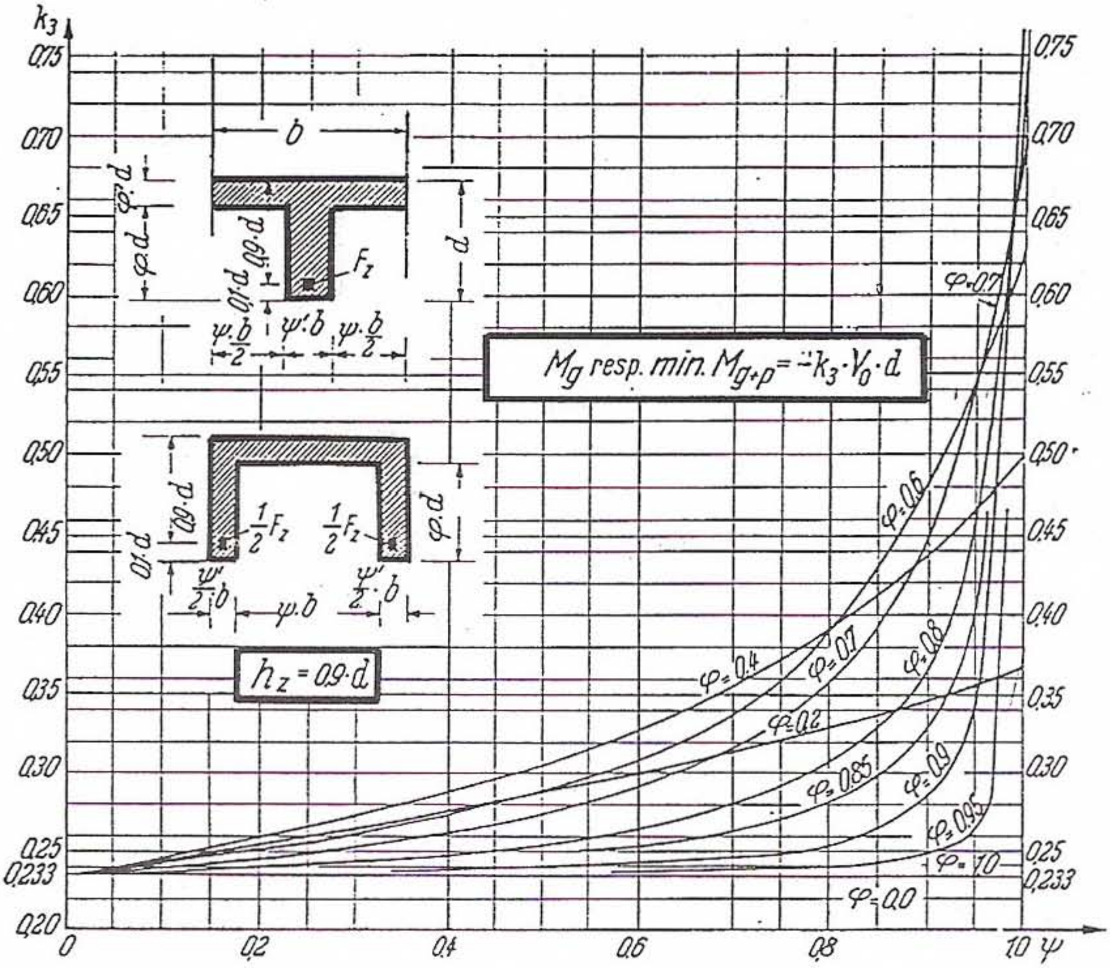
I.a. k_1 – Curves : The required depth $d = k_1 \sqrt{M_{max} / b . f_{top}}$ $\left(k_1 = 1 / \sqrt{\frac{1}{3}(1 - \varphi^3 . \psi) - \alpha\, k_2}\right)$

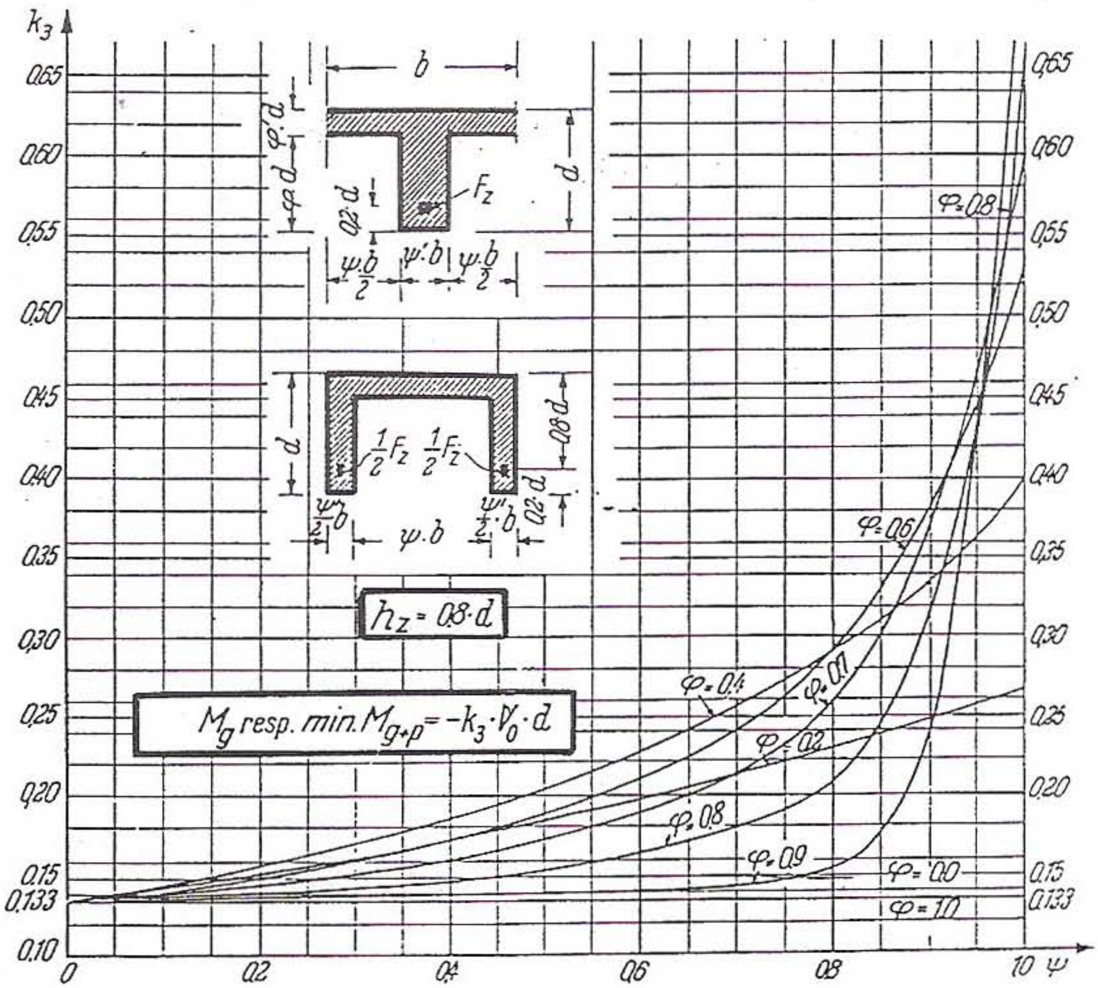




I. LEONHARDT 's Curves For Dimensioning of Prestressed Concrete T–Sections

I.b. k_3 – Curves : M_g or min $M_{g+p} = k_3 . F_o . d$ where $k_3 = 1 - \alpha - \frac{k_4}{3} (\varphi^3 \cdot \psi + \psi')$

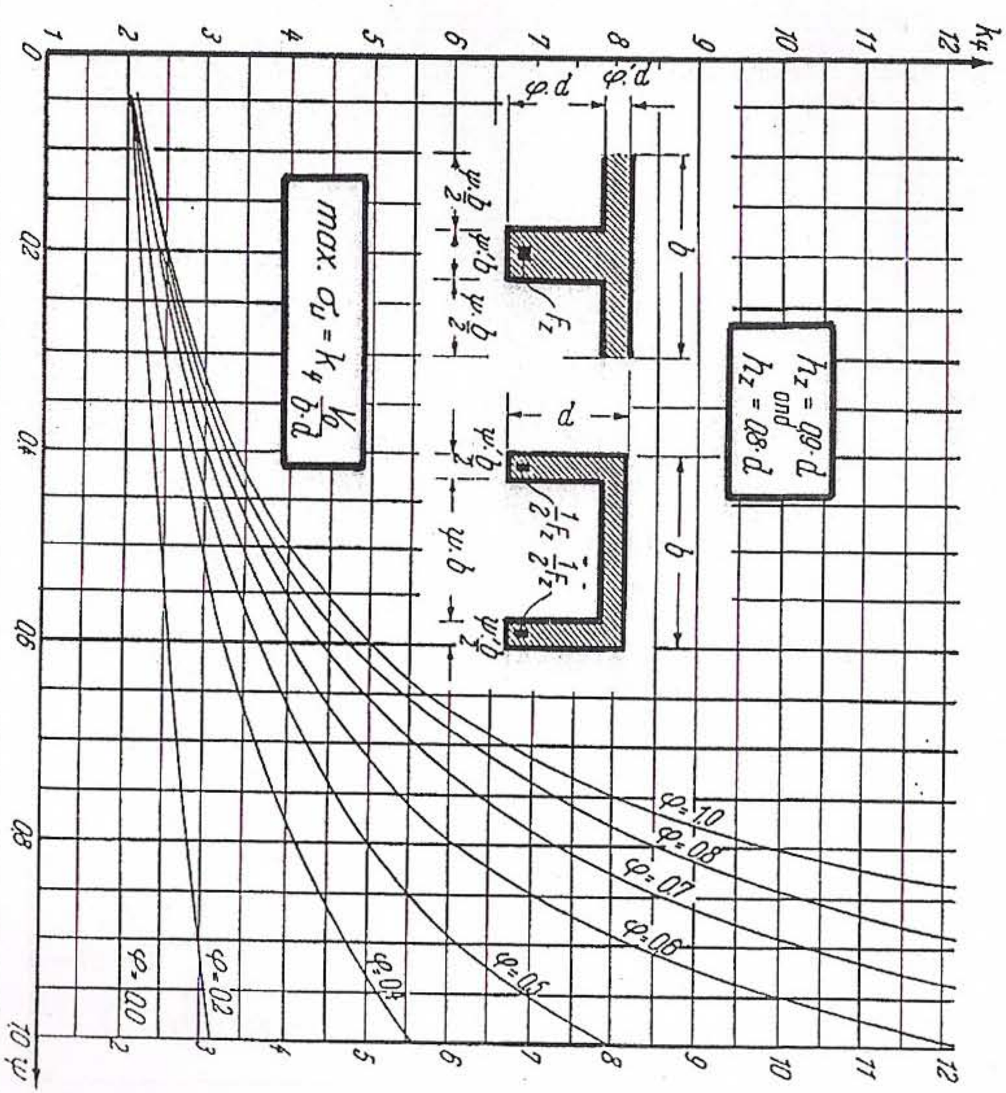




I. LEONHARDT 's Curves For Dimensioning of Prestressed Concrete T–Sections

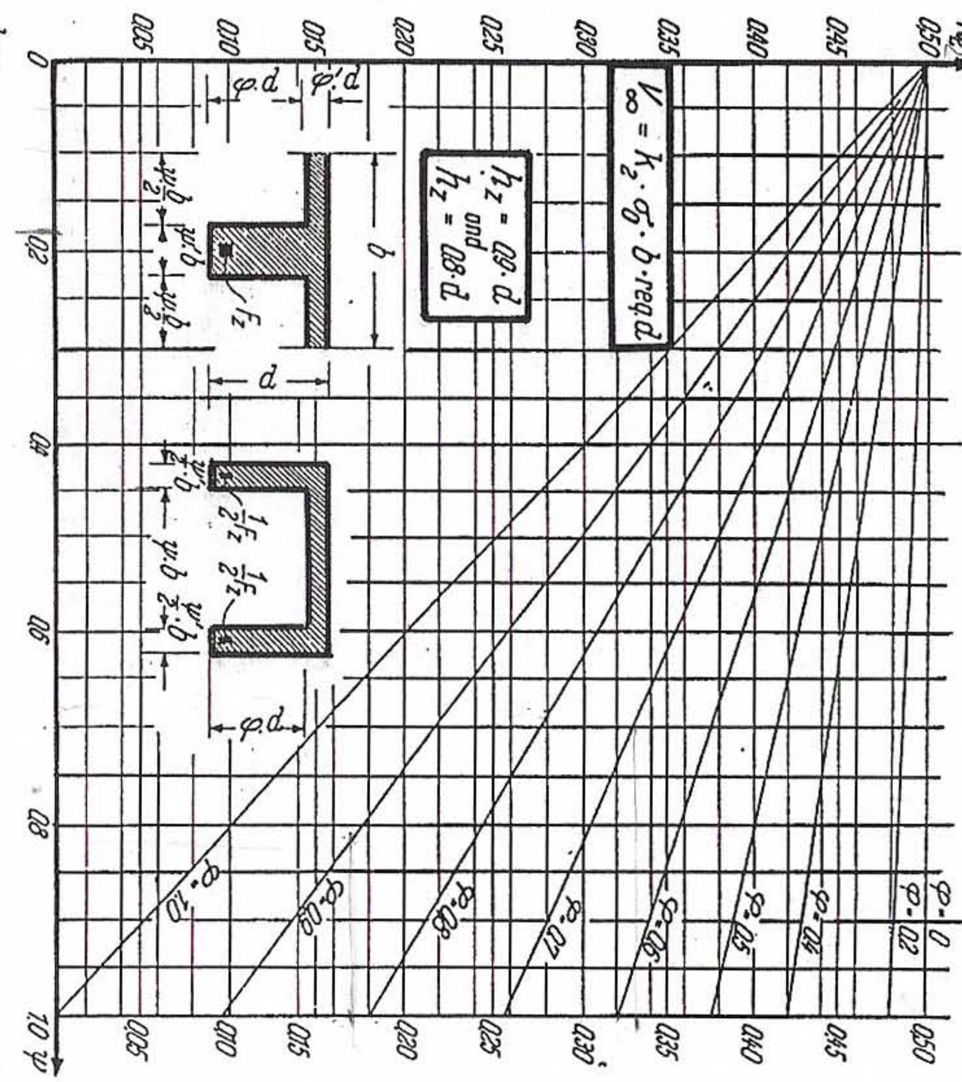
I.d. k_4 – Curves : Max $f_{top} = k_4 . F_o / b.d$

$(k_4 = 2/(\varphi'^2 . \psi + \psi'))$



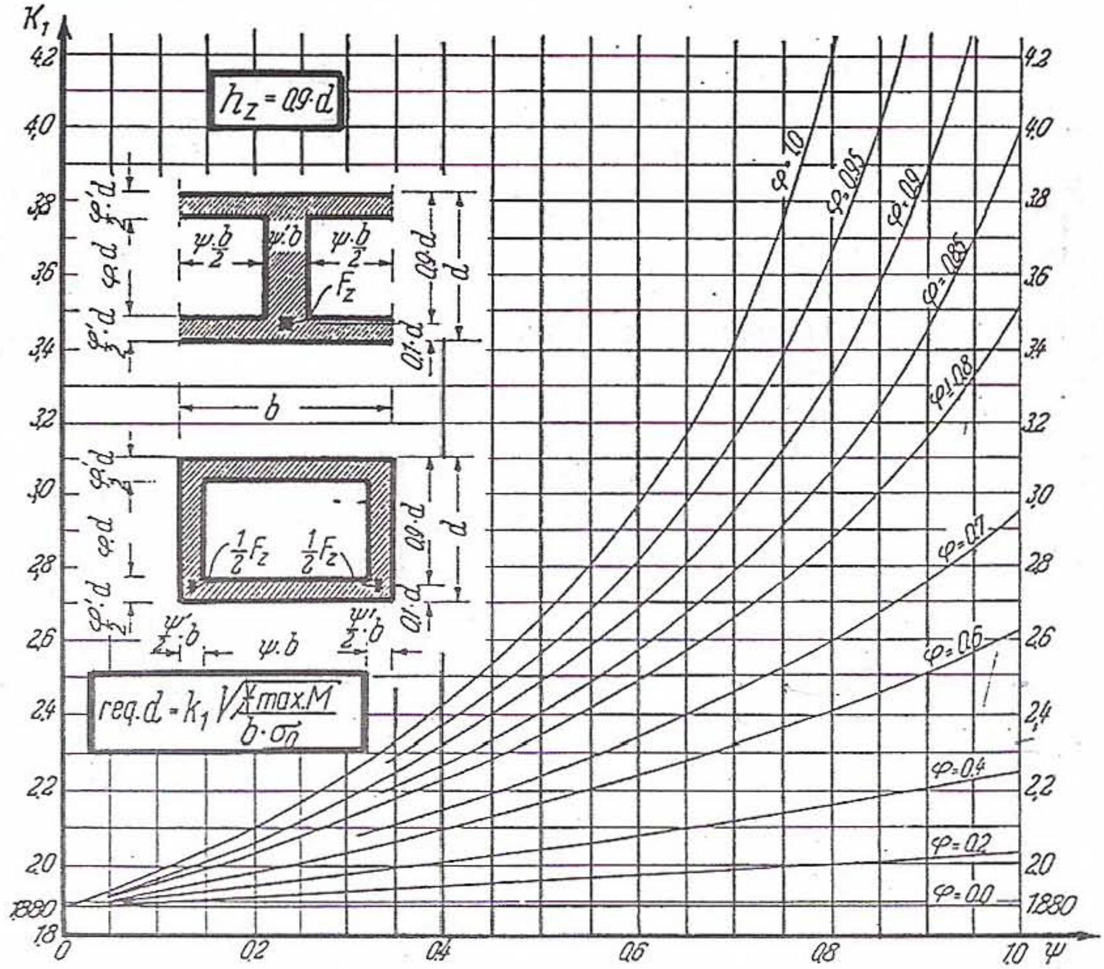
I.c. k_2 – Curves : $F_\infty = k_2 . bd . f_{top}$

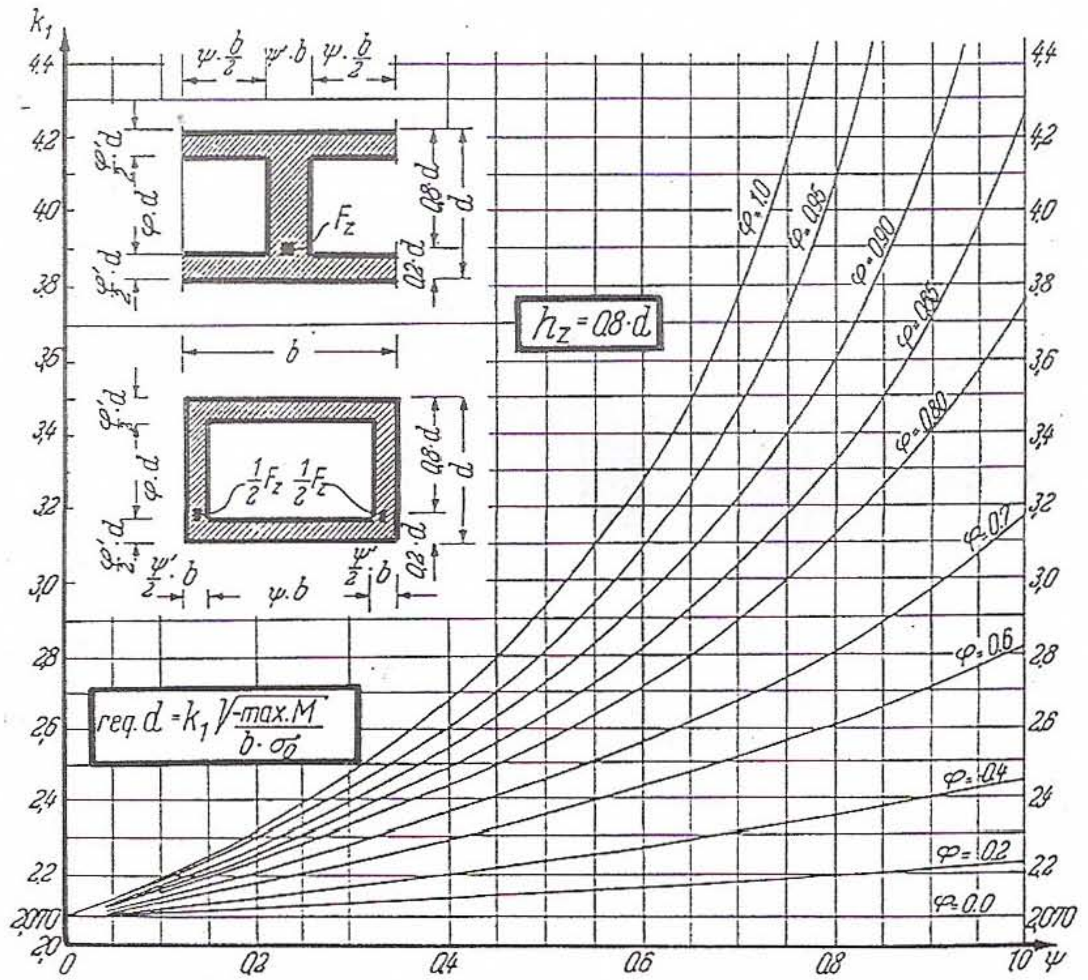
$(k_2 = (1 - \varphi^2 . \psi)/2)$



II. LEONHARDT 's Curves For Dimensioning of Prestressed Concrete Box & I–Sections

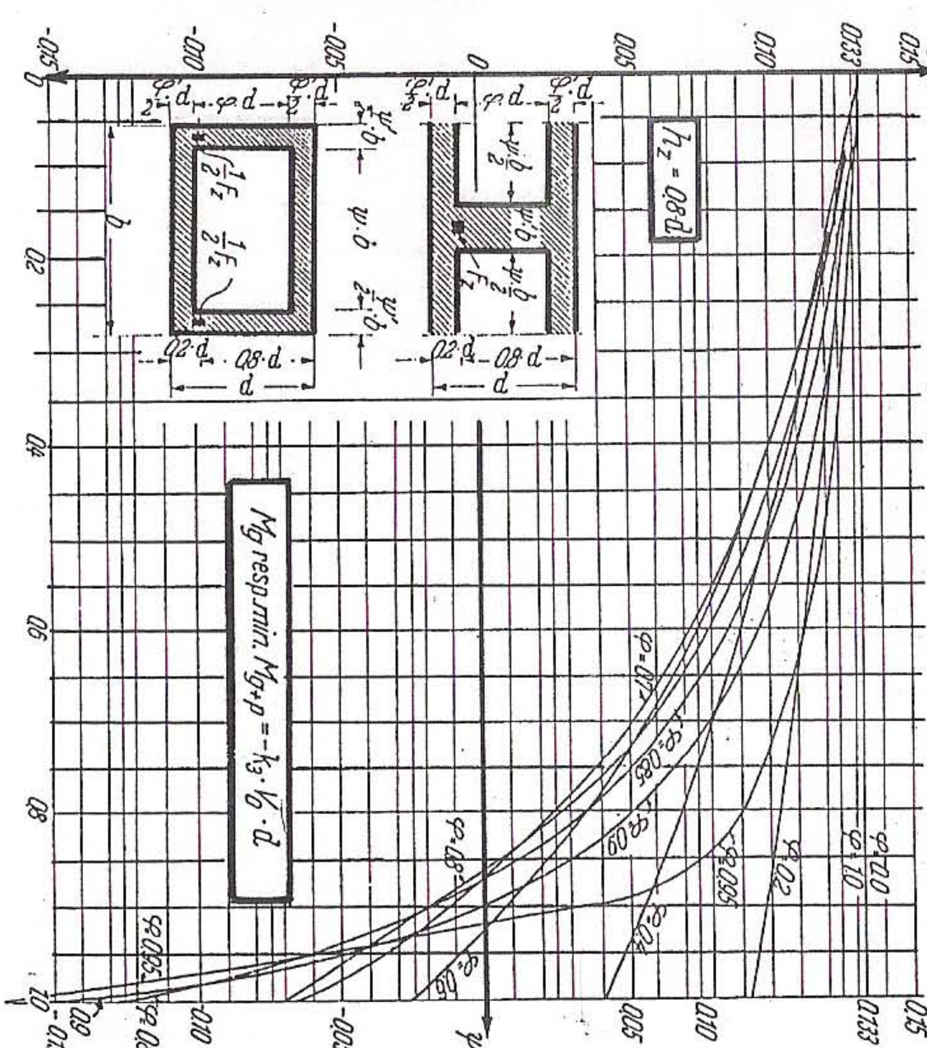
II.a. k_1 – Curves : The required depth $d = k_1\sqrt{M_{max} / bf_{top}}$ $\left(k_1 = 1/\sqrt{\frac{1}{12}(1-\varphi^3 \cdot \psi)+k_2(.5-a)}\right)$

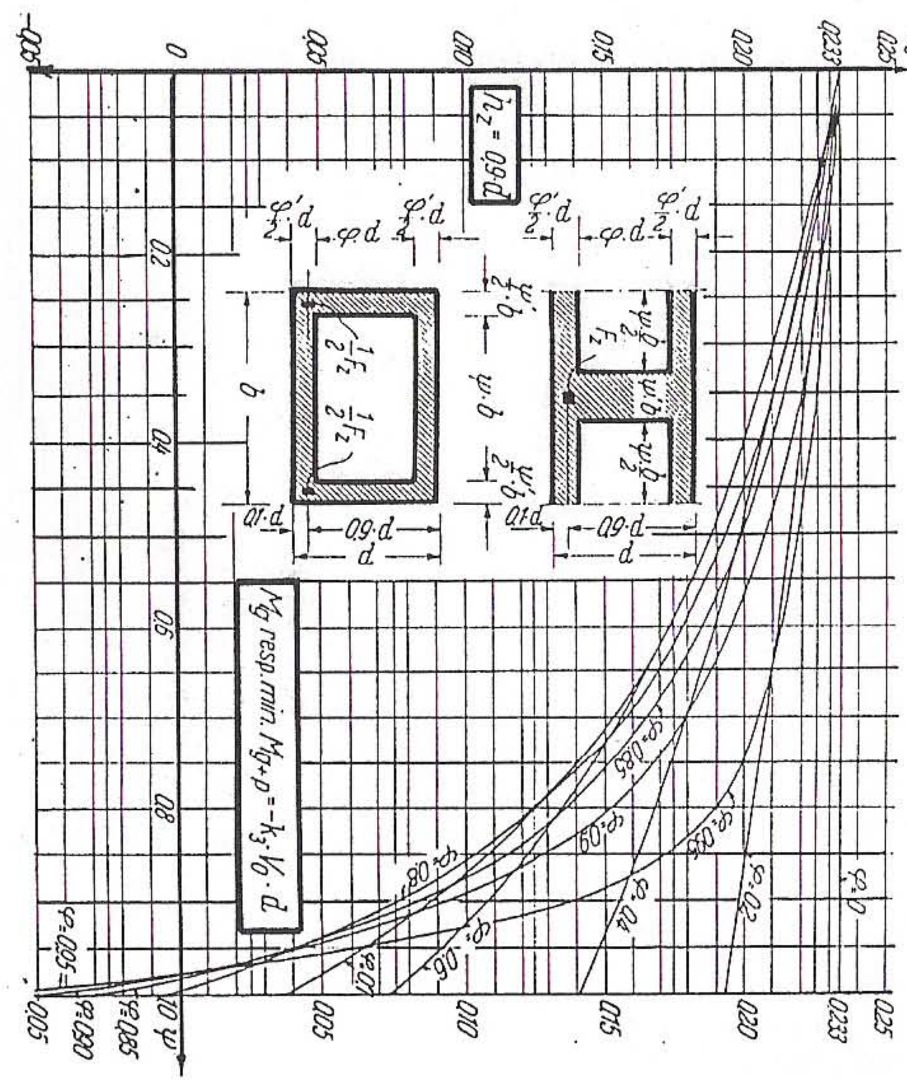




II. LEONHARDT 's Curves For Dimensioning of Prestressed Concrete Box & I–Sections

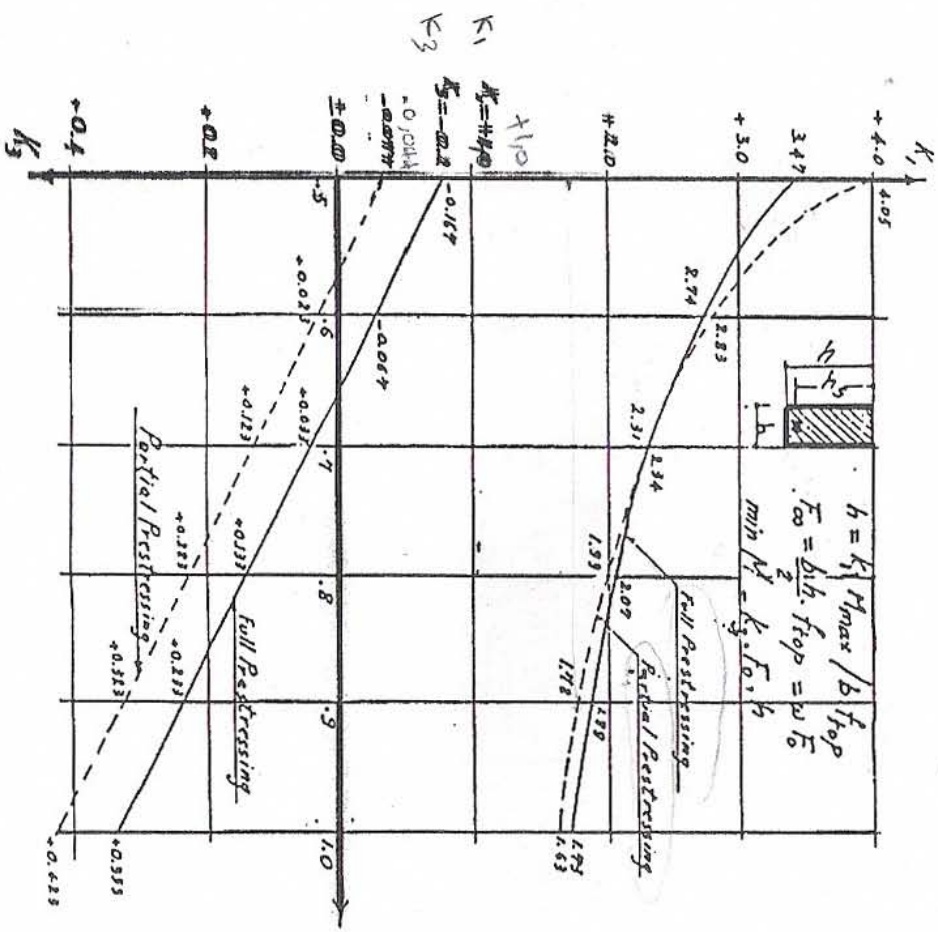
II.b. k_3 – Curves : M_g or min $M_{g+p} = k_3 \cdot F_0 \cdot d$, where $k_3 = 0.5 - \alpha - \frac{1-\varphi^3 \cdot \psi}{12\,k_2}$





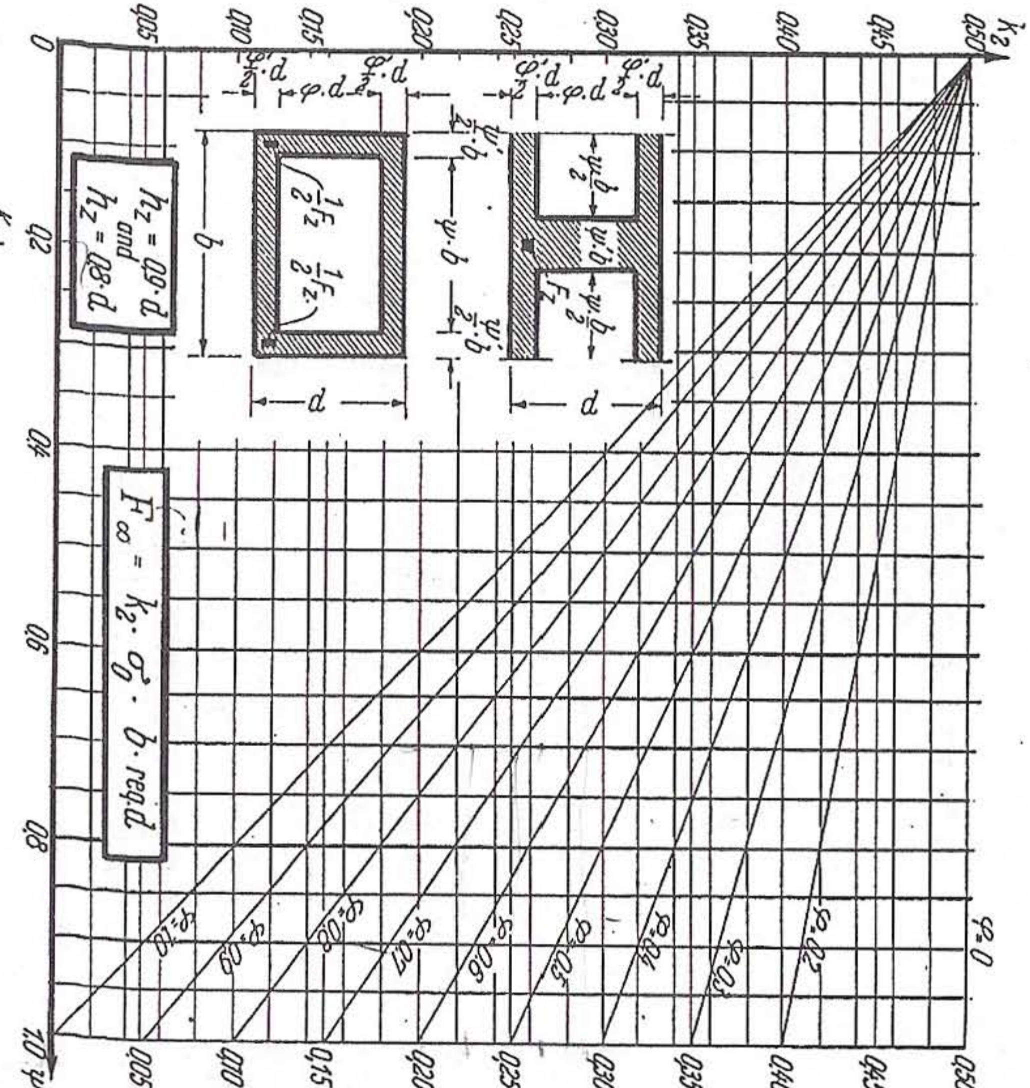
III. Dimensioning of Presressed Rectangular Sections

$d = k_1 \sqrt{M_{max} / b f_{top}}$, $F_\infty = \frac{1}{2}\, b d . f_{top} = \omega F_o$, $\min M = k_3 . F_o . d$



II. Dimensioning of Presressed Box & I–Sections

II.c. k_2 – Curves : $F_\infty = k_2\, b.d.f_{top}$, $k = \frac{1-\varphi\psi}{2}$



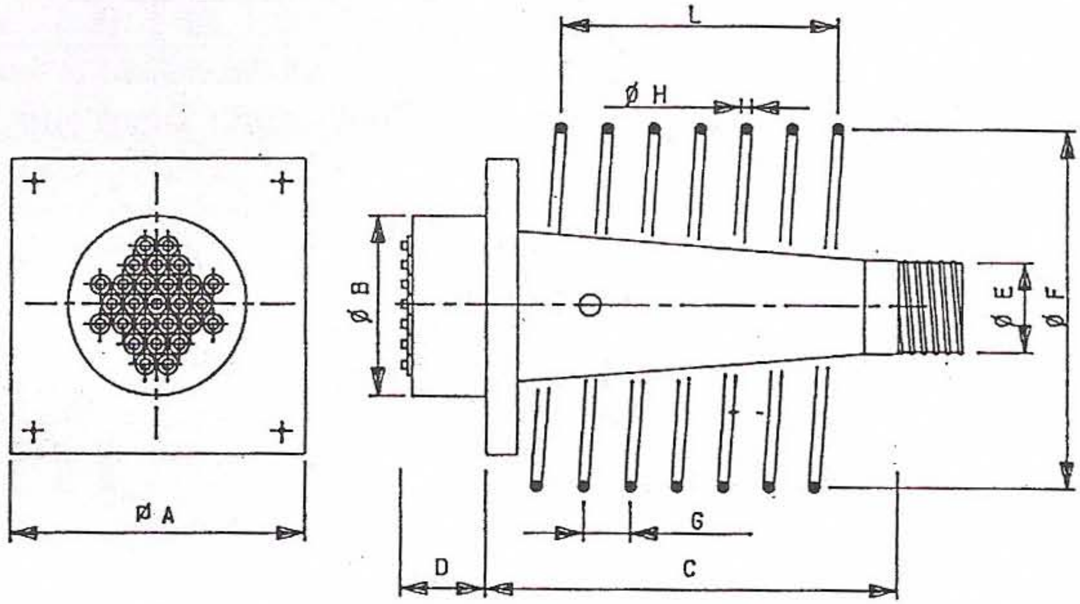
Properties of Prestressing Steel Strands (Freyssinet System)

GRADE	NOMINAL DIAMETER		NOMINAL AREA	NOMINAL WEIGHT	BREAKING STRENGTH	MIN. LOAD AT 1% EXSTENSION
	in``	mm	mm2	kg/m	kN	kN
250	0.5``	12.7	92.90	0.73	160.1	136.2
270	0.5``	12.7	98.71	0.775	183.7	156.1
250	0.6``	15.24	139.35	1.094	240.2	204.2
270	0.6``	15.24	140.00	1.102	260.7	221.5

Allowable Prestressing Force For Tendons (KN):-

	GRADE 250			GRADE 270		
UNIT TYPE	100%	80%	75%	100%	80%	75%
12ϕ13	1921	1537	1440	2204	1764	1653
19ϕ13	3042	2434	2281	3490	2792	2617
27ϕ13	4323	3458	3242	4960	3968	3720
7ϕ15	1681	1345	1260	1825	1460	1369
12ϕ15	2882	2305	2161	3128	2503	2346
19ϕ15	4563	3651	3422	4953	3962	3715

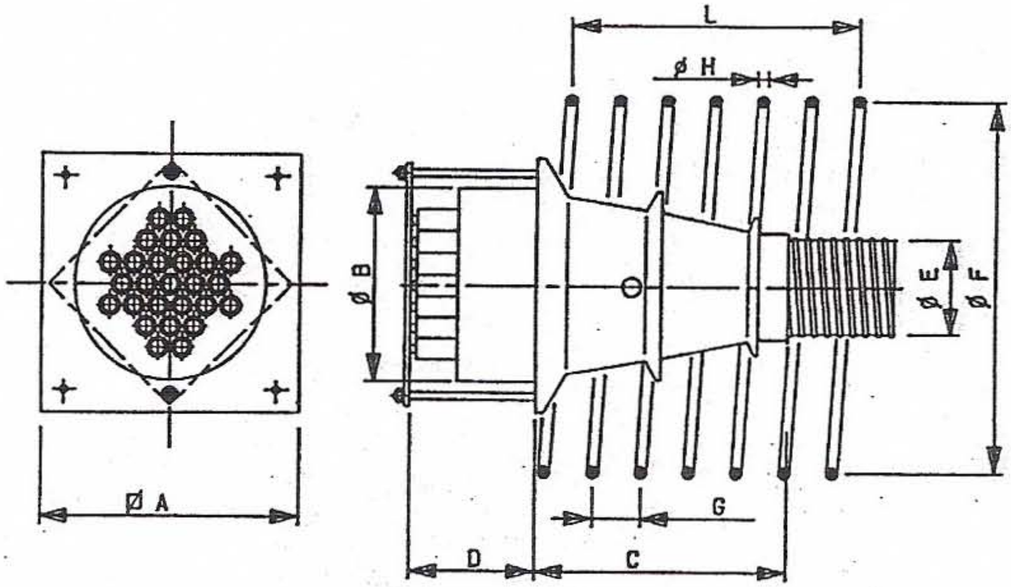
Properties of Stressing anchorages:- 1 - Live end. Type (A)



UNIT TYPE		A	B	C	D	E I.D./O.D.	F	G	H	L
12ϕ13	7ϕ15	240	150	300	90	65/71	250	50	14	300
19ϕ13	12ϕ15	300	180	450	105	80/86	330	50	14	300
27ϕ13	19ϕ15	360	230	550	120	100/106	390	60	16	360
31ϕ13	22ϕ15	390	230	550	130	110/116	430	60	18	360

All dimensions in mm

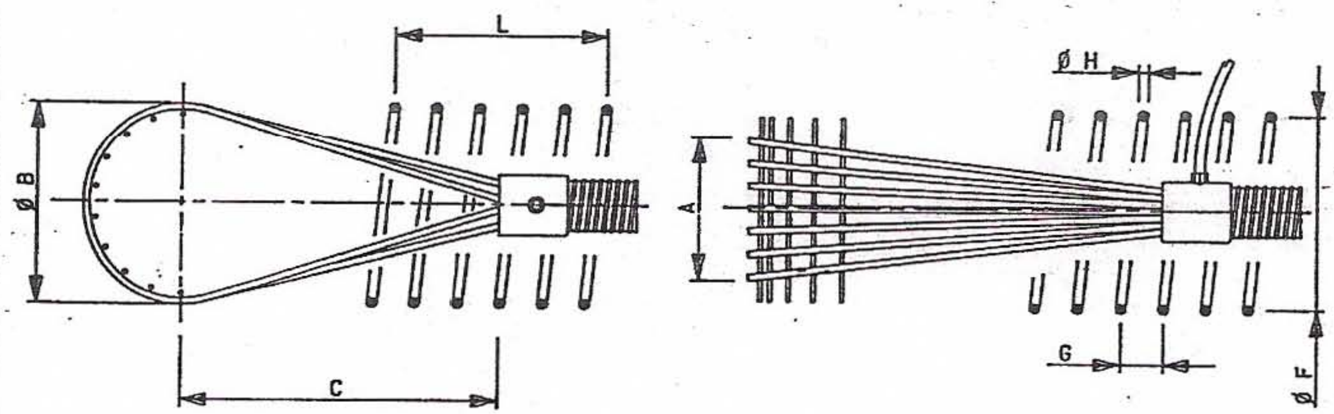
Properties of Stressing anchorages:- 1 - Live end. Type (B)



UNIT TYPE		A	B	C	D	E I.D./0.D.	F	G	H	L
12ϕ13	7ϕ15	195	150	190	145	65/71	250	50	14	300
19ϕ13	12ϕ15	245	180	250	160	80/86	330	50	14	300
27ϕ13	19ϕ15	290	230	310	175	100/106	390	60	16	360
31ϕ13	22ϕ15	310	230	340	185	110/116	430	60	18	360

All dimensions in mm

Properties of Stressing anchorages:- 2- Blind - end (Dead end)

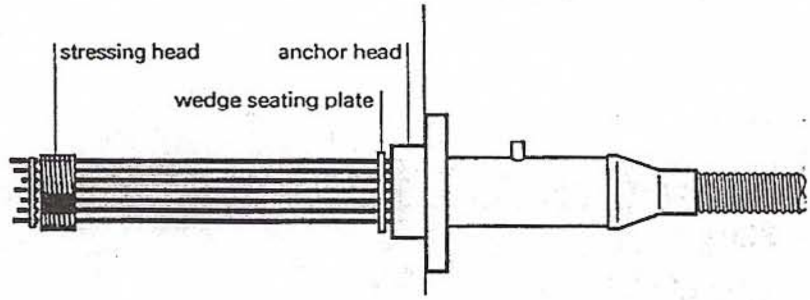


UNIT TYPE	A	B	C	F	G	H	L
12ϕ13	250	350	800	250	60	12	360
19ϕ13	420	350	800	300	60	14	420
7ϕ15	160	400	850	250	60	12	360
12ϕ15	250	400	850	300	60	14	420
19ϕ15	440	400	850	350	75	16	525

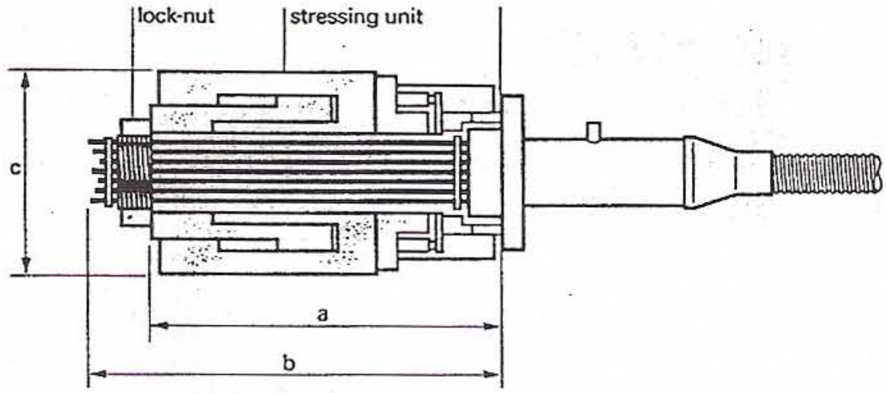
All dimensions in mm

1- Properties of prestressing Tendons:-

Strands installed in anchor and stressing head



Prestressing operation



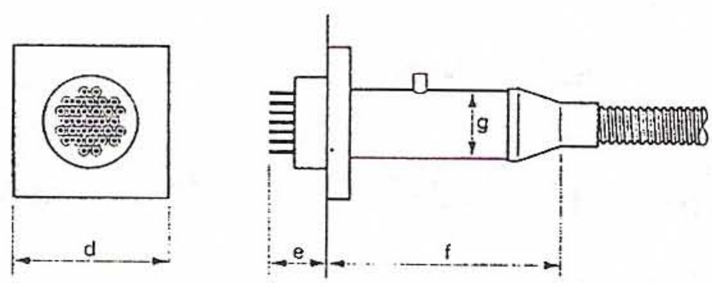
	Strand dia. 0.5``				Strand dia. 0.6``			
Number of strands	7	12	19	31	4	7	12	19
Area (cm2)	7.0	12.0	19.0	31.0	6.0	10.5	18.0	28.50
Weight of Strand bundle kg/m	5.5	9.42	14.92	24.34	4.71	8.4	14.13	22.37
Ultimate tensileforce (t)	130.9	224.4	355.3	579.7	112.2	196.4	336.6	533.0
Stressing force at 0.8 u.t.s.(t)	104.7	179.5	284.2	463.8	89.8	157.1	269.3	426.4

Area of SingleStrand 0.5`` (13mm) = 1.00 cm2 = 100mm2

Area of Single Strand 0.6`` (15mm) = 1.50 cm2 = 150mm2

Guaranteed Ultimate Tensile Strength = 1870 N/mm2 = 18.70 t/cm2

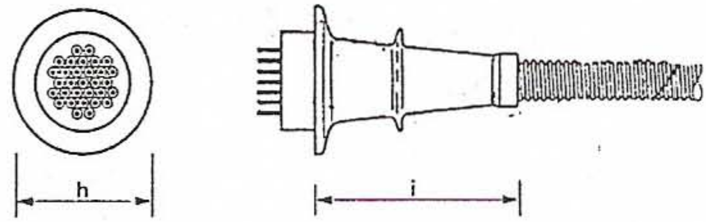
1- Properties of .Stressing Anchorage:-



		Strand dia. 0.5``				Strand dia. 0.6``			
number of strands		7	12	19	31	4	7	12	19
anchorage	typeM	705	1205	1905	3105	406	706	1206	1906
plate size	d 1)	200	265	330	425	180	240	315	395
plate size	d 2)	175	220	270	345	175	220	270	345
anchor length	e	80	90	100	110	90	90	100	110
trumpetlength	f	230	400	550	750	230	300	560	720
trumpet dia	g	79	112	142	182	74	96	138	174

All dimensions in mm

Properties of Bearing Plate castings:-

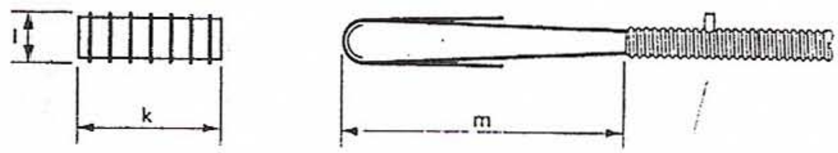


		strand dia. 0.5``					strand dia. 0.6``				
number of strands		7	12	19	31	42	4	7	12	19	31
Plate diameter	h	175	230	285	360	420	175	220	275	345	440
trumpet length	i	250	400	475	550	700	250	300	400	500	650

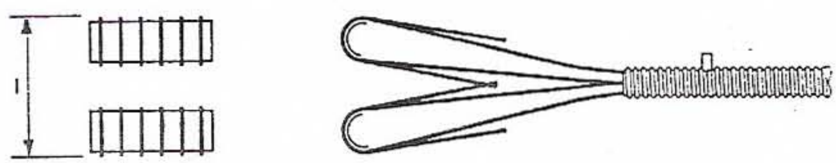
All dimensions in mm

Properties of Fixed anchorages:- Fixed Loop anchorages

Anchorage with up to 7 strands



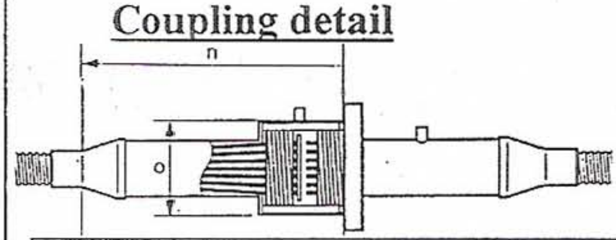
Anchorage with 8 to 19 strands

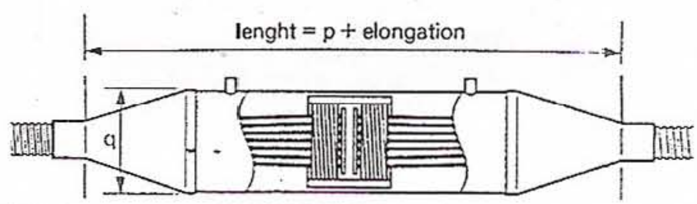


		strand dia. 0.5``			strand dia. 0.6``		
number of strands		7	12	19	4	7	12
anchorage	type E	705	1205	1905	406	706	1206
side length	K	350	300	500	240	420	360
side length	I	125	330	330	155	155	400
anchor length	m	700	700	900	900	900	900

All dimensions in mm

Coupling detail





			strand dia. 0.5``			strand dia. 0.6``			
number of strands		7	12	19	31	4	7	12	19
fixed coupling	type K	705	1205	1905	3105	406	706	1206	1906
length	n	390	570	750	930	410	480	710	900
diameter	o	155	200	245	305	155	195	245	300
movable coupling	type KC	705	1205	1905	3105	406	706	1206	1906
length=p+elongation	p	610	960	1290	1620	630	770	1210	1560
Diameter	q	155	200	245	305	155	195	245	300

All dimensions in mm

Mechanical properties of stress-relieved7 wires strands:

Nominal diameter (mm)	Nominal area (mm^2)	Tensile strength f_{pu} (N/mm^2)		Yeild stress f_{py} (N/mm^2)	
		Grade 270	Grade 250	Grade 270	Grade 250
23.22	6.35	---	40.00	---	34.00
37.42	7.94	---	64.50	---	54.70
51.61	9.53	102.3	89.00	87.0	75.60
69.68	11.11	137.9	120.1	117.2	102.3
92.90	12.70	183.7	160.1	156.1	136.2
139.35	15.24	260.7	240.2	221.5	204.2

* % of relaxation is determined after 1000 hour in case of low relaxation wires or strands.

** Yield stress of low relaxation wires and strands shall not be less than 90% of tensile strength.

Mechanical properties for strands in British standards:

Strand type	Nominal diameter (mm)	Nominal area (mm^2)	Tensile strength (KN)	Proof strength 0.10 % (KN)	standard
7 Standard strand	9.3	52.3	92	78.00	BS 5896
	11.0	71.0	125	106.00	
	12.5	94.2	164	139.00	
	15.2	138.2	232	197.00	
Super 7 – wire strands	8.0	38.0	70	59.00	BS 5896
	9.6	55.0	102	89.00	
	11.3	75.0	139	118.00	
	12.9	100.0	186	158.00	
Drawn 7 –wires strand	15.7	150.0	265	225.00	BS 5896
	12.7	112.0	209	179.00	
	15.2	165.0	300	255.00	
	18.0	223.0	380	323.00	
19 wires strand	18.0	210.0	370	319.50	BS 4757
	25.4	423.0	659	560.15	
	28.6	535.0	823	699.55	
	31.8	660.0	979	832.15	

Appendix

Conversion Factors
From Metric System to System International (SI)

Metric system		SI
Kg (Force)	=	9.81 N
Kg (Force).m	=	9.81 N. m
Kg (Force) / m	=	9.81 N / m
Kg (Force) / cm^2	=	0.0981 N / mm^2
Kg (Force) / m^2	=	9.81 N / m^2
Kg (Force) / cm^3	=	9.81 N / m^3
0.102 Kg (Force)	=	1.00 N
0.102 Kg (Force).m	=	1.0 N. m
0.102 Kg (Force) / m	=	1.0 N / m
0.102 Kg (Force) / cm^2	=	1.0 N / mm^2
0.102 Kg (Force) / m^2	=	1.0 N / m^2
0.102 Kg (Force) / m^3	=	1.0 N / m^3

Note:
For simplicity, it is considered that:

1.0 Kg (Force) = 10 N

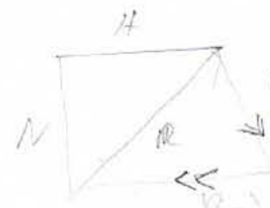
Kg/cm^2	N/mm^2	Eqn No
$E_c = 14000\sqrt{f_{cu}}$	$E_c = 4400\sqrt{f_{cu}}$	2-1
$\mu_{min} = A_s / bd = 11/f_y$	$\mu_{min} = A_s / bd = 1.1/f_y$	4-9
$q_{u\,max} = 2.2\sqrt{f_{cu}/\gamma_c}$	$q_{u\,max} = 0.7\sqrt{f_{cu}/\gamma_c}$	4-15
$q_{tu} < 0.19\sqrt{f_{cu}/\gamma_c}$	$q_{tu} < 0.06\sqrt{f_{cu}/\gamma_c}$	4-17
$q_{cu} = 0.75\sqrt{f_{cu}/\gamma_c}$	$q_{cu} = 0.24\sqrt{f_{cu}/\gamma_c}$	4-18
$\delta_c = 1 + 0.007(P_u / A_c)$	$\delta_c = 1 + 0.07(P_u / A_c)$	4-19
$\delta_t = 1 - 0.02(T_u / A_c)$	$\delta_t = 1 - 0.2(T_u / A_c)$	4-20
$q_{sub} \le 0.75\sqrt{f_{cu}/\gamma_c}$	$q_{sub} \le 0.24\sqrt{f_{cu}/\gamma_c}$	4-27
$\mu_{min} = \frac{A_s}{bS} = \frac{4}{(f_y)}$	$\mu_{min} = \frac{A_s}{bS} = \frac{0.4}{(f_y)}$	4-28
$\mu_{min} = A_{st\,min}/(b.s) = (4/(f_y)(q_u/q_{cu})$	$\mu_{min} = A_{st\,min}/(b.s) = (0.4/(f_y)(q_u/q_{cu})$	4-29
$q_{cu} = 0.5\sqrt{f_{cu}/\gamma_c} \ge q_u$	$q_{cu} = 0.16\sqrt{f_{cu}/\gamma_c} \ge q_u$	4-30
$q_{cup} = 2.5[\frac{\alpha.d}{b_o} + 0.2]\sqrt{f_{cu}/\gamma_c}$	$q_{cup} = 0.8[\frac{\alpha.d}{b_o} + 0.2]\sqrt{f_{cu}/\gamma_c}$	4-32-a
$q_{cup} = [0.5 + (a/b)]\sqrt{f_{cu}/\gamma_c}$	$q_{cup} = 0.316[0.5 + (a/b)]\sqrt{f_{cu}/\gamma_c}$	4-32-b
$q_{cup} \le \sqrt{f_{cu}/\gamma_c}$	$q_{cup} \le 0.316\sqrt{f_{cu}/\gamma_c}$	4-33

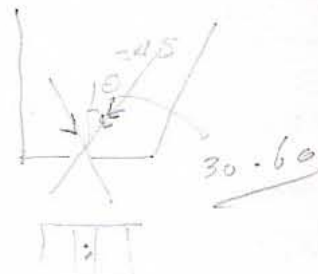
Kg/cm^2	N/mm^2	Eqn No
$q_{cup} \le 1.45\sqrt{f_{cu}/\gamma_c}$	$q_{cup} \le 0.46\sqrt{f_{cu}/\gamma_c}$	4-44
$q_{tu(max)} = 2.2\delta_{ti}\sqrt{f_{cu}/\gamma_c}$	$q_{tu(max)} = 0.7\delta_{ti}\sqrt{f_{cu}/\gamma_c}$	4-48-a
$q_{u\,(max)} = 2.2\delta_{si}\sqrt{f_{cu}/\gamma_c}$	$q_{u\,(max)} = 0.7\,\delta_{si}\sqrt{f_{cu}/\gamma_c}$	4-48-b
$(2A_{str} + A_{st}) \ge 3.5\, S.b/(fyst/\gamma_s)$	$(2A_{str} + A_{st}) \ge 0.35\, S.b/(fyst/\gamma_s)$	4-52
$A_{sl\ min} = \frac{1.45\sqrt{(f_{cu}/\gamma_c)}\, A_{cp}}{f_y} - (\frac{A_{str}}{S})\, P_h\, (\frac{f_{yst}}{f_y})$	$A_{sl\ min} = \frac{0.46\sqrt{(f_{cu}/\gamma_c)}\, A_{cp}}{f_y} - (\frac{A_{str}}{S})\, P_h\, (\frac{f_{yst}}{f_y})$	4-53-b
$M_{tu} = \left(A^2 cp/P_{cp}\right)\sqrt{f_{cu}/\gamma_c}$	$M_{tu} = 0.316\left(A^2 cp/P_{cp}\right)\sqrt{f_{cu}/\gamma_c}$	4-54
$f_{bu} = 0.95\sqrt{f_{cu}/\gamma_c}$	$f_{bu} = 0.30\sqrt{f_{cu}/\gamma_c}$	4-57
$t = \frac{L_n\left(0.8 + f_y/15000\right)}{36 + 9\beta}$	$t = \frac{L_n\left(0.8 + f_y/1500\right)}{36 + 9\beta}$	4-61
$f_{ctr} = 1.90\sqrt{f_{cu}}$	$f_{ctr} = 0.6\sqrt{f_{cu}}$	4-64
$f_{ps} = f_{pe} + 700 + (f_{cu}/125\,\mu_p)$	$f_{ps} = f_{pe} + 70 + (f_{cu}/125\,\mu_p)$	10-9
$f_{ps} = f_{pe} + 700 + (f_{cu}/375\,\mu_p)$	$f_{ps} = f_{pe} + 70 + (f_{cu}/375\,\mu_p)$	10-10
$q_{ci} = 0.142\sqrt{f_{cu}} + q_d + q_i\, M_{cr}/M_{max}$	$q_{ci} = 0.045\sqrt{f_{cu}} + q_d + q_i\, M_{cr}/M_{max}$	10-17
$M_{cr} = (I/y_t)\,(1.90\sqrt{f_{cu}} + f_{pce} - f_{cd})$	$M_{cr} = (I/y_t)\,(0.60\sqrt{f_{cu}} + f_{pce} - f_{cd})$	10-18
$q_{cw} = 0.85\sqrt{f_{cu}} + 0.30\, f_{pcc} + q_{pv}$	$q_{cw} = 0.27\sqrt{f_{cu}} + 0.30\, f_{pcc} + q_{pv}$	10-19
$q_{tu} = 0.19\sqrt{f_{cu}/\gamma_c}\sqrt{1 + f_{pcc}/(0.3\sqrt{f_{cu}})}$	$q_{tu} = 0.06\sqrt{f_{cu}/\gamma_c}\sqrt{1 + f_{pcc}/(0.3\sqrt{f_{cu}})}$	10-21

Kg/cm^2	N/mm^2	Eqn No
$A_{sLmin}=(1.45\sqrt{f_{cu}/\gamma_c}\,A_{cp}/f_y)-(A_{str}/s)P_h(f_{yst}/f_y)$	$A_{sLmin}=(0.46\sqrt{f_{cu}/\gamma_c}\,A_{cp}/f_y)-(A_{str}/s)P_h(f_{yst}/f_y)$	10-25
$M_{tu}=(A^2{}_{cp}/P_{cp})\sqrt{f_{cu}/\gamma_c}\sqrt{1+f_{pcc}/(0.95\sqrt{f_{cu}})}$ kg/cm² $M_{tu}=0.316(A^2{}_{cp}/P_{cp})\sqrt{f_{cu}/\gamma_c}\sqrt{1+f_{pcc}/(0.3\sqrt{f_{cu}})}$ N/mm²		10-26

Design of cross bar support

$A_{s1} = \frac{R_1'}{f_s'}$, $A_{s2} = \frac{R_2'}{f_s'}$





$f_s' = 700 \rightarrow 24/35$

$= 1000 \rightarrow 36/52$

$d = \frac{N/2 \tan\theta + H.a/y_{ct}}{0.005\, a.b + A_{st}}$

$a = 8\ \emptyset$, A_s ...

$y_{ct} = 0.87 \times (d)$

spiral

$V_{sp} = \frac{\pi \times A_{st} \times D_k}{s} = \frac{\pi \times 0.503 \times 15}{8} = 2.945$

$N = f_{co} \times A_k + 0.44 f_y A_{sc} + 0.92 f_{ysp} V_{sp}$

$\frac{\pi\, 15^2}{4} + 0.44 \times 3600 . A_{sc} + 0.92 \times 2400 \times 2.945$

short cantilever

$d = k_1 \sqrt{\frac{M}{b}}$, $A_s = \frac{R.a}{f_s . d'}$, $d' = 0.87 \times d$

$A_s' = \frac{A_s}{4}$

30 → 60 cross bar
60 → 90 spiral
90 → ∞ lead

Hinges: $N = ✓$, $M = 0.0$; $N = F_{co}\, b.t\, H$

$N = f_{co}\, b\, t (1 + h\mu)$ at $M = 0.0$